

- Q.1.** Using the nodal analysis technique, derive (but DO NOT solve) the equations in the matrix form for determining the nodal voltages in the circuit of Fig. 1(a).

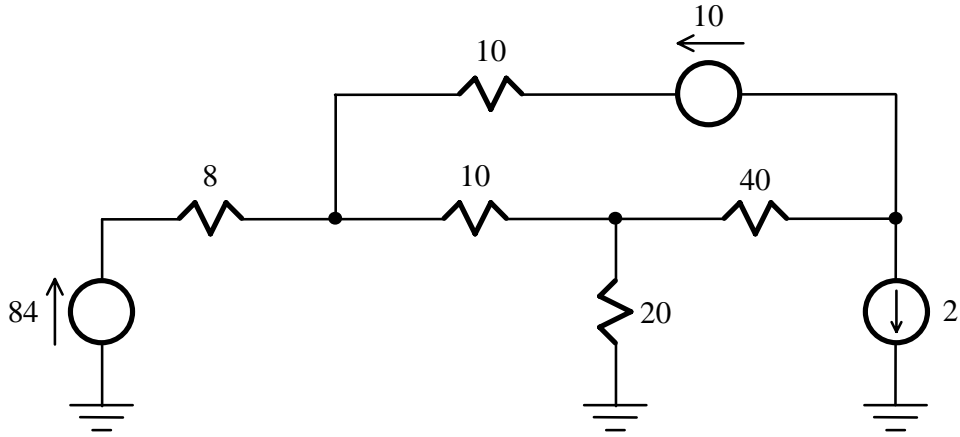


Fig. 1(a)

(20 Marks)

Numbering the nodes in the circuit by 1, 2 and 3 from left to right, and applying KVL:

$$\frac{v_1 - 84}{8} + \frac{v_1 - v_2}{20} + \frac{v_1 - v_3 - 10}{10} = 0 \Rightarrow 11v_1 - 2v_2 - 4v_3 = 460$$

$$\frac{v_2}{20} + \frac{v_2 - v_1}{10} + \frac{v_2 - v_3}{40} = 0 \Rightarrow -4v_1 + 7v_2 - v_3 = 0$$

$$\frac{v_3 - v_2}{40} + 2 + \frac{v_3 - v_1 + 10}{10} = 0 \Rightarrow -4v_1 - v_2 + 5v_3 = -120$$

$$\begin{bmatrix} 11 & -2 & -4 \\ -4 & 7 & -1 \\ -4 & -1 & 5 \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{bmatrix} 460 \\ 0 \\ -120 \end{bmatrix} \Rightarrow \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{bmatrix} 60.81 \\ 39.39 \\ 32.53 \end{bmatrix}$$

- Q.2.** Determine the Thevenin and Norton equivalent circuits as seen from terminals A and B of the network of Fig. 1(c). What is the maximum power that can be obtained from these two terminals?

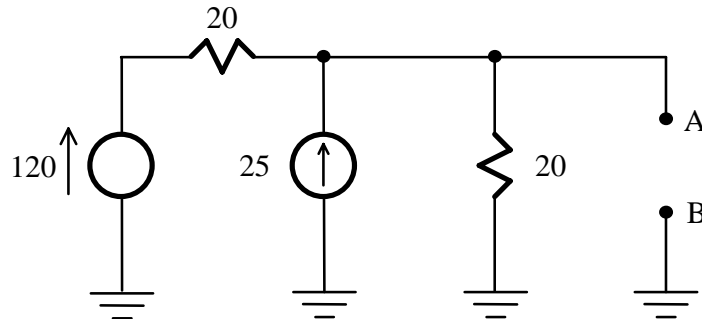


Fig. 1(c)

(20 Marks)

Replacing all independent sources with their internal resistances, the resistance across A and B is

$$R = 20 \parallel 20 = 10$$

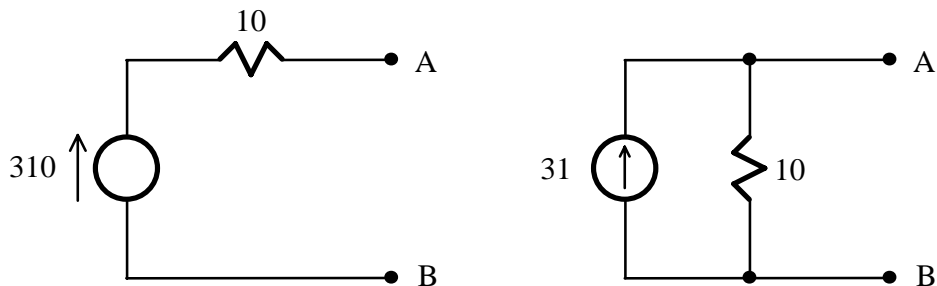
Using superposition, the open circuit voltage across A and B is

$$v_{AB} = 120 \left(\frac{20}{20+20} \right) + 25 \left(\frac{20}{20+20} \right) 20 = 310$$

and the short circuit current is

$$i = \frac{310}{10} = 31$$

Thus, the equivalent circuits are



The maximum power that can be obtained is

$$p = \frac{310^2}{4(10)} = 2402.5$$

Q.3. A 5 kW electric motor is operating at a lagging power factor of 0.5. If the input voltage is

$$v(t) = 250 \cos(\omega t + 15^\circ)$$

determine the apparent power, and find the phasor and sinusoidal expression for the input current.

(20 Marks)

Letting V and I to be the voltage and current phasor, the apparent power is

$$\frac{5000}{0.5} = 10000 \text{ VA} = |VI| = |V| \cdot |I|$$

where

$$V = \frac{250}{\sqrt{2}} e^{j15^\circ}$$

and

$$|I| = \frac{10000}{|V|} = \frac{10000\sqrt{2}}{250} = 40\sqrt{2}$$

With 0.5 lagging power factor

$$\arg(I) - \arg(V) = -\cos^{-1}(0.5)$$

Thus

$$I = 40\sqrt{2} e^{j(15^\circ - \cos^{-1} 0.5)}$$

$$\begin{aligned} i(t) &= 40\sqrt{2}\sqrt{2} \cos(\omega t + 15^\circ - \cos^{-1} 0.5) \\ &= 80 \cos(\omega t + 15^\circ - \cos^{-1} 0.5) \end{aligned}$$

- Q.4.** In the circuit of Fig. 2(b), the current $i(t)$ is the excitation and the voltage $v(t)$ is the response. Determine the frequency response of the circuit. Derive (but DO NOT solve) an equation for finding the "resonant" frequency at which the frequency response becomes purely real.

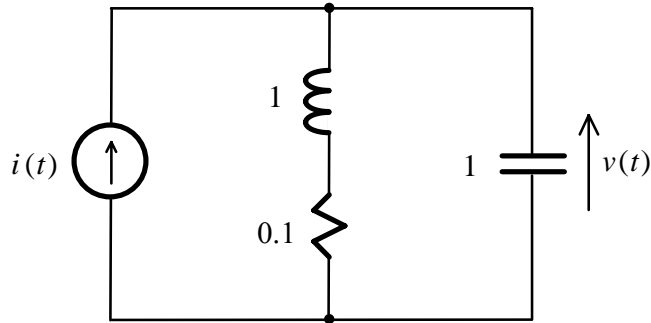


Fig. 2(b)

(20 Marks)

Using phasor analysis

$$\begin{aligned}
 H(f) &= \frac{V}{I} = \frac{1}{j\omega} \left(\frac{0.1 + j\omega}{0.1 + j\omega + \frac{1}{j\omega}} \right) \\
 &= \frac{0.1 + j\omega}{1 + j0.1\omega - \omega^2} = \frac{0.1 + j\omega}{(1 - \omega^2) + j0.1\omega} = \frac{0.1 + j\omega(0.99 - \omega^2)}{(1 - \omega^2)^2 + (0.1\omega)^2}
 \end{aligned}$$

Thus, we have $\omega = 0$, $\omega = 0.995$.

Alternatively, we can calculate the phase response

$$\arg[H(f)] = \tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{0.1\omega}{1 - \omega^2}\right)$$

The resonant frequency is therefore given by

$$\tan^{-1}(10\omega) = \tan^{-1}\left(\frac{0.1\omega}{1 - \omega^2}\right)$$

- Q.5.** A series RLC resonant circuit is to be designed for use in a communication receiver. Based on measurements using an oscilloscope, the coil that is available is found to have an inductance of 25.3 mH and a resistance of 2 Ω. Determine the value of the capacitor that will give a resonant frequency of 1 kHz. If a Q factor of 90 is required, will the coil be good enough?

(20 Marks)

The value of the capacitor needed is

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi 1000)^2 (25.3)10^{-3}} = 1\mu\text{F}$$

The Q factor without using any external resistor is

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi 1000 (25.3)10^{-3}}{2} = 25.3\pi = 79.48$$

Since this is less than 90, the coil is not good enough.