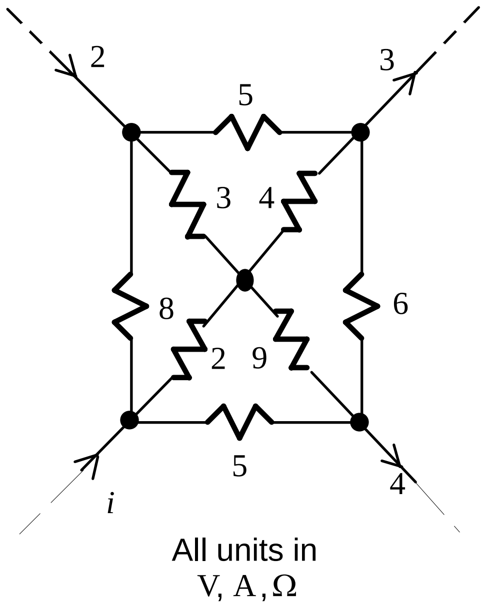
E

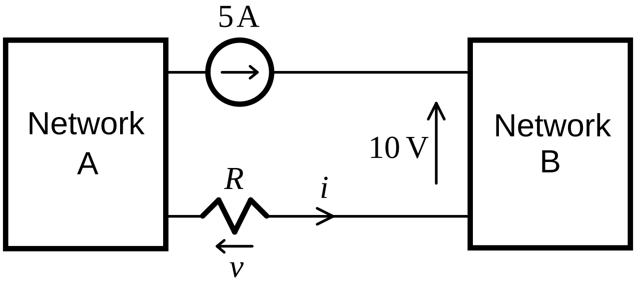
TUTORIAL PROBLEMS

E.1 KCL, KVL, Power and Energy

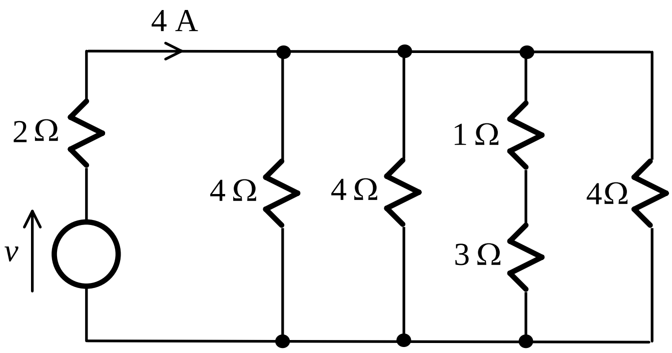
Q.1 Determine the current i in the following circuit.



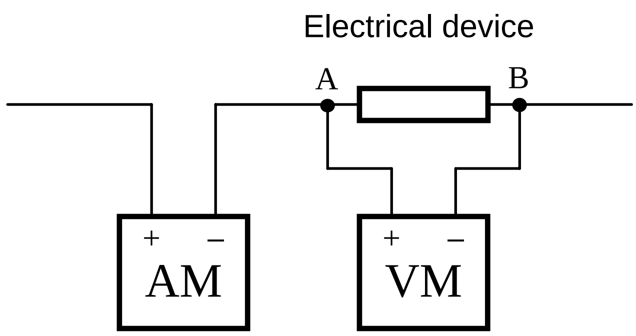
Q.2 Determine the current i and the voltage v in the following circuit for $R = 2\,\Omega$ and $R = 50\,\mathrm{K}\Omega$.



Q.3 Determine the source voltage v and the voltage across the 3Ω resistor in the following circuit.



Q.4 The Ammeter AM and voltmeter VM, connected as shown below, measure current and voltage, respectively. The ammeter will give a positive reading if the current flowing into its "+" terminal is positive.



Determine if the electrical device is consuming or supplying power when

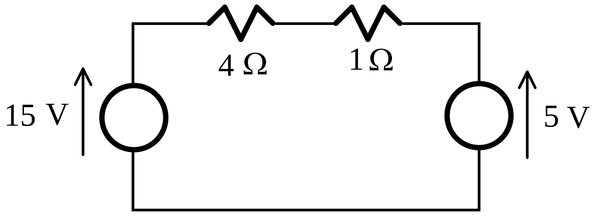
(a) both meters give positive readings;

(b) both meters give negative readings; and

(c) one meter gives a positive reading while the other gives a negative reading.

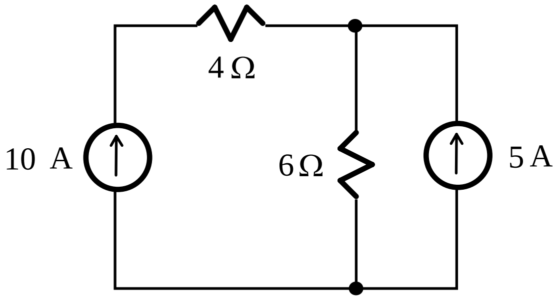
(d) one meter or both meters give zero readings.

Q.5 Determine the current in the following circuit.



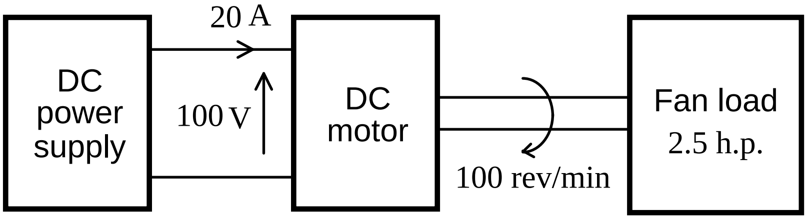
How much power is each component consuming or supplying?

Q.6 Determine the voltages and currents in the following circuit.



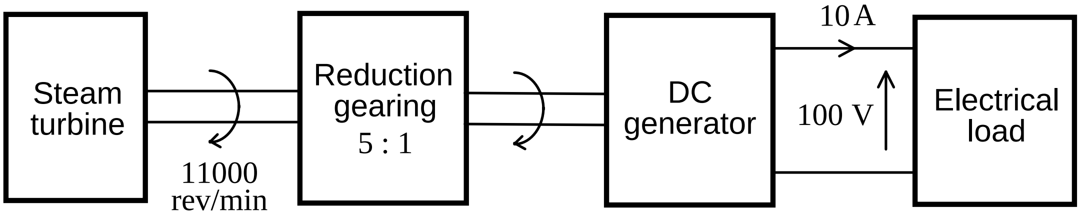
Is the $10\,\text{A}$ current source consuming or supplying power?

Q.7 In the following system, determine the efficiency of the motor and the torque on the fan shaft?



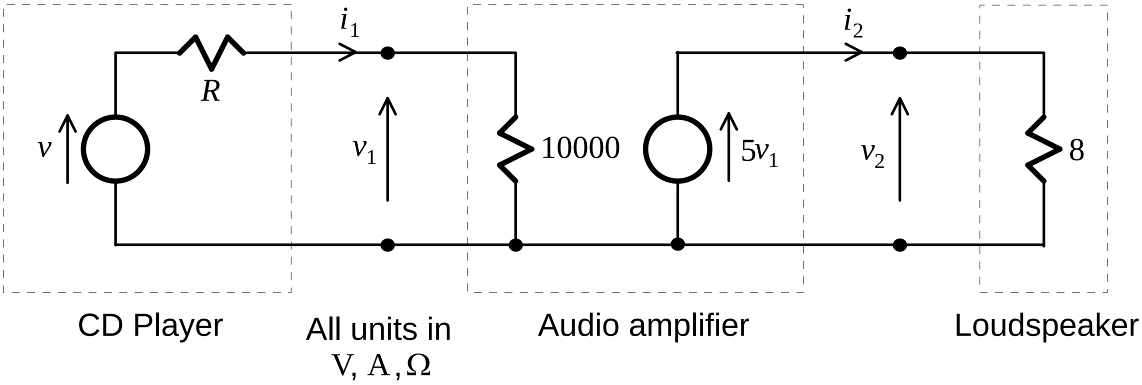
How much energy is lost in the motor per minute? (Note that $1\,\text{h.p.} = 746\,\text{W}$.)

Q.8 In the following system, the efficiency of the generator is 0.9.



Determine the torque in its shaft.

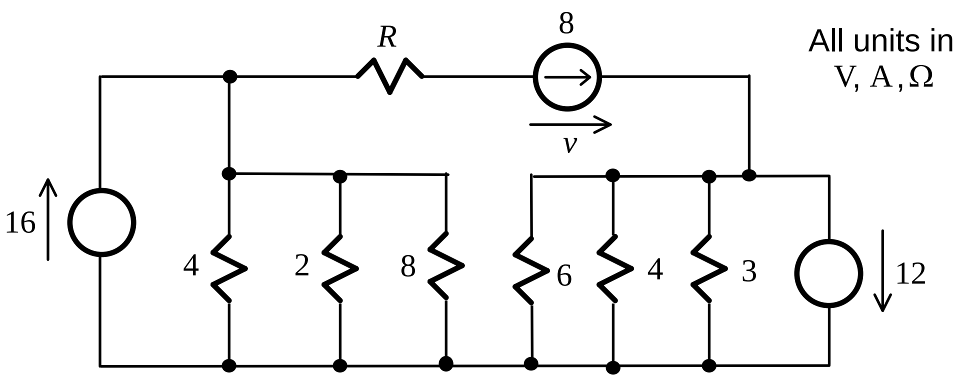
Q.9 The following figure shows the equivalent circuit of an audio amplifier when connected to an input audio source (CD player) and a load (loudspeaker).



Calculate the voltage gain $|v_2/v_1|$, current gain $|i_2/i_1|$ and power gain $|(v_2 i_2)/(v_1 i_1)|$ of the amplifier in dB. What is the relationship between these gains? Show that these gains will be equal to one another when the load resistance is equal to input resistance of the amplifier. Do you expect this to be the case in practice?

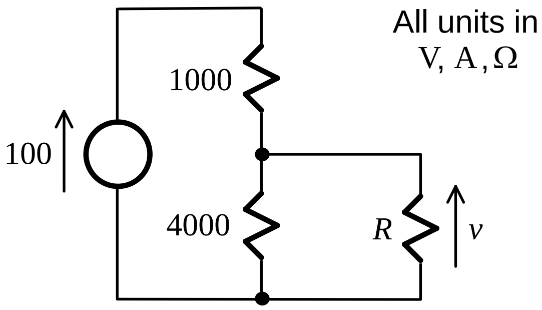
E.2 KCL, KVL and Grounding

Q.1 Determine the current in each branch of the following circuit by inspection.



If the voltage v is $-16\,\mathrm{V}$, determine R.

Q.2 The following circuit shows a common voltage divider for obtaining a certain voltage v across a load resistor R.



In finding v, a novice ($=1$st year student) may forget (okay, if it's only for this time) to include the loading effects of R. To understand these effects, determine v and the current in R when

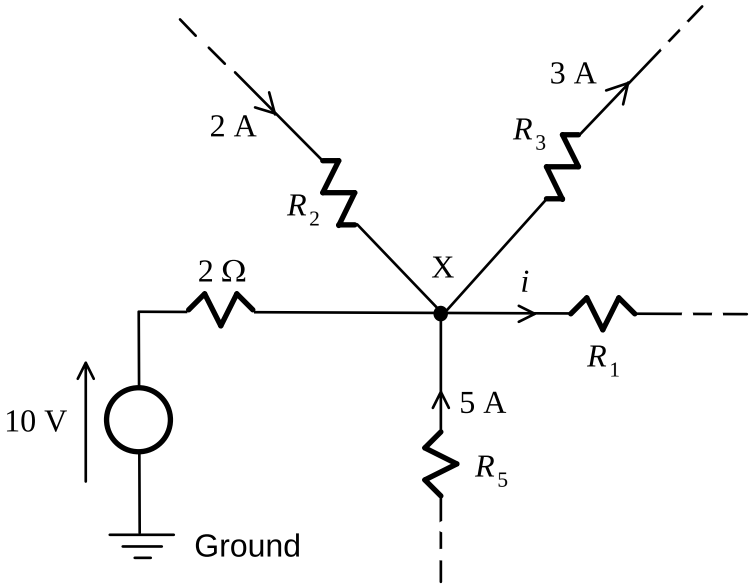
(a) $R = \infty$ (open circuit or no load situation);

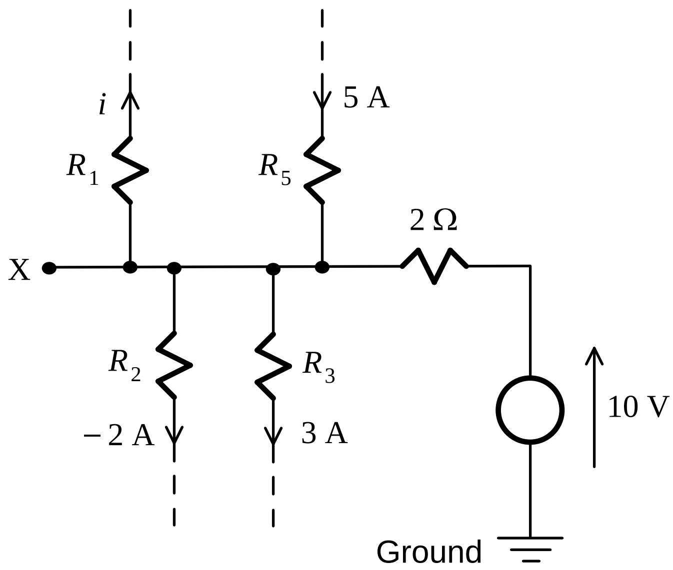
(b) $R = 8000\Omega$;

(c) $R = 200\Omega$; and

(d) $R = 0$ (short circuit situation).

Q.3 Are the two circuits shown below equivalent? Why?

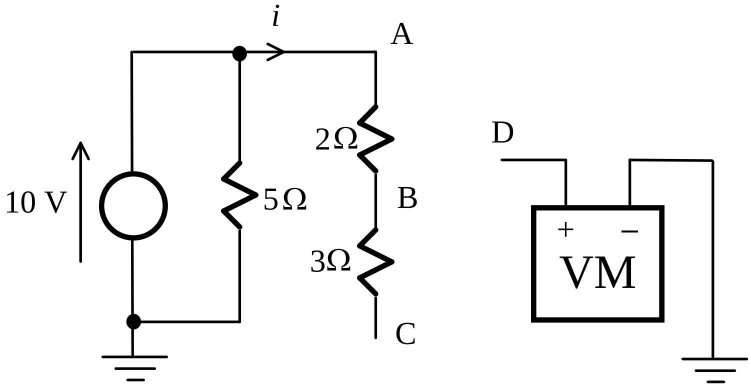




As is done in the above circuits, voltages in electrical systems are often measured with respect to a reference or ground potential. In practice, the ground will be connected to a large number of components and will run through the entire system using thick conductors. It is sometimes connected to the chassis of the system or the earth pin of the 3–pin power plug.

Assuming that there are other components (not shown) connected to ground, determine the current flowing through the voltage source and the potential of node X with respect to ground when $i = 2\,\mathrm{A}$ and $i = -3\,\mathrm{A}$. Why is the ground node seems to be taking/giving current all the time and still Kirchhoff's current law is not violated?

Q.4 In the following circuit, the voltmeter VM is used to measure the voltages of various points with respect to ground.



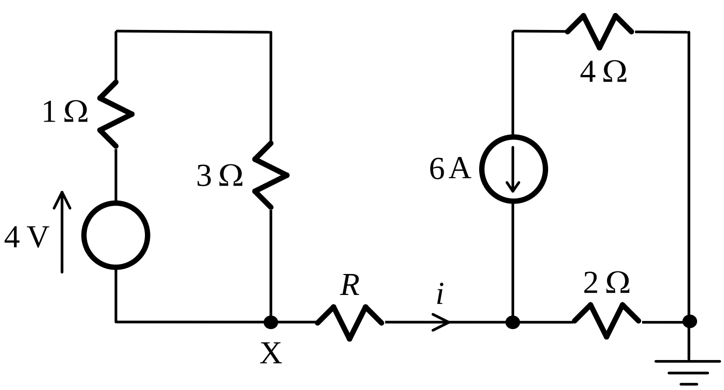
Assuming that VM does not take any current, determine the current i and the voltmeter reading when

(a) Point C is grounded and Point B is connected to Point D;

(b) there is no connection for Point C and Point B is connected to Point D; and

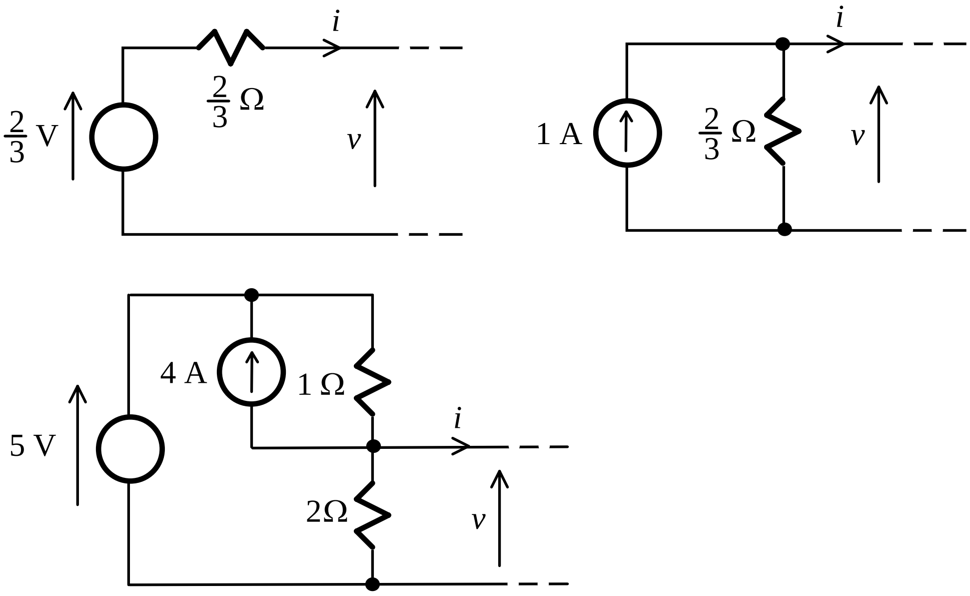
(c) Point B is grounded and Point C is connected to Point D.

Q.5 Determine the current i and the potential of node X with respect to ground in the following circuit.



How will these be changed if the circuit is not grounded or if the circuit is grounded at Point X only?

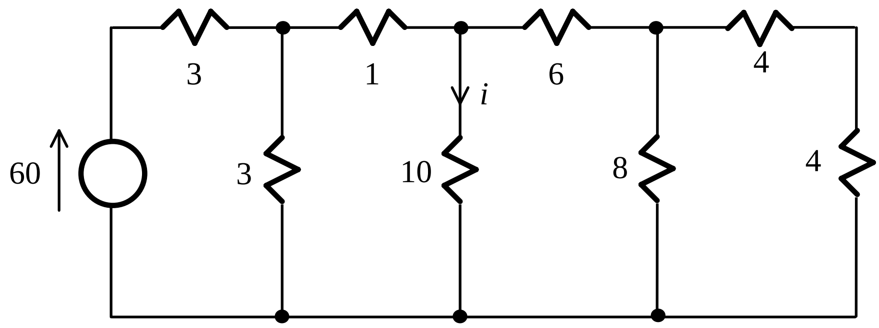
Q.6 Determine the relationships between v and i for the following three circuits.



What can you say about these three circuits?

E.3 DC Circuit Analysis I

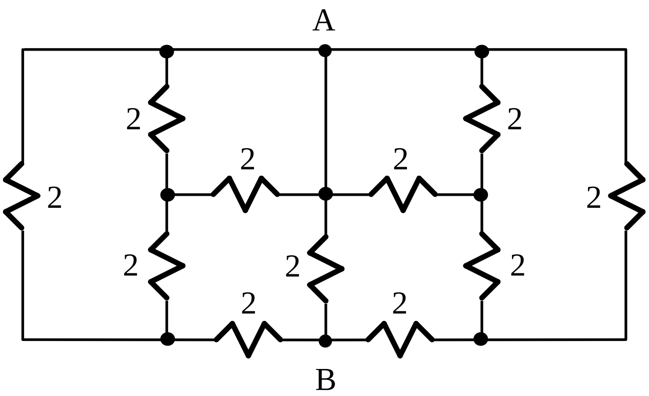
Q.1 Find the source current in the following ladder network by reducing the series and parallel combinations to a single resistor.



All units in
V, A, Ω

Determine the current i by using current division.

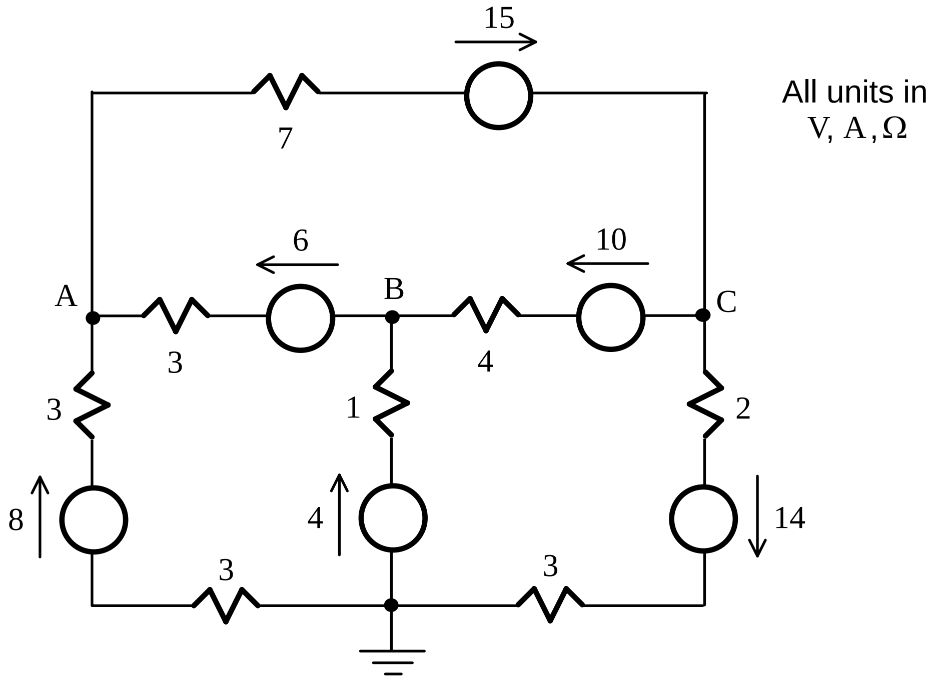
Q.2 Determine the equivalent resistance that would be measured between Points A and B of the following circuit.



All units in
V, A, Ω

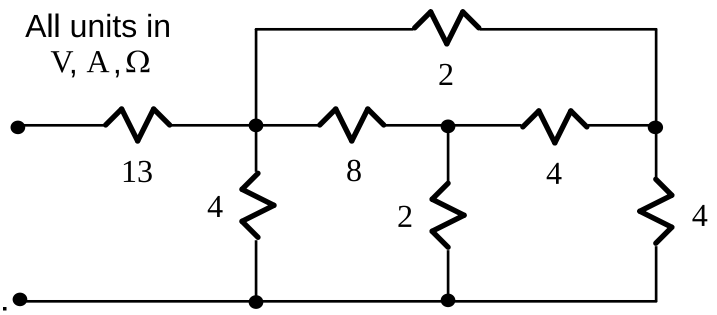
What will the result be if the outer two resistors are short-circuited?

Q.3 Formulate the equations for obtaining the loop currents of the following circuit by using mesh analysis. Determine the voltages of nodes A, B and C with respect to ground in terms of these currents.

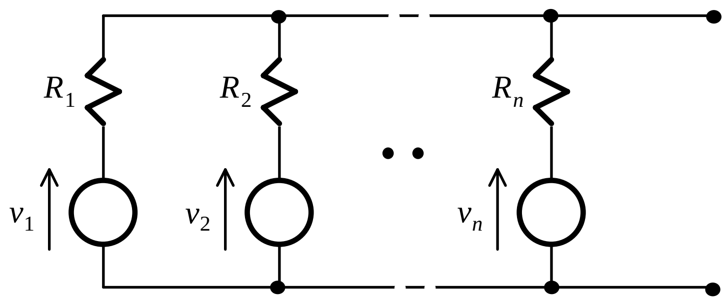


Using nodal analysis, derive the equations for obtaining the voltages of the three nodes.

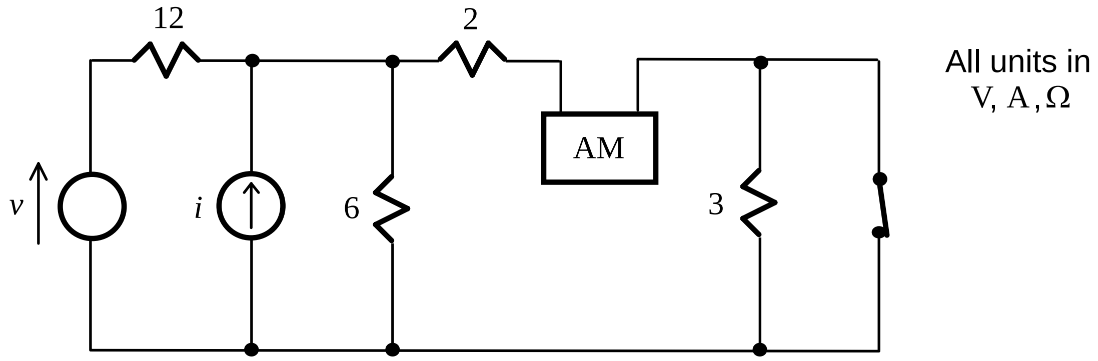
Q.4 Using nodal analysis, calculate the equivalent resistance of the following 2-terminal network.



Q.5 Determine Norton's and Thevenin's equivalent circuits for the following 2-terminal network.

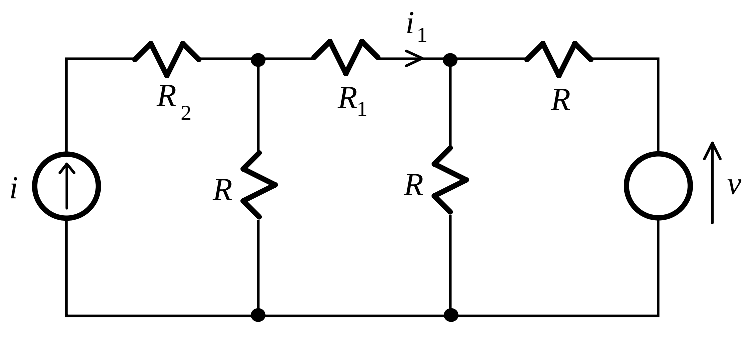


Q.6 In the following circuit, the ammeter AM reads 1A when the switch is closed.



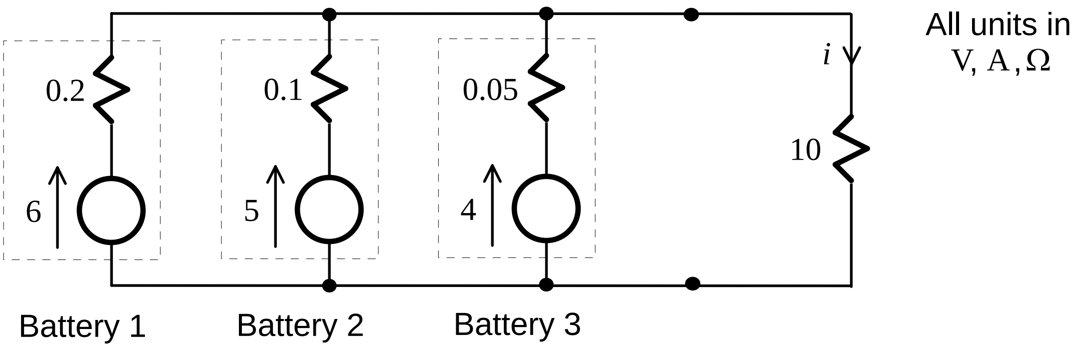
Using Norton's equivalent circuit, determine the reading when the switch is open.

Q.7 Using Thevenin's equivalent circuit, determine the current i_1 In the following circuit.



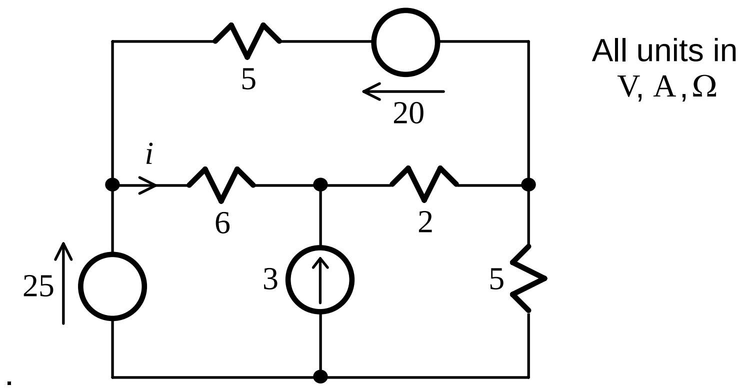
E.4 DC Circuit Analysis II

Q.1 The following circuit shows three batteries connected in parallel and supplying power to a 10Ω load.

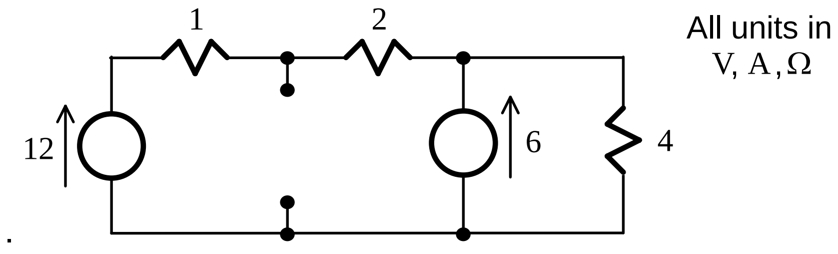


Using superposition, determine the component of the load current i due to each battery. Hence, calculate the load current.

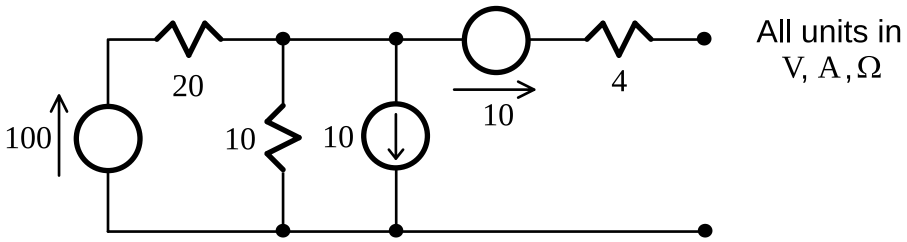
Q.2 Determine the current i in the following circuit by using superposition.



Q.3 Determine the maximum power that can be obtained from the two terminals of the following circuit.

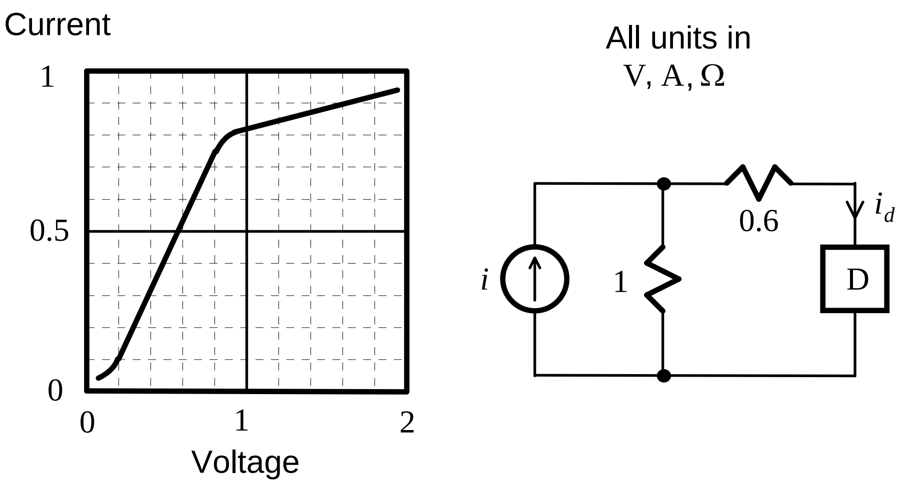


Q.4 Determine the resistor that will draw a current of $2\,\mathrm{A}$ when connected across the two terminals of the following network.



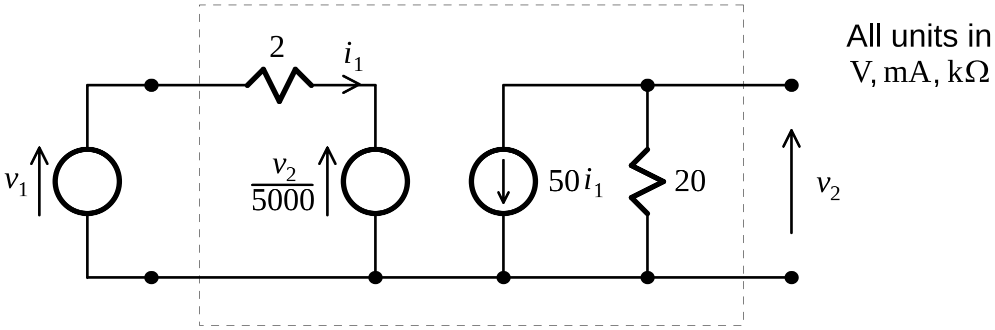
What resistor will absorb the maximum power? Calculate this power.

Q.5 The following figure shows the voltage-current curve of an electrical device D and a circuit in which the device is employed.



Determine the device current i_d if $i = 1.6\,\mathrm{A}$. At what value of i will the power dissipated in D be $0.6\,\mathrm{W}$?

Q.6 The following figure shows the equivalent circuit of a transistor amplifier connected to amplify the magnitude of the voltage v_1 from an input voltage source. The main activities of the transistor are represented by the two dependent sources, while the substrate and other contact resistances are represented by the two resistors.



Transistor amplifier

Determine the output voltage v_2 of the amplifier in terms of v_1. What is the gain of the amplifier in terms of voltage magnitudes? Give your answer in dB. Determine the Thevenin's equivalent circuit of the system as seen from the output terminals.

E.5 AC Circuit Analysis I

Q.1 Write down the peak and rms values, frequency, phase, complex and phasor representations of the following ac quantities.

(a) $5\sqrt{2}\sin(\omega t)$

(b) $5\sqrt{2}\cos(\omega t)$

(c) $10\sqrt{2}\sin(20t + 30^0)$

(d) $120\sqrt{2}\cos(314t - 45^0)$

(e) $-50\sin\left(4t - \frac{\pi}{3}\right)$

(f) $0.25\cos(2t + 100^0)$

Q.2 Taking the frequency to be $50\,\text{Hz}$, write down the sinusoidal voltages and currents corresponding to the following phasors.

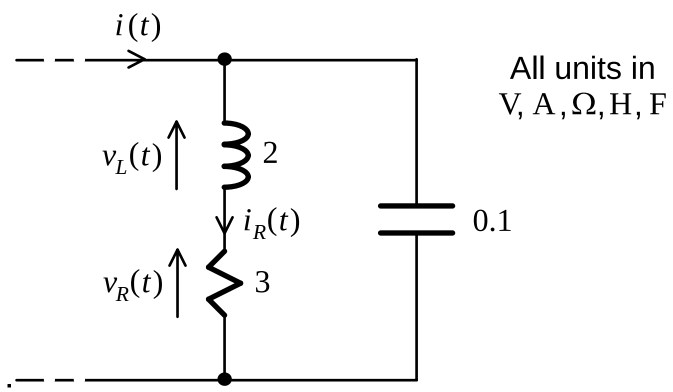
(a) $\frac{100}{\sqrt{2}} e^{j30^0}\ \text{V}$

(b) $115e^{j\pi/3}\ \text{V}$

(c) $-0.12e^{-j\pi/4}\ \text{A}$

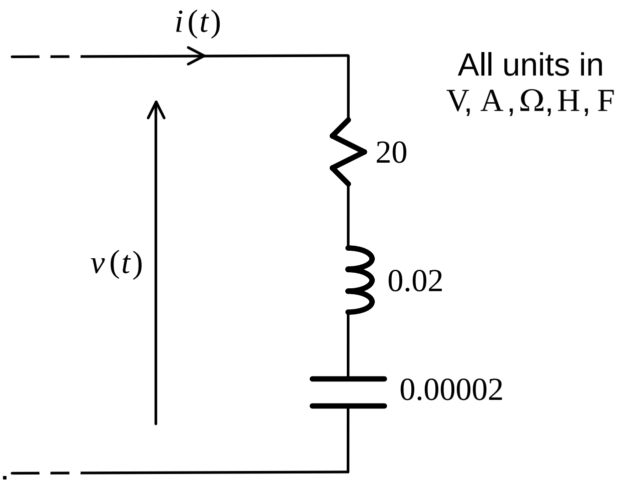
(d) $-0.69\underline{/60^o}\ \text{A}$

Q.3 In the following ac circuit, $v_R(t) = 12\sqrt{2}\cos(2t)\ \text{V}$.



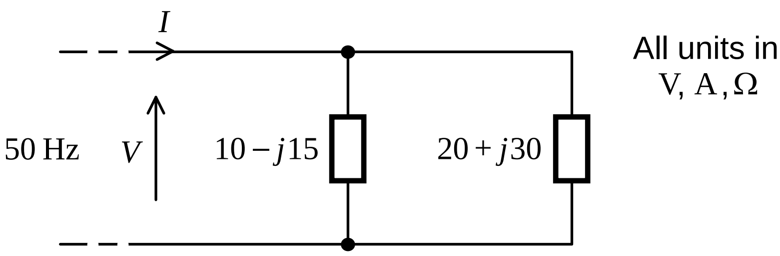
Determine $i_R(t)$, $v_L(t)$ and $i(t)$ by using phasor analysis.

Q.4 In the following series RLC circuit, $v(t) = 50\sqrt{2}\cos\left(1250t + 30^0\right)$ V. Determine the phasors of the voltages across the three components and that of the current $i(t)$.



Draw a phasor diagram showing all these phasors in the complex plane.

Q.5 The following figure shows an ac circuit with two parallel branches. Each branch consists of two series components: RL (resistor and inductor) or RC (resistor and capacitor) or LC (inductor and capacitor). From the impedances given, find the components and their values in the circuit.



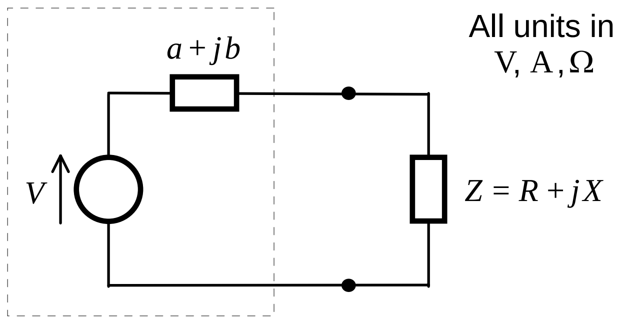
Calculate the admittance (reciprocal of impedance) of the entire circuit and the phase angle between I and V. What is the power factor of the circuit?

E.6 AC Circuit Analysis II

Q.1 A 4Ω resistor is connected in series with a coil across a $20\,\mathrm{V}$ $50\,\mathrm{Hz}$ supply. The voltage across the resistor is $9\,\mathrm{V}$ and that across the coil is $14\,\mathrm{V}$. If the coil can be represented as an ideal inductor L in series with a resistor R (accounting for the winding resistance and other losses), calculate the values of L and R, the power absorbed by the coil, and the power factor of the whole circuit.

Q.2 A load connected across a $2000\,\mathrm{V}$ $50\,\mathrm{Hz}$ line draws $10\,\mathrm{kW}$ at a lagging power factor of 0.5. Determine the current taken by the load. To improve the power factor, a capacitor C is now connected in parallel with the load. Determine the value of C so that the overall power factor is $0.9\,\mathrm{lagging}$, unity and $0.8\,\mathrm{leading}$.

Q.3 The following diagram shows the Thevenin's equivalent circuit of an ac system when connected to a load with impedance $Z = R + jX$.



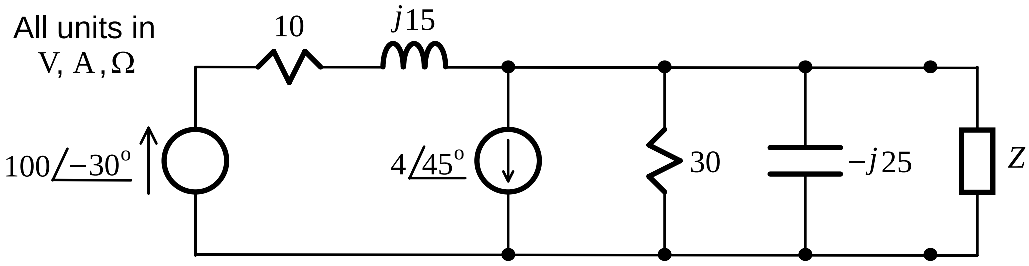
Electrical system

Show that R and X should be given by

$$Z = R + jX = (a + jb)^* = a - jb$$

in order for the load to absorb the maximum power. Determine this maximum power.

Q.4 Using Norton's equivalent circuit, determine the maximum power that can be absorbed by the load with impedance Z in the following ac system.



Q.5 A certain tune circuit consists of a $100\ \mu\text{H}$ coil with a resistance of 10Ω (representing losses due to thermal resistance and other effects) connected in series with a $100\,\text{pF}$ capacitor. Determine the resonant frequency, Q factor, bandwidth and 3dB cutoff frequencies of the circuit.

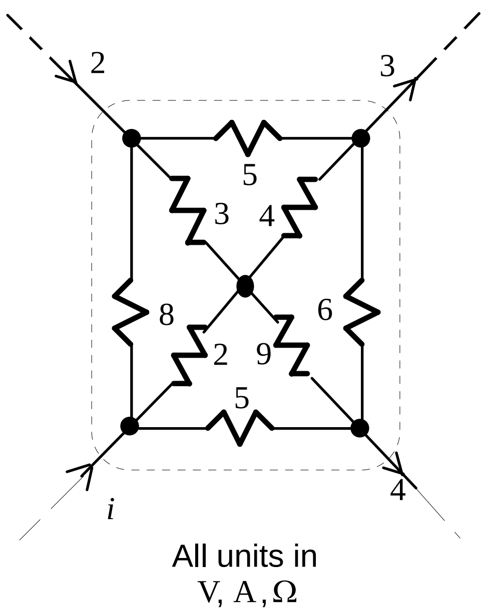
Q.6 A variable tuning capacitor in a radio receiver has a maximum capacitance of $500\,\text{pF}$ and a minimum capacitance of $20\,\text{pF}$. What inductance is required so that the lowest frequency to which the circuit can be tuned is $666\,\text{kHz}$? What is the highest frequency to which this circuit can be tuned?

F

Tutorial Solutions

F.1 KCL, KVL, Power and Energy

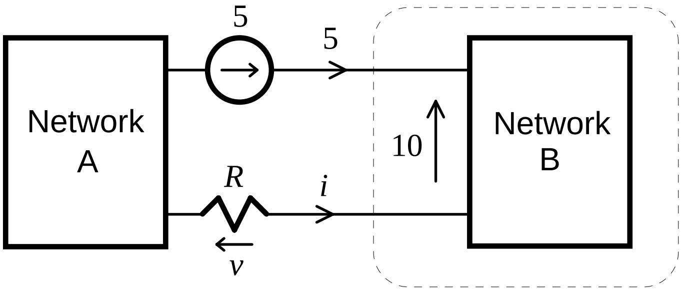
Q.1



Applying KCL to the dotted surface:

$$i + 2 = 3 + 4 \;\Rightarrow\; i = 5$$

Q.2



All units in
V, A, Ω

Applying KCL to the dotted surface:

$$5 + i = 0 \;\Rightarrow\; i = -5\text{A}$$

regardless of the value of R. For $R = 2\Omega$,

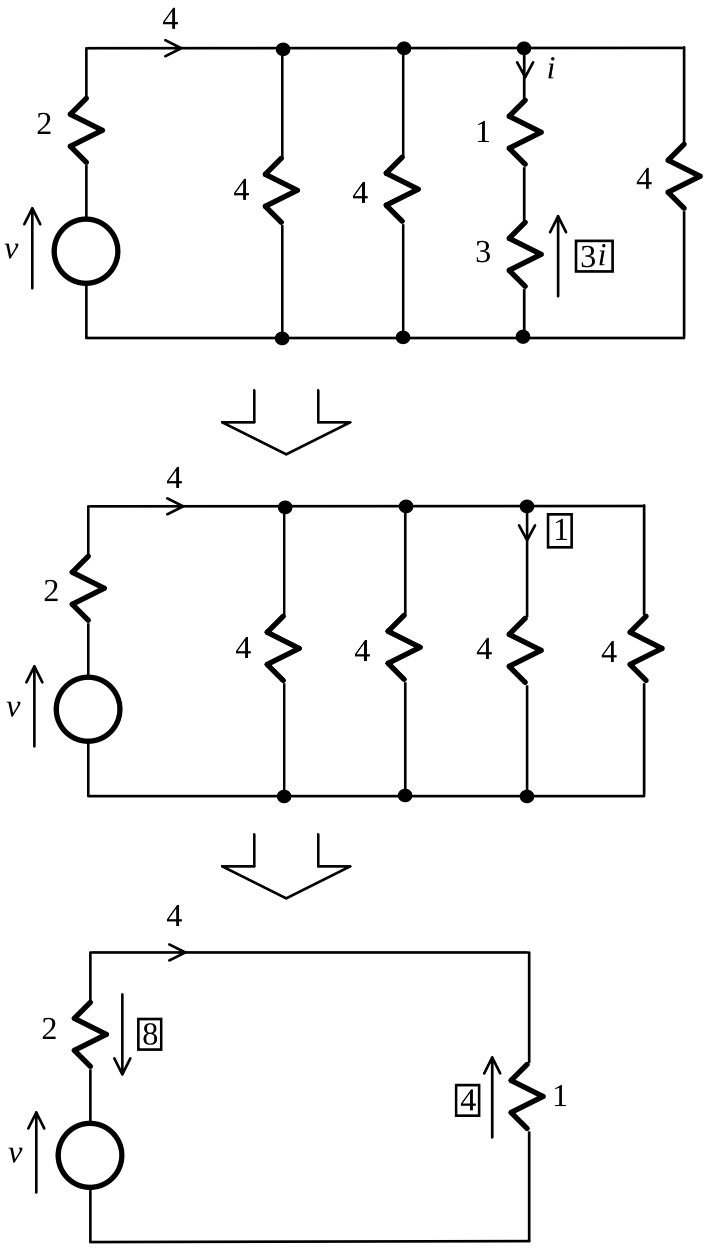
$$v = iR = -5 \times 2 = -10\,\text{V}$$

For $R = 50\,\text{K}\Omega$,

$$v = iR = -5 \times 50000 = -250\,\text{KV}$$

Q.3

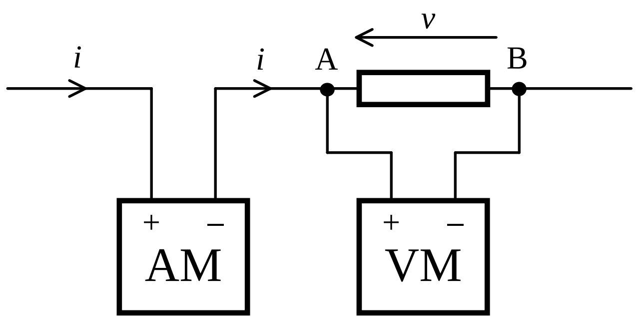
All units in
V, A, Ω



$v = 8 + 4 = 12\,\mathrm{V}$; $i = 1$; voltage across 3Ω resistor $= 3i = 3\,\mathrm{V}$

Q.4

(a) Both meters give positive readings



Since the arrows for v and i are in opposite directions

Power consumed $= vi$

Also, AM will give a positive reading if i is positive, while VM will give a positive readings if v is positive.

Since both i and v are positive in this case, vi is positive and power is consumed.

(b) Both meters give negative readings

Both i and v are negative, vi is positive and power is consumed.

(c) One meter gives a positive reading and the other gives a negative reading

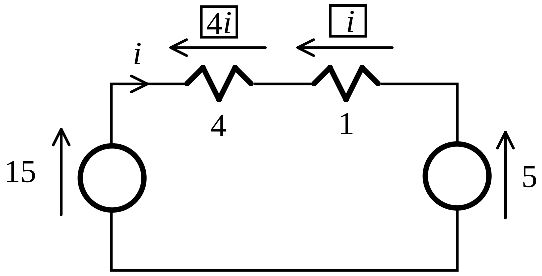
i and v have opposite signs, vi is negative and power is supplied by the device.

(d) One meter or both meter give zero readings

Power neither is consumed nor supplied by the device.

Q.5

Current in circuit



Applying KVL:

$15 = 4i + i + 5 \;\Rightarrow\; i = 2$

Power consumed/supplied

If the voltage and current arrows are in opposite directions,

Power consumed $= (\text{voltage})(\text{current})$

Thus:

Power consumed by 4Ω resistor $= i(4i) = 16\,\text{W}$

Power consumed by 1Ω resistor $= i(i) = 4\,\text{W}$

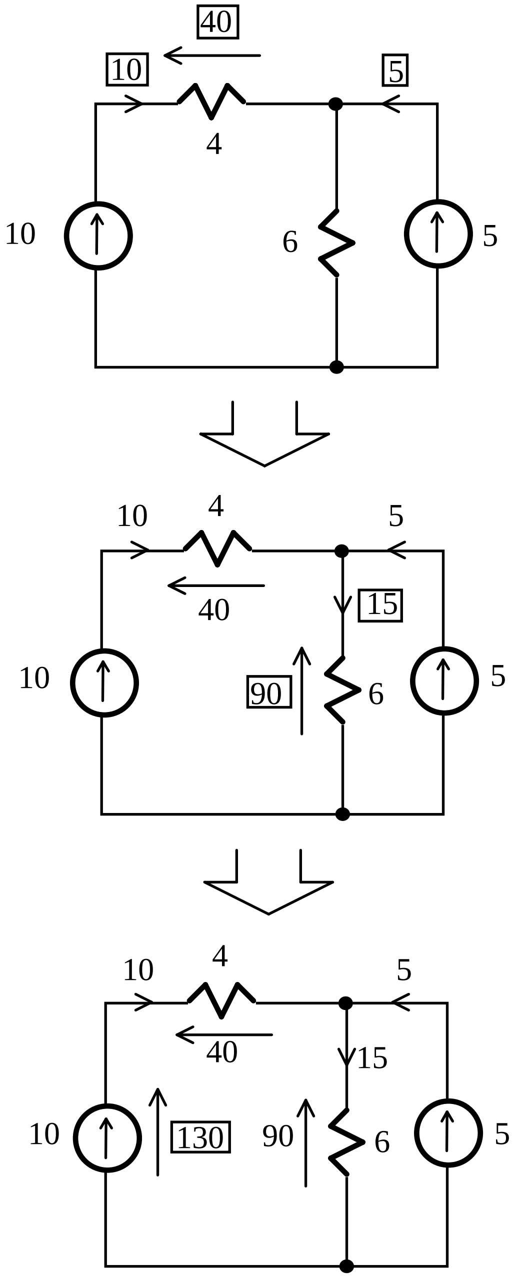
Power consumed by $5\,\text{V}$ source $= i(5) = 10\,\text{W}$

Power consumed by $15\,\text{V}$ source $= -i(15) = -30\,\text{W}$

or

Power supplied by $15\,\text{V}$ source$= 30\,\text{W}$

Q.6



All units in
V, A, Ω

The $10\,\text{A}$ current source is supplying a power of $(130)(10) = 1300\text{W}$.

Q.7

Efficiency

Electrical power supplied $= (100)(20) = 2000\,\mathrm{W}$

Mechanical power delivered $= (2.5\,\mathrm{h.p.})(746\,\mathrm{W/h.p.}) = 1865\,\mathrm{W}$

Efficiency $= \dfrac{1865}{2000} = 93.25\%$

Torque

Motor speed $= (100\,\mathrm{rev/min})\left(\dfrac{2\pi}{60}\dfrac{\mathrm{rad/rev}}{\mathrm{s/min}}\right) = \dfrac{20\pi}{6}\,\mathrm{rad/s}$

Torque $= \dfrac{\text{Mechanical power delivered}}{\text{motor speed}} = \dfrac{1865}{20\pi/6} = 178.1\,\mathrm{Nm}$

Energy lost

Power lost $= 2000 - 1865 = 135\,\mathrm{W}$

Energy lost per $\min = (135)(60) = 8100\,\mathrm{J}$

Q.8

Generator output power $= (100)(10) = 1000\,\mathrm{W}$

Generator input power $= \dfrac{1000}{0.9} = 1111.1\,\mathrm{W}$

Generator shaft speed $= \left(\dfrac{11000}{5}\,\mathrm{rev/min}\right)\left(\dfrac{2\pi}{60}\dfrac{\mathrm{rad/rev}}{\mathrm{s/min}}\right) = 230.4\,\mathrm{rad/s}$

Torque $= \dfrac{1111.1}{230.4} = 4.82\,\mathrm{Nm}$

Q.9

Voltage, current and power gains for system

Voltage gain $= g_v = \left|\frac{v_2}{v_1}\right| = \left|\frac{5v_1}{v_1}\right|$

$= 5 = 20\log(5)\,\text{dB} = 14\,\text{dB}$

Current gain $= g_i = \left|\frac{i_2}{i_1}\right| = \left|\frac{v_2/8}{v_1/10000}\right|$

$= \left|\frac{5v_1/8}{v_1/10000}\right| = 6250 = 20\log(6250)\,\text{dB} = 76\,\text{dB}$

Power gain $= g_p = \left|\frac{v_2 i_2}{v_1 i_1}\right| = g_v g_i$

$= (5)(6250) = 10\log[(5)(6250)]\,\text{dB}$

$= \frac{1}{2}[20\log(5) + 20\log(6250)]\,\text{dB} = \frac{14+76}{2}\,\text{dB} = 45\,\text{dB}$

Relationship between these gains

$g_p = g_v g_i$

$(g_p\ \text{dB}) = \frac{(g_v\ \text{dB}) + (g_i\ \text{dB})}{2}$

$g_p = g_v = g_i$ if load resistance equals amplifier's input resistance

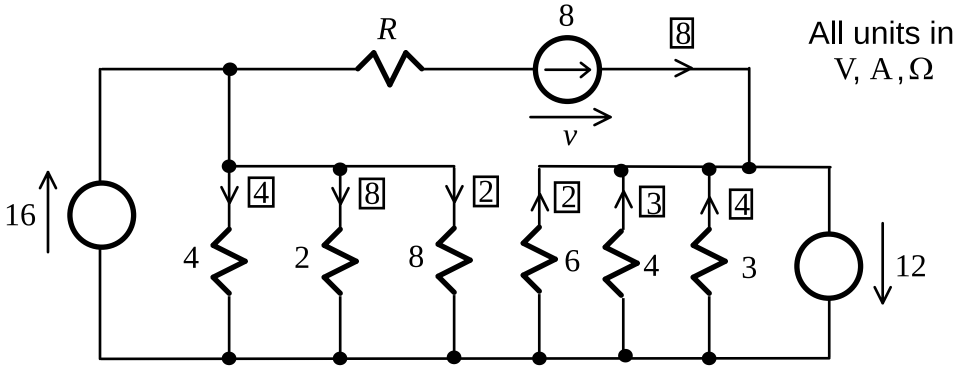
Audio amplifier

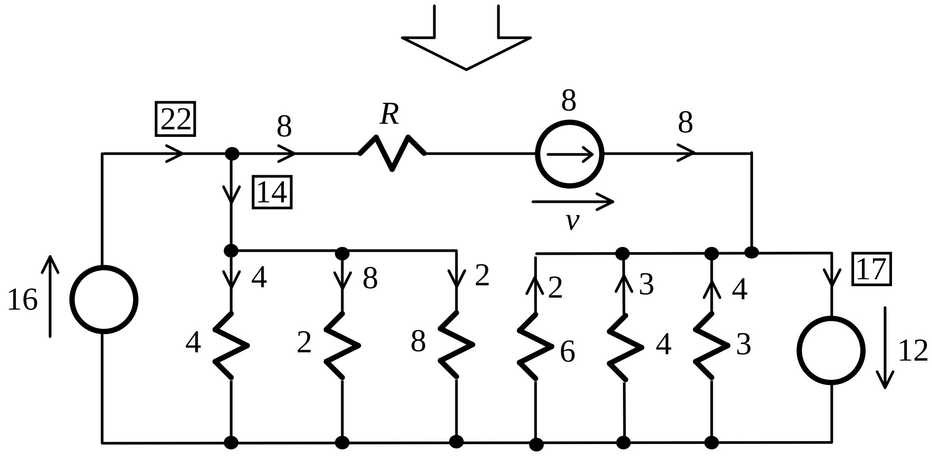
Most loudspeakers have resistances in the order of a few Ω. However, in order not to load the CD player or other audio input equipment, the input resistance of the amplifier will have to be large and is usually greater than many $\text{k}\Omega$.

F.2 KCL, KVL and Grounding

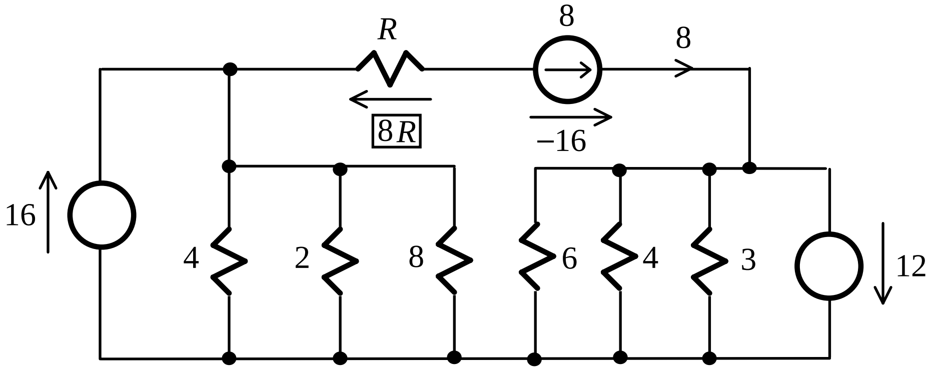
Q.1

Currents





Value for R

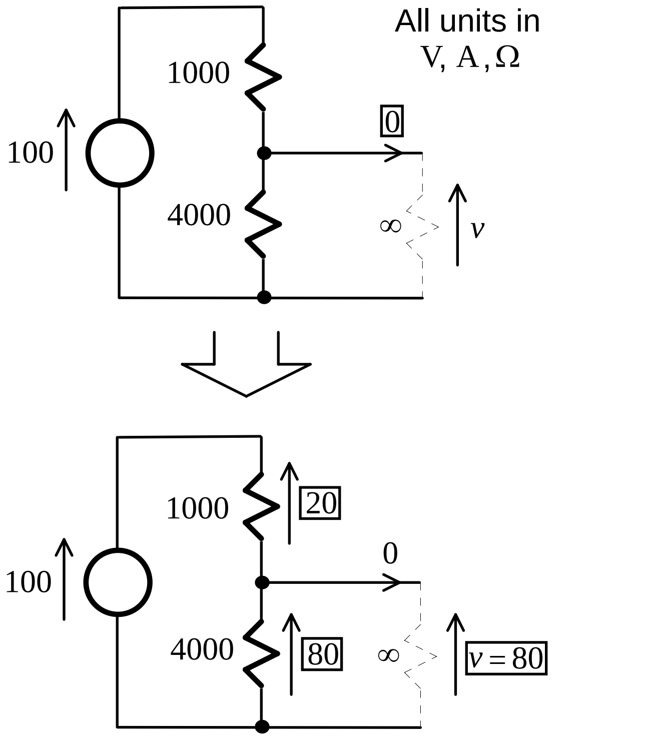


Applying KVL to the loop with the sources and R:

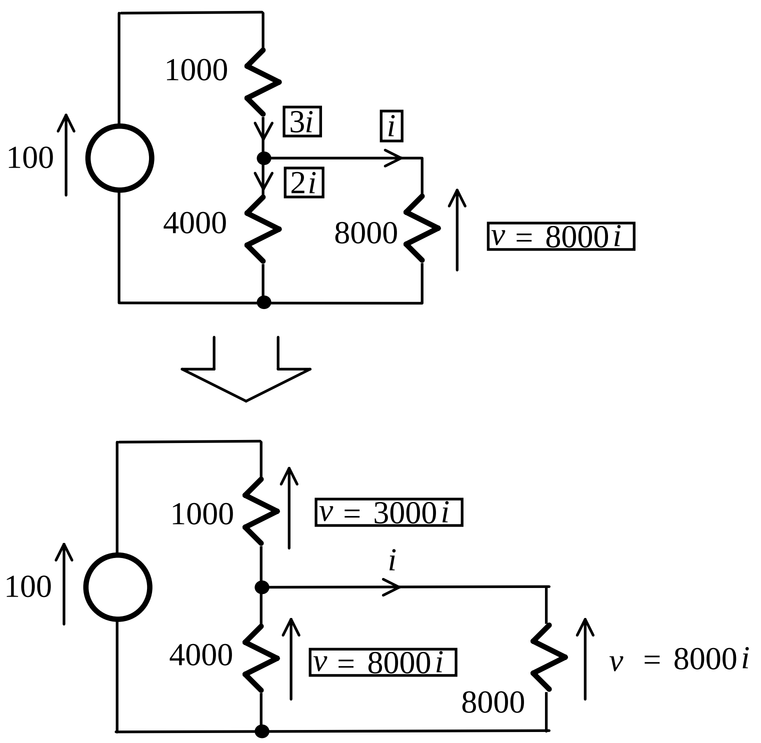
$$16 - 8R - 16 + 12 = 0 \Rightarrow R = 1.5$$

Q.2

(a) $R=\infty$ *(open circuit or no load situation)*



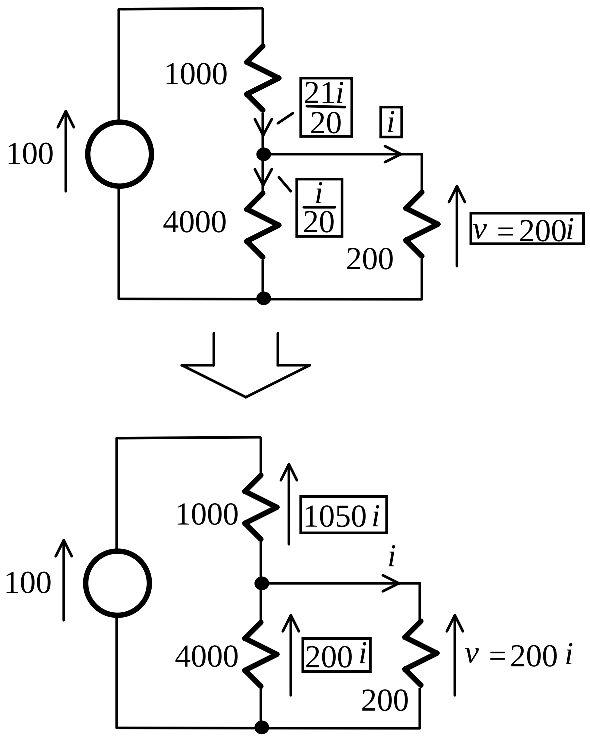
(b) $R=8000\Omega$



$$100=3000i+8000i \;\Rightarrow\; i=\frac{1}{110}\text{A}$$

$v = 8000i = 72.73\text{V}$

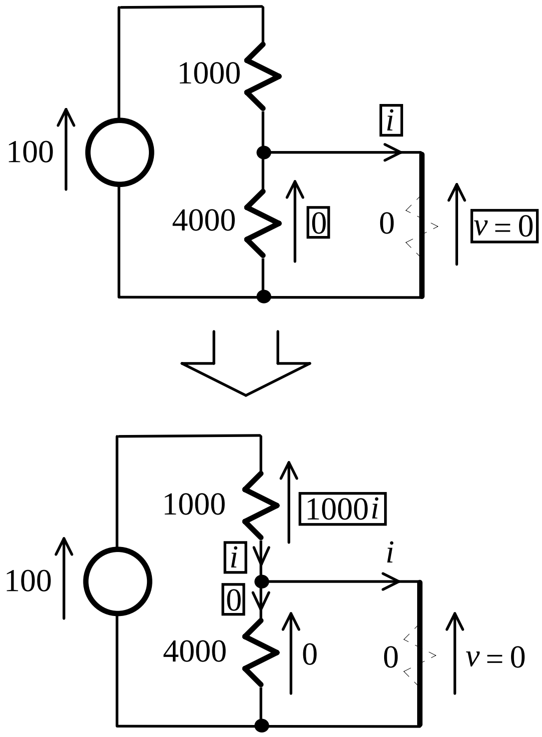
(c) $R = 200\Omega$



$$100 = 1050i + 200i \;\Rightarrow\; 100 = 1250i \;\Rightarrow\; i = \frac{2}{25}\text{A}$$

$v = 200i = 16\,\text{V}$

(d) $R = 0$ *(short circuit)*



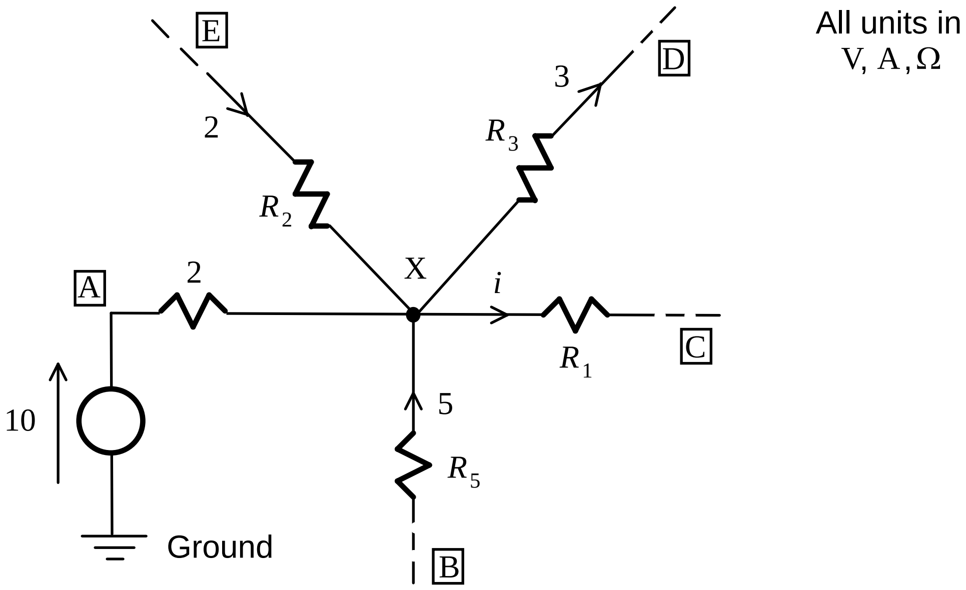
$100 = 1000i \;\Rightarrow\; i = 0.1\text{A}$

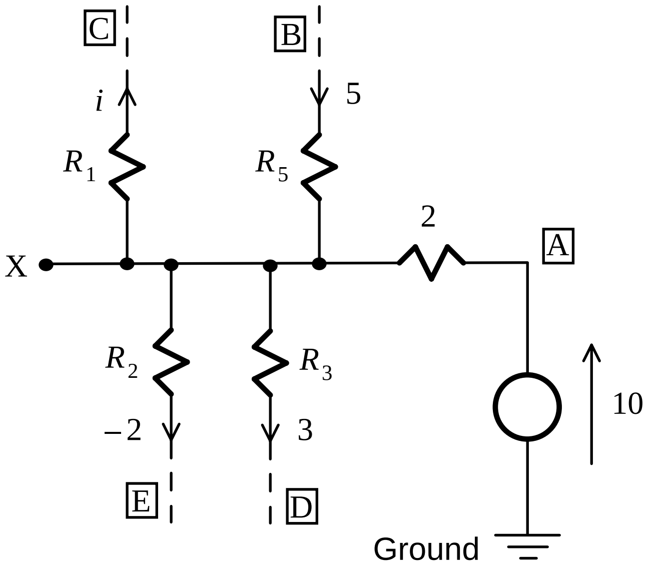
It may be slightly faster to derive two general formulas for v and i and then substitute the values for R.

Q.3

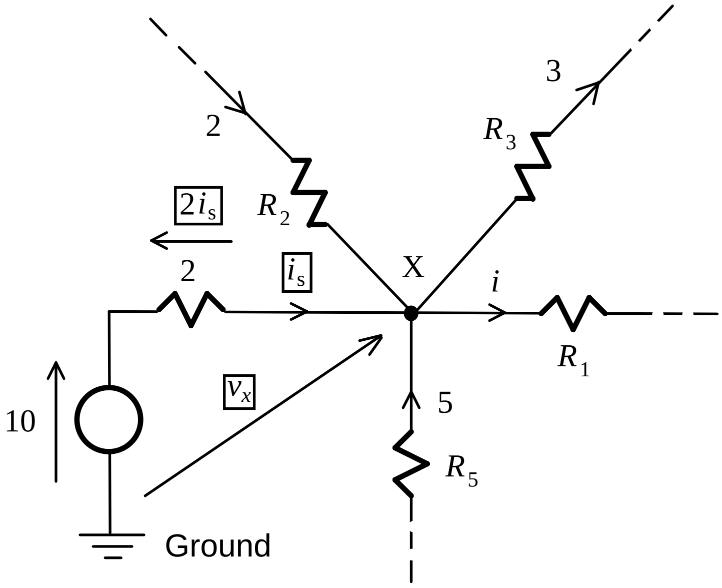
Equivalence of circuits

The two circuits are equivalent because the connections (topology), elements and currents between the various nodes are identical:





Current and voltage



Applying KCL to node X and then KVL:

$i_s = i + 3 - 2 - 5 = i - 4$

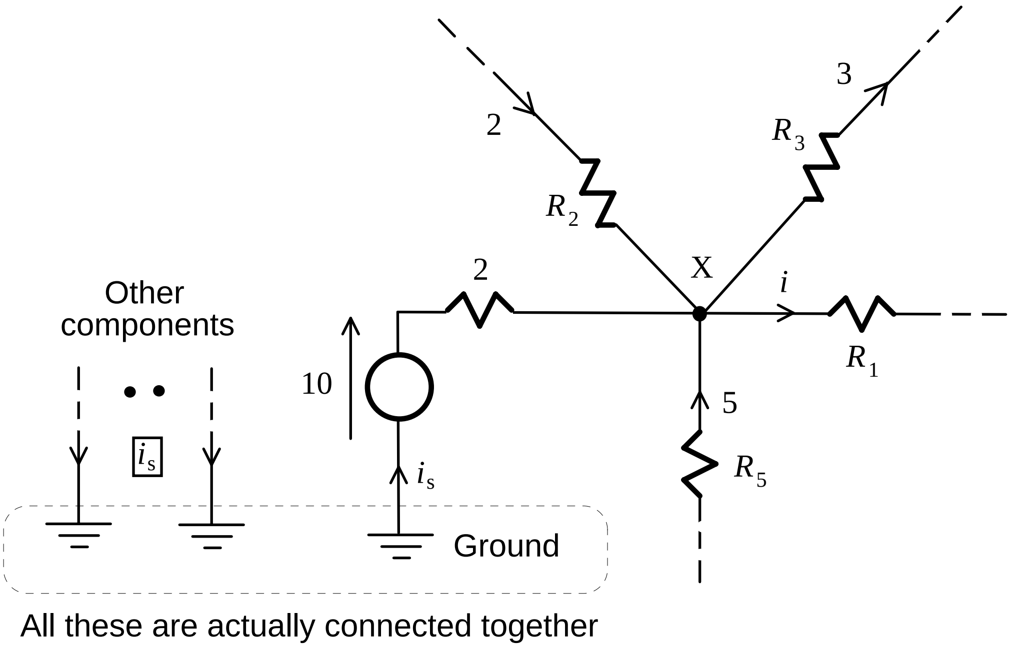
$v_x = 10 - 2i_s = 10 - 2(i - 4) = 18 - 2i$

When $i = 2$, $i_s = -2\,\text{A}$ and $v_x = 14\,\text{V}$

When $i = -3$, $i_s = -7\,\text{A}$ and $v_x = 24\,\text{V}$

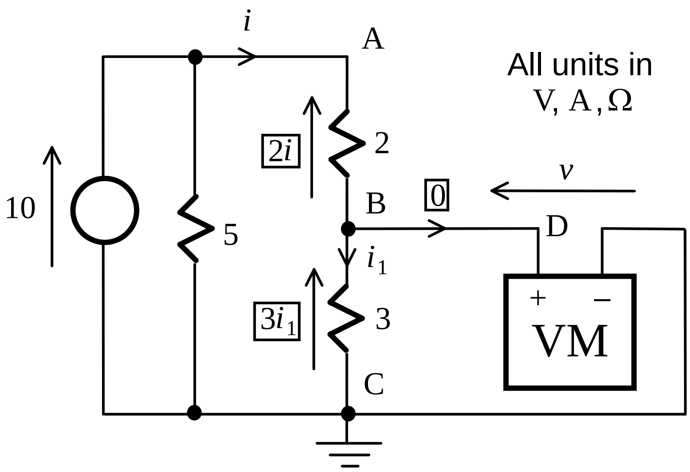
KCL for ground node

Since there may be other components connected to ground, the application of KCL must include all the other connections not shown in the original diagram. The implication is that all these other components must be delivering a combined current of i_s to ground:



Q.4

(a) Point C *grounded and Point* B *connected to Point* D



Applying KCL to node B:

$$i_1 = i - 0 = i$$

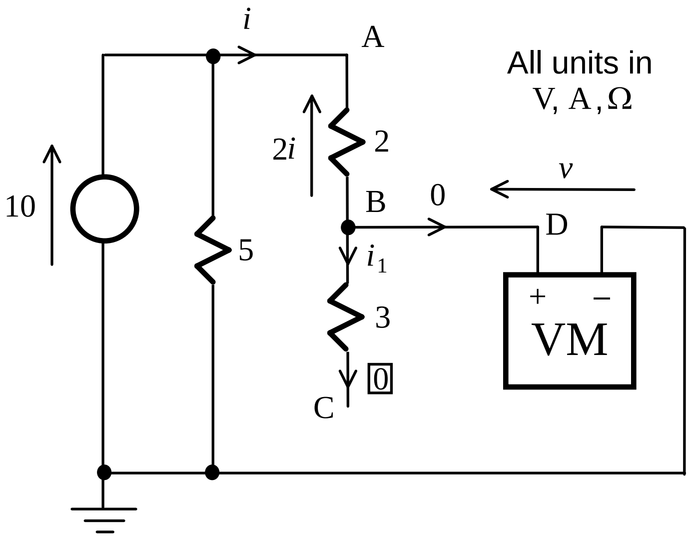
Applying KVL to loop with voltage source, A, B and C:

$$10 = 2i + 3i_1 = 5i \;\Rightarrow\; i = 2\,\text{A}$$

Applying KVL to loop with B, C and VM:

$$v = 3i_1 = 3i = 6\,\text{V}$$

(b) No connection for Point C *and Point* B *connected to Point* D



Applying KCL to node C and then to node B:

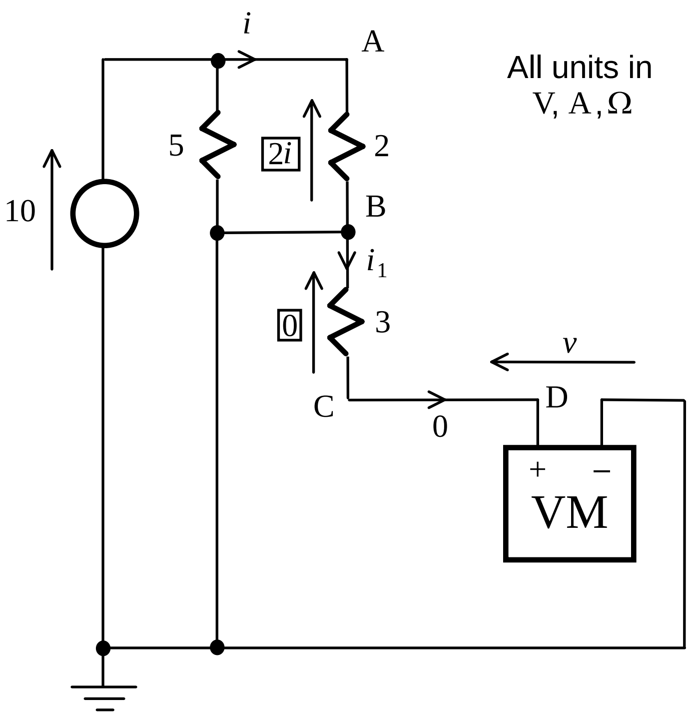
$$i_1 = 0\,\text{A}$$

$$i = i_1 + 0 = 0\,\text{A}$$

Applying KVL to loop with voltage source, A, B and VM:

$$10 = 2i + v \;\Rightarrow\; v = 10\,\text{V}$$

(c) Point B *grounded and Point* C *connected to Point* D



Applying KVL to loop with voltage source, A and B:

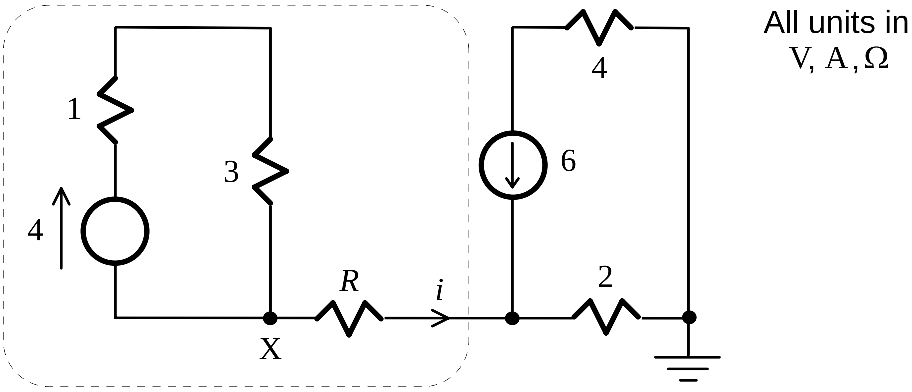
$$10 = 2i \;\Rightarrow\; i = 5\,\mathrm{A}$$

Applying KVL to loop with B, C, D and VM:

$$v + 3i_1 = 0 \;\Rightarrow\; v = 0\,\mathrm{V}$$

Q.5

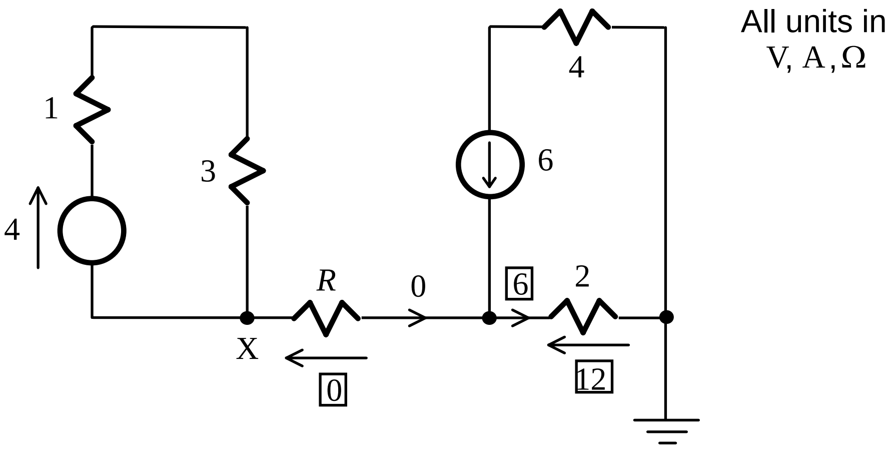
Original circuit



Applying KCL to the dotted surface:

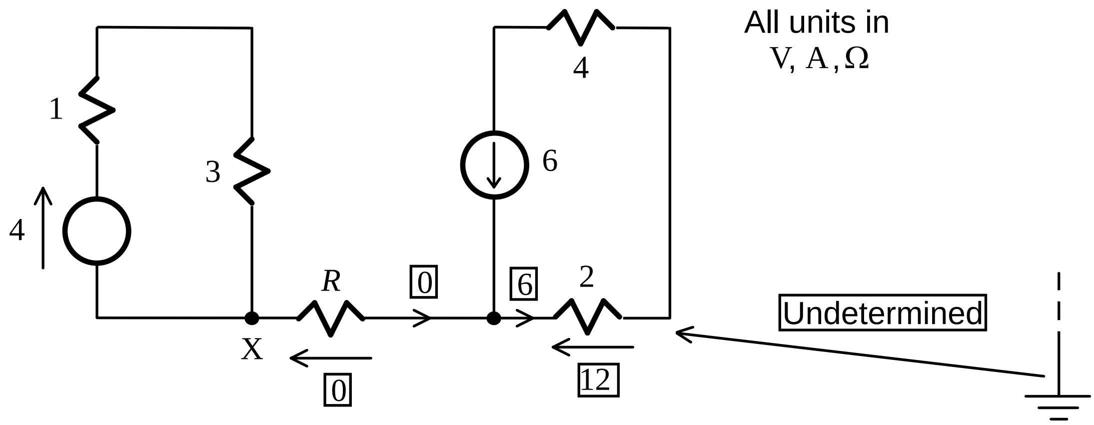
$$i = 0$$

Thus:



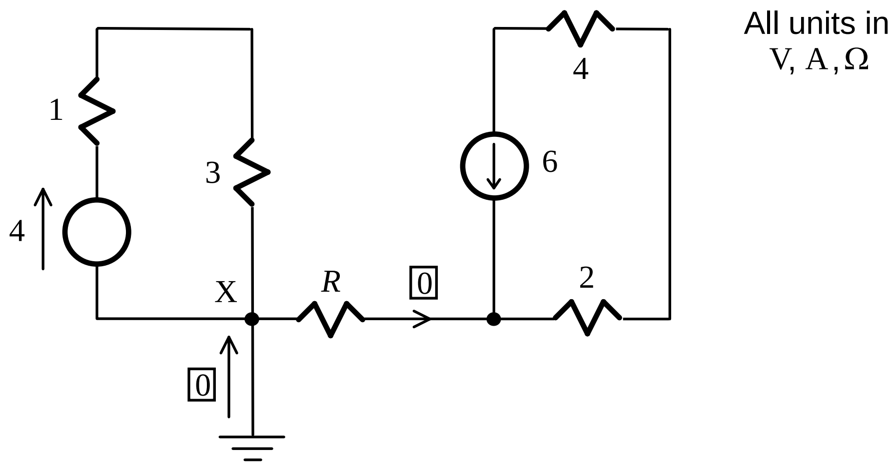
Potential of node X wrt ground $= 12\,\mathrm{V}$

When the circuit is not grounded



The potential of node X wrt ground cannot be determined. In practice, its value will depend on factors such as the existence of static charges and other electrical and magnetic effects.

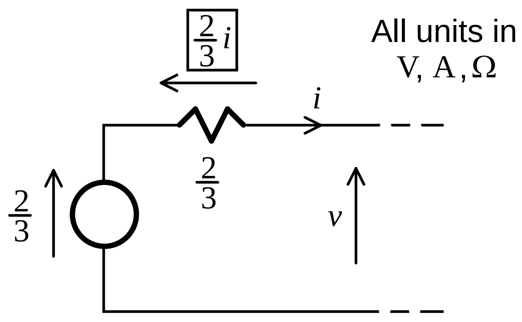
When Point X is grounded



The potential of node X wrt ground is now 0.

Q.6

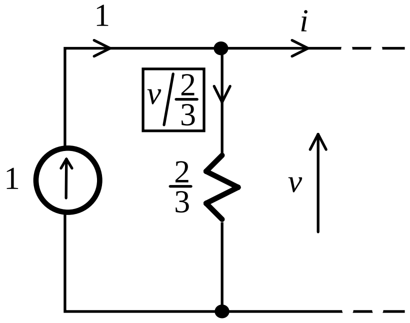
First circuit



Applying KVL:

$$\frac{2}{3}=v+\frac{2i}{3}$$

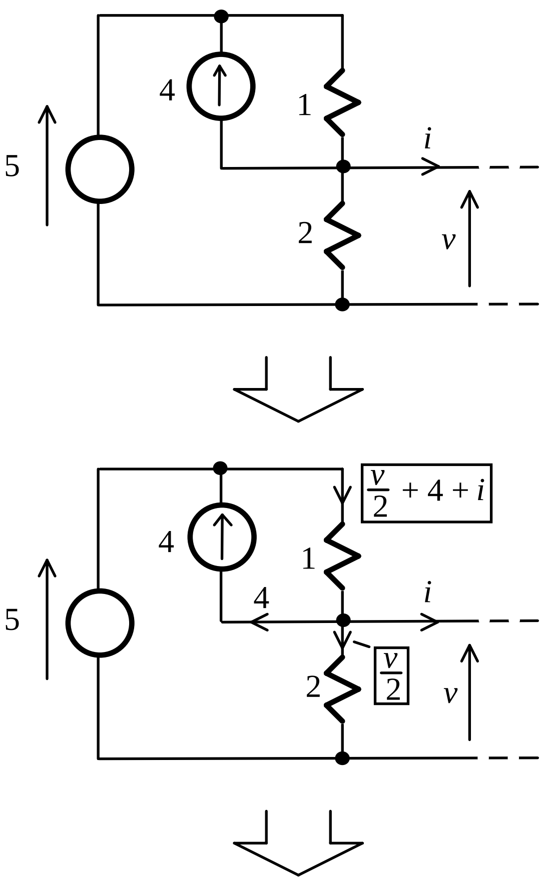
Second circuit

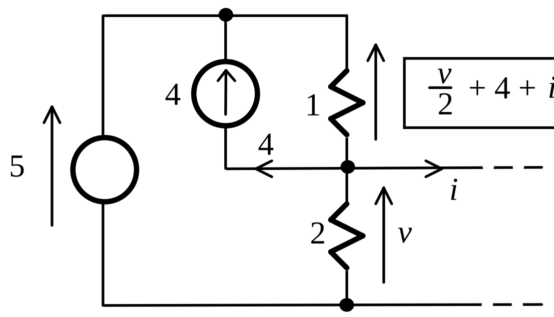


Applying KCL:

$$1=\frac{v}{2/3}+i \;\Rightarrow\; \frac{2}{3}=v+\frac{2i}{3}$$

Third circuit





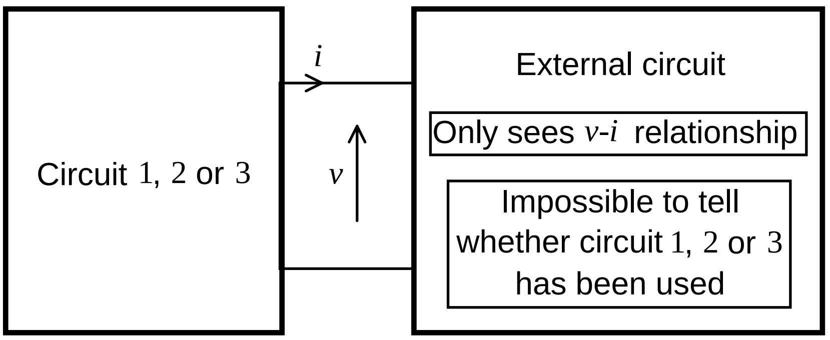
From KVL:

$$5 = \frac{v}{2} + 4 + i + v = \frac{3v}{2} + 4 + i$$

$$1 = \frac{3v}{2} + i \Rightarrow \frac{2}{3} = v + \frac{2i}{3}$$

Equivalence

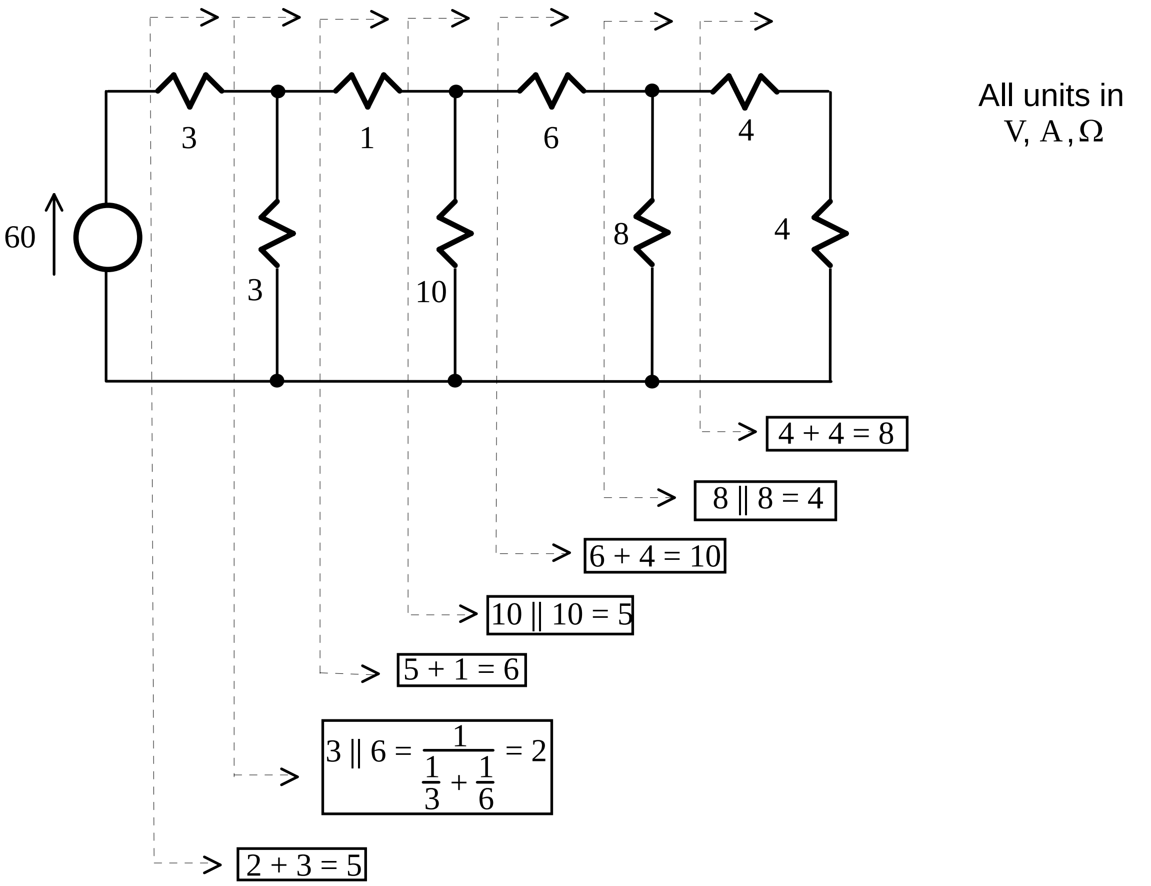
The three circuits are different in circuit topology and the components used. However, they have the same voltage-current relationship and are electrically equivalent from a voltage-current point of view. It is impossible for an external circuit connected to the outputs of these circuits to tell which circuit has actually been used:



F.3 DC Circuit Analysis I

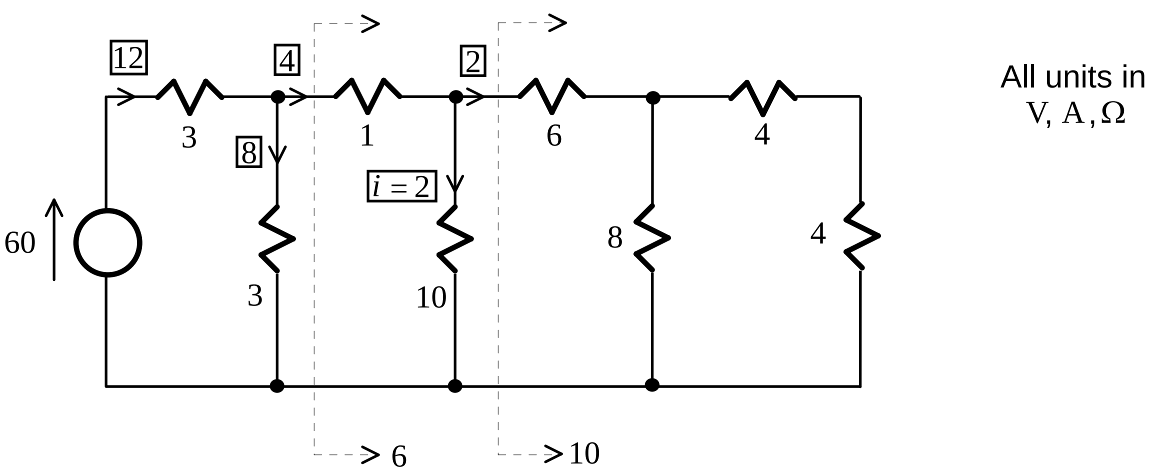
Q.1

Source current



Source current $= \dfrac{60}{5} = 12\,\text{A}$

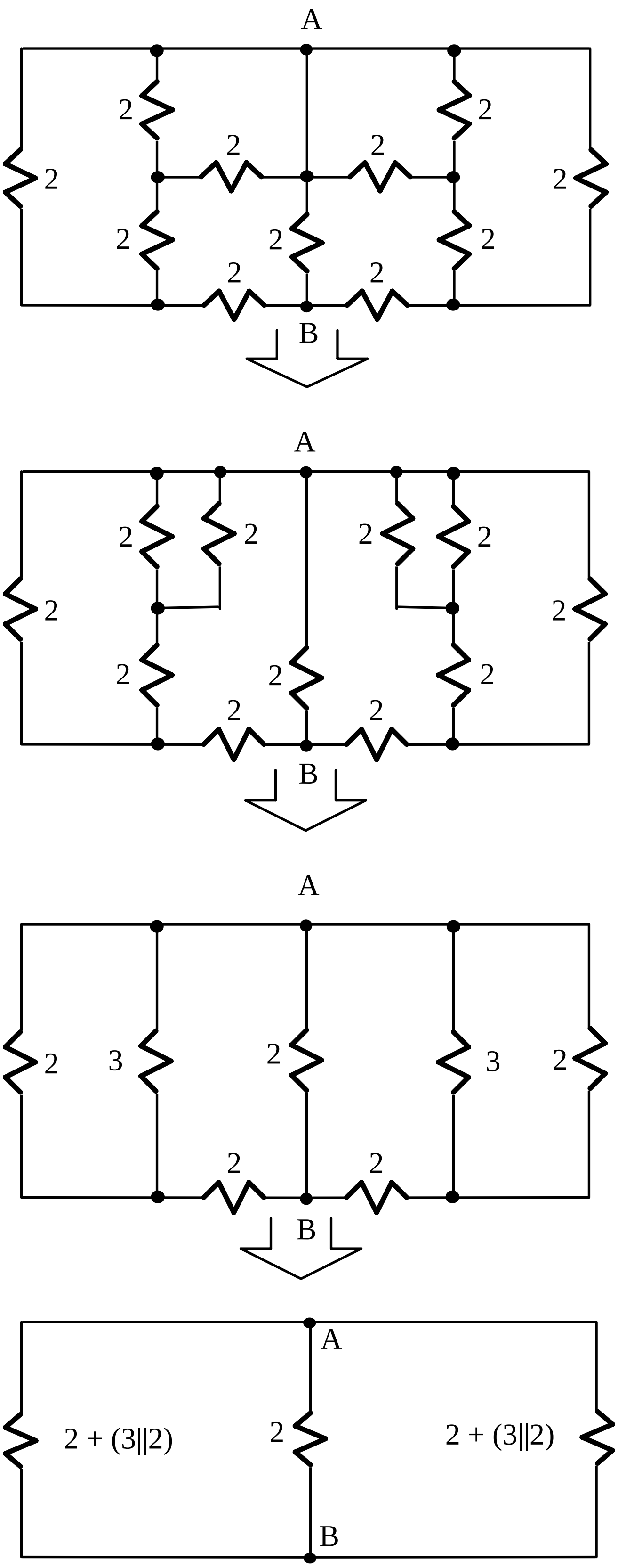
Value for i



Q.2

Original circuit

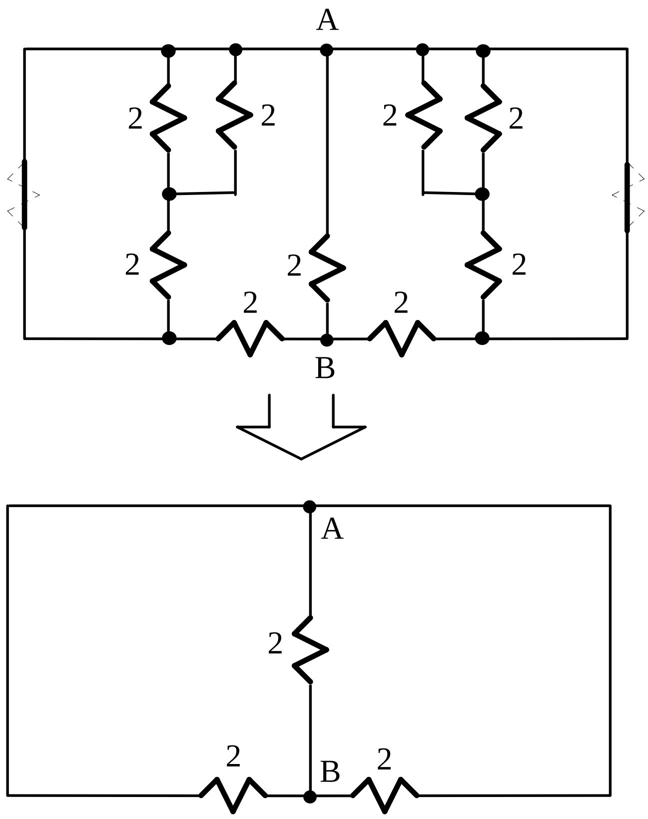
All units in V, A, Ω



$$2+(3\|2)=2+\frac{1}{\frac{1}{3}+\frac{1}{2}}=2+\frac{6}{5}=3.2$$

Equivalent resistance $=2\,||\,3.2\,||\,3.2=2\,||\,1.6=\dfrac{1}{0.5+0.625}=0.889\,\Omega$

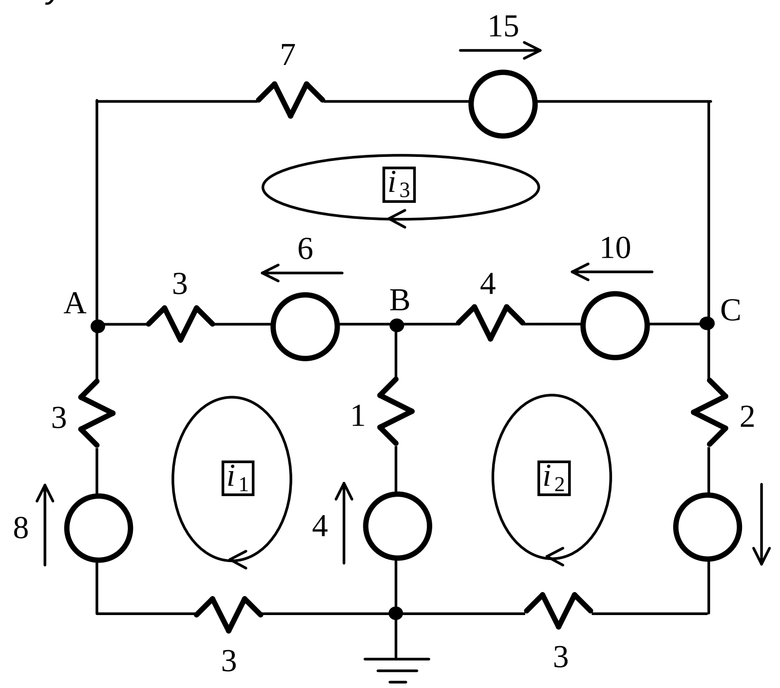
When outer two resistors are short-circuited



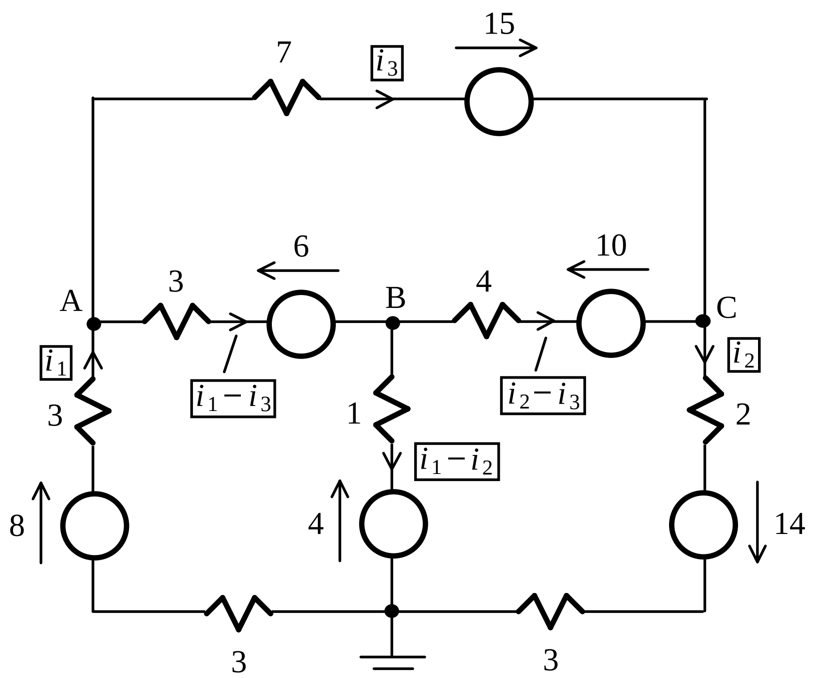
Equivalent resistance $=2\|\,2\,||\,2=\dfrac{2}{3}=0.666\,\Omega$

Q.3

Mesh analysis



All units in
V, A, Ω



Applying KVL for the three loops shown:

$$8-3i_1-3(i_1-i_3)-6-(i_1-i_2)-4-3i_1=0$$
$$14-3i_2+4-(i_2-i_1)-4(i_2-i_3)-10-2i_2=0$$
$$15+10-4(i_3-i_2)+6-3(i_3-i_1)-7i_3=0$$

Simplifying:

$$-2 = 10i_1 - i_2 - 3i_3$$

$$8 = -i_1 + 10i_2 - 4i_3$$

$$31 = -3i_1 - 4i_2 + 14i_3$$

In matrix form:

$$\begin{bmatrix} 10 & -1 & -3 \\ -1 & 10 & -4 \\ -3 & -4 & 14 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \\ 31 \end{bmatrix}$$

Solving (actually not required in this question):

$$-2 + 10(8) = 10i_1 - i_2 - 3i_3 + 10(-i_1 + 10i_2 - 4i_3) \quad \Rightarrow \quad 78 = 99i_2 - 43i_3$$

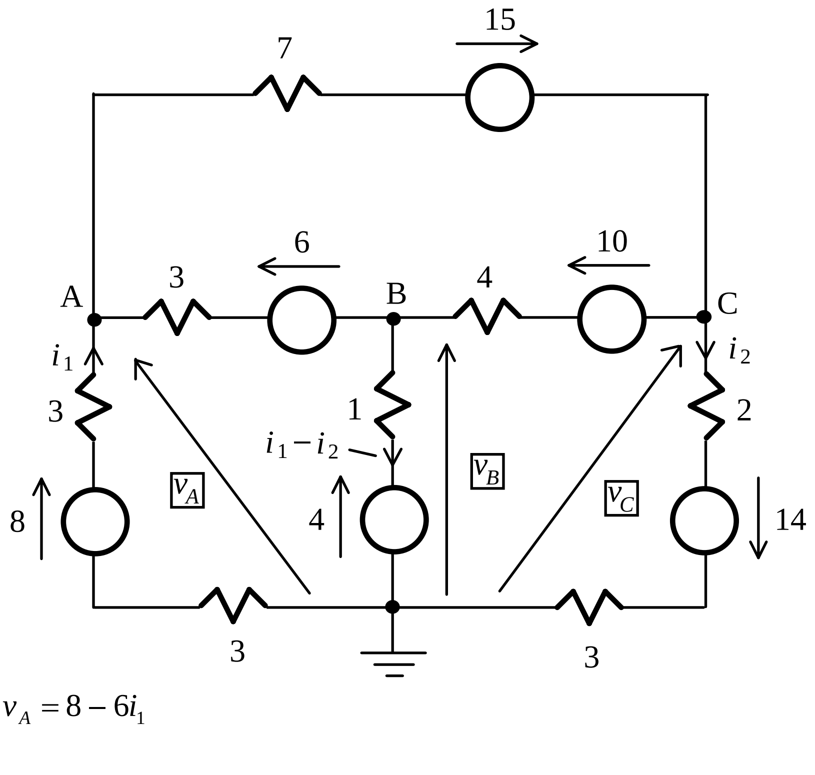
$$31 - 3(8) = -3i_1 - 4i_2 + 14i_3 - 3(-i_1 + 10i_2 - 4i_3) \;\Rightarrow\; 7 = -34i_2 + 26i_3$$

$$26(78) + 43(7) = 26(99i_2 - 43i_3) + 43(-34i_2 + 26i_3) \;\Rightarrow\; i_2 = \frac{26(78) + 43(7)}{26(99) - 43(34)} = \frac{2329}{1112}$$

$$34(78) + 99(7) = 34(99i_2 - 43i_3) + 99(-34i_2 + 26i_3) \;\Rightarrow\; i_3 = \frac{34(78) + 99(7)}{-34(43) + 99(26)} = \frac{3345}{1112}$$

$$i_1 = -8 + 10i_2 - 4i_3 = -8 + \frac{23290}{1112} - \frac{4(3345)}{1112} = \frac{1014}{1112}$$

Voltages of nodes A, B and C with respect to ground:

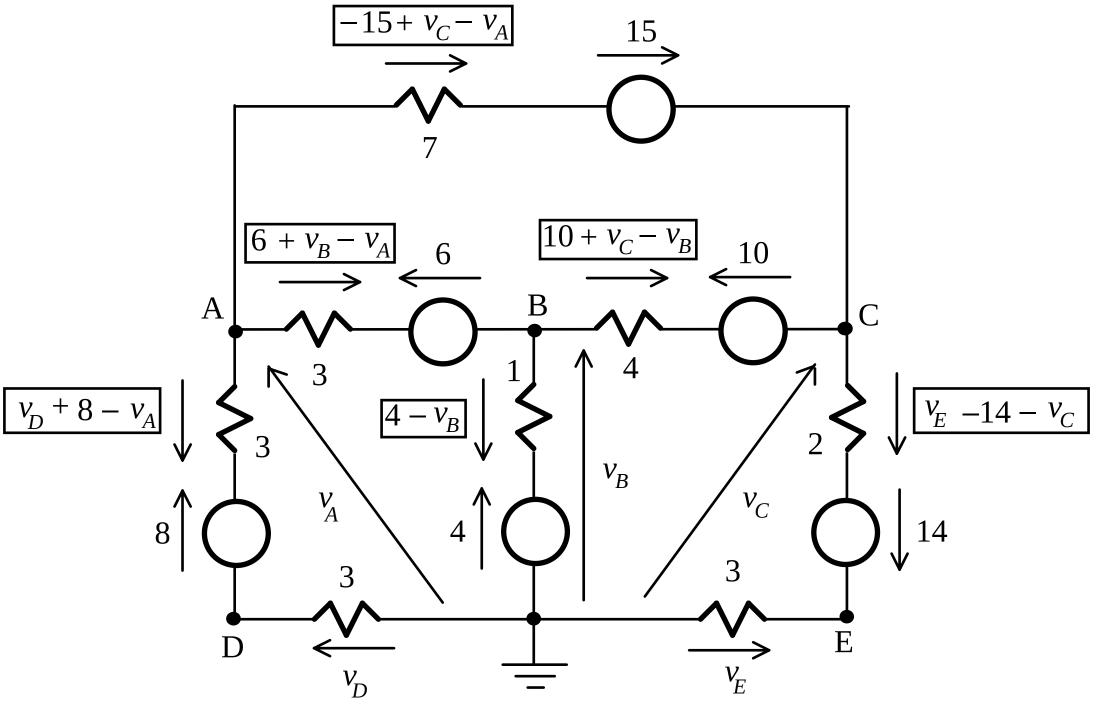


$$v_A = 8 - 6i_1$$

$$v_B = 4 + i_1 - i_2$$

$$v_C = -14 + 5i_2$$

Nodal analysis



Applying KCL to nodes A, B, C, D and E:

$$\frac{v_D + 8 - v_A}{3} + \frac{6 + v_B - v_A}{3} + \frac{v_C - v_A - 15}{7} = 0$$

$$\frac{4 - v_B}{1} + \frac{v_A - v_B - 6}{3} + \frac{v_C - v_B + 10}{4} = 0$$

$$\frac{v_E - 14 - v_C}{2} + \frac{v_A - v_C + 15}{7} + \frac{v_B - v_C - 10}{4} = 0$$

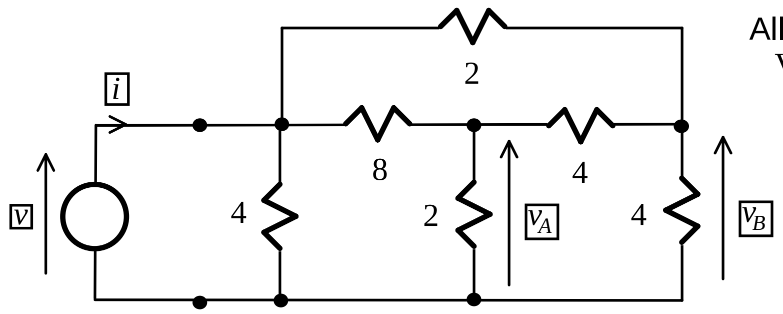
$$\frac{v_D + 8 - v_A}{3} + \frac{v_D}{3} = 0$$

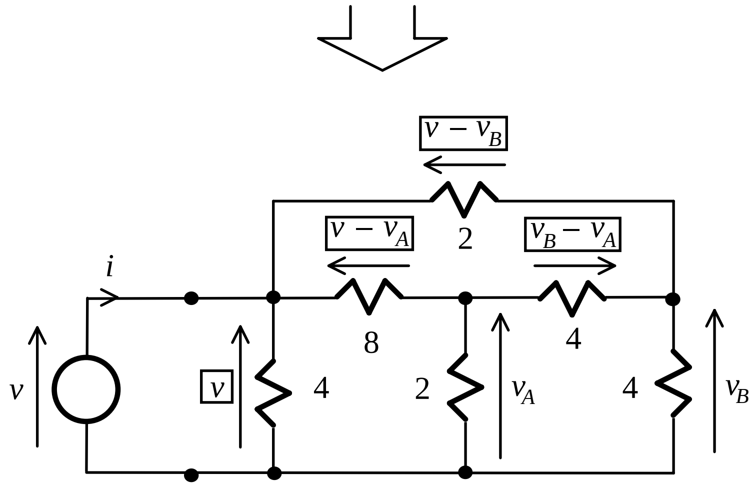
$$\frac{v_E - 14 - v_C}{2} + \frac{v_E}{3} = 0$$

In matrix form:

$$\begin{bmatrix} 1/3+1/3+1/7 & -1/3 & -1/7 & -1/7 & 0 \\ -1/3 & 1+1/3+1/4 & -1/4 & 0 & 0 \\ -1/7 & -1/4 & 1/2+1/7+1/4 & 0 & -1/2 \\ `/3 & 0 & 0 & -1/3-1/3 & 0 \\ 0 & 0 & 1/2 & 0 & -1/2-1/3 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \end{bmatrix} = \begin{bmatrix} 8/3+6/3-15/7 \\ 1/4-2+10/4 \\ -7+15/7-10/4 \\ 8/3 \\ -14/2 \end{bmatrix}$$

Q.4





Applying KCL:

$$i = \frac{v}{4} + \frac{v - v_A}{8} + \frac{v - v_B}{2}$$

$$\frac{v_A}{2} + \frac{v_A - v}{8} + \frac{v_A - v_B}{4} = 0 \;\Rightarrow\; 4v_A + v_A - v + 2v_A - 2v_B = 7v_A - 2v_B - v = 0$$

$$\frac{v_B}{4} + \frac{v_B - v_A}{4} + \frac{v_B - v}{2} = 0 \;\Rightarrow\; v_B + v_B - v_A + 2v_B - 2v = -v_A + 4v_B - 2v = 0$$

Eliminating v_A and v_B:

$$\left(7v_A - 2v_B - v\right) + 7\left(-v_A + 4v_B - 2v\right) = 0 \;\Rightarrow\; 26v_B - 15v = 0 \;\Rightarrow\; v_B = \frac{15}{26}v$$

$$2\left(7v_A - 2v_B - v\right) + \left(-v_A + 4v_B - 2v\right) = 0 \;\Rightarrow\; 13v_A - 4v = 0 \;\Rightarrow\; v_A = \frac{4}{13}v$$

$$i = \frac{v}{4} + \frac{v - v_A}{8} + \frac{v - v_B}{2} = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right)v - \frac{1}{8}\left(\frac{4}{13}\right)v - \frac{1}{2}\left(\frac{15}{26}\right)v = \frac{57}{104}v$$

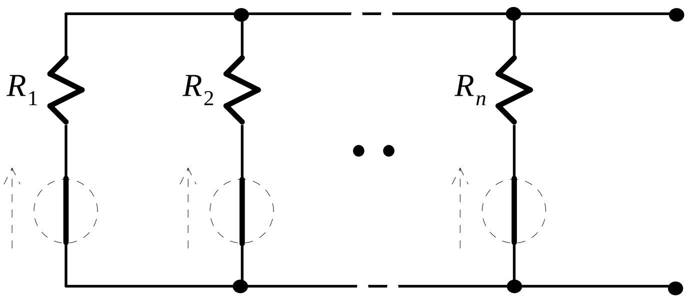
The equivalent resistance without the 13Ω resistor is therefore

$$\frac{v}{i} = \frac{104}{57} = 1.82\,\Omega$$

and the equivalent resistance with the 13Ω series resistor $= 14.82\,\Omega$

Q.5

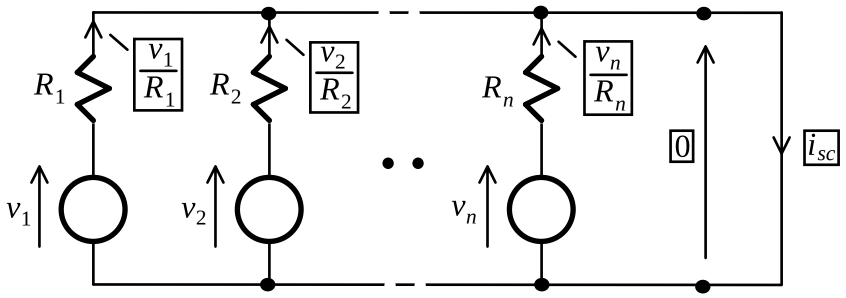
Equivalent resistance



Since the resistors are in parallel, the equivalent resistance R is

$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

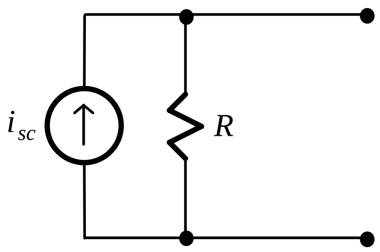
Short circuit current



Applying KCL:

$$i_{sc} = \frac{v_1}{R_1} + \cdots + \frac{v_n}{R_n}$$

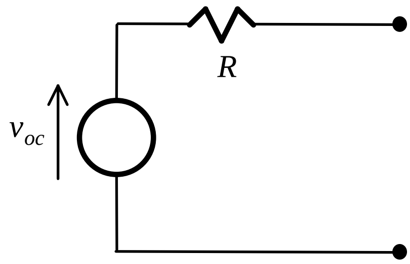
Norton's equivalent circuit



$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

$$i_{sc} = \frac{v_1}{R_1} + \cdots + \frac{v_n}{R_n}$$

Thevenin's equivalent circuit

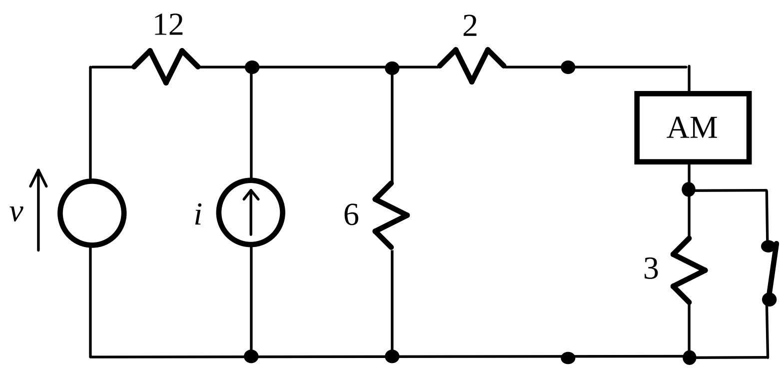


$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

$$v_{oc} = Ri_{sc} = \frac{\frac{v_1}{R_1} + \cdots + \frac{v_n}{R_n}}{\frac{1}{R_1} + \cdots + \frac{1}{R_n}}$$

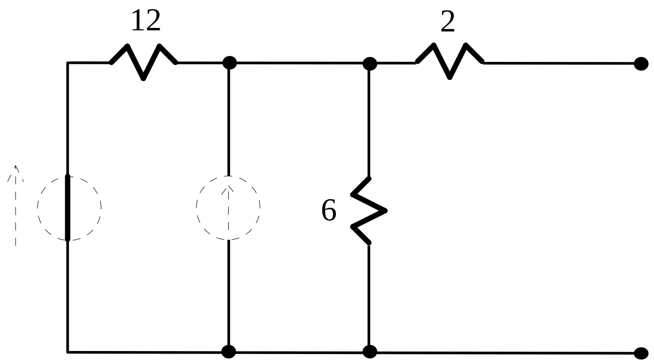
Q.6

Re-drawing original circuit



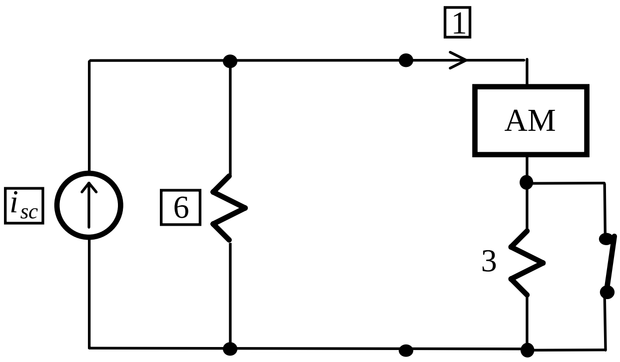
All units in V, A, Ω

Equivalent resistance without 3Ω resistor



Equivalent resistance $R = 2 + (6||12) = 2 + \frac{1}{\frac{1}{6}+\frac{1}{12}} = 2 + 4 = 6\Omega$

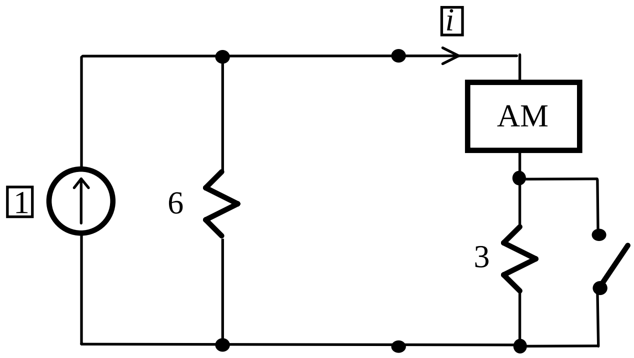
Using Norton's equivalent circuit



Since AM reads 1A

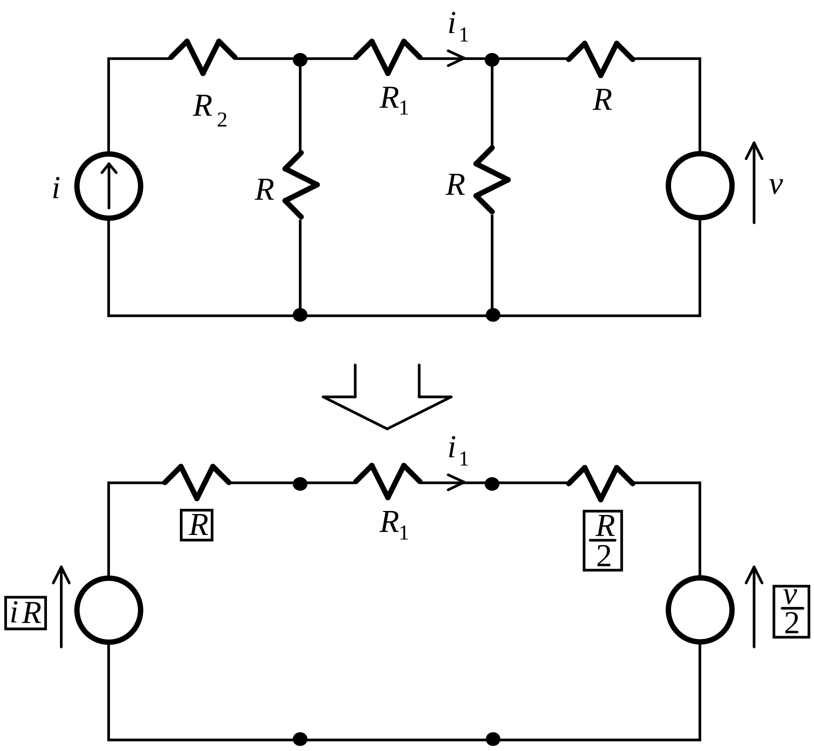
$i_{sc} = 1\text{A}$

Thus, when the switch is open



$$i = 1 \times \left(\frac{6}{6+3} \right) = \frac{2}{3} \mathrm{A}$$

Q.7

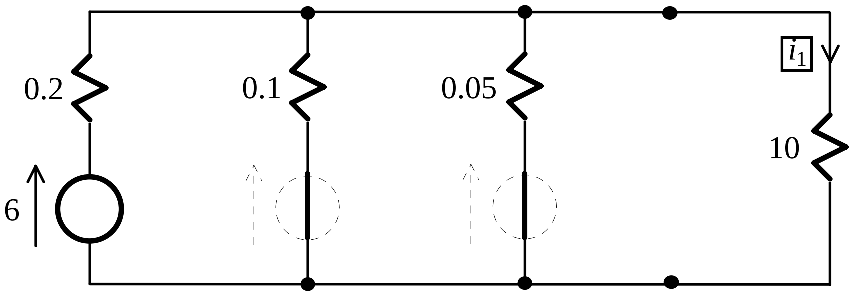


$$i_1 = \frac{iR - \frac{v}{2}}{\frac{3R}{2} + R_1} = \frac{2iR - v}{3R + 2R_1}$$

F.4 DC Circuit Analysis II

Q.1

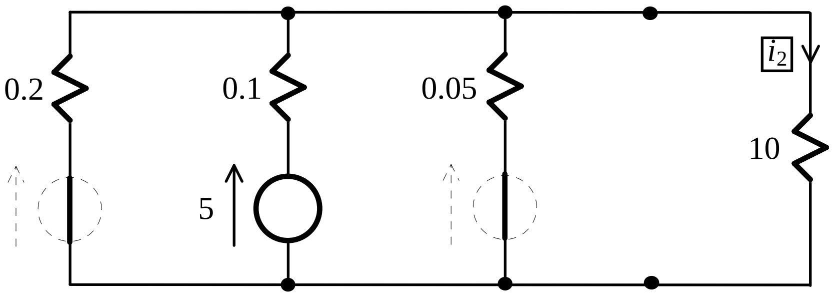
Load current due to Battery 1



$$\text{Current from source} = \frac{6}{0.2 + 0.1||0.05||10} = \frac{6}{0.2 + \frac{1}{10+20+0.1}} = \frac{6}{0.2 + 0.033} = 25.75$$

$$i_1 = 25.75 \times \left(\frac{0.1||0.05}{10 + 0.1||0.05} \right) = 25.75 \times \left(\frac{\frac{1}{10+20}}{10 + \frac{1}{10+20}} \right) = 25.75 \times \left(\frac{0.033}{10.033} \right) = 0.0847\,\text{A}$$

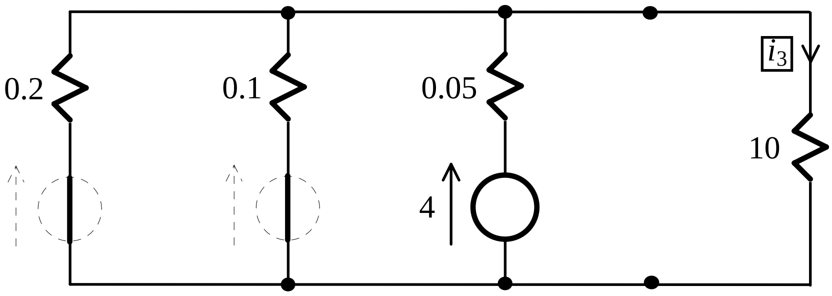
Load current due to Battery 2



$$\text{Current from source} = \frac{5}{0.1 + 0.2||0.05||10} = \frac{5}{0.1 + \frac{1}{5+20+0.1}} = \frac{5}{0.1 + 0.04} = 35.71$$

$$i_2 = 35.71 \times \left(\frac{0.2||0.05}{10 + 0.2||0.05} \right) = 35.71 \times \left(\frac{\frac{1}{5+20}}{10 + \frac{1}{5+20}} \right) = 35.71 \times \left(\frac{0.04}{10.04} \right) = 0.1423\,\text{A}$$

Load current due to Battery 3



$$\text{Current from source} = \frac{4}{0.05 + 0.2||0.1||10} = \frac{4}{0.05 + \frac{1}{5+10+0.1}} = \frac{4}{0.05 + 0.066} = 34.48$$

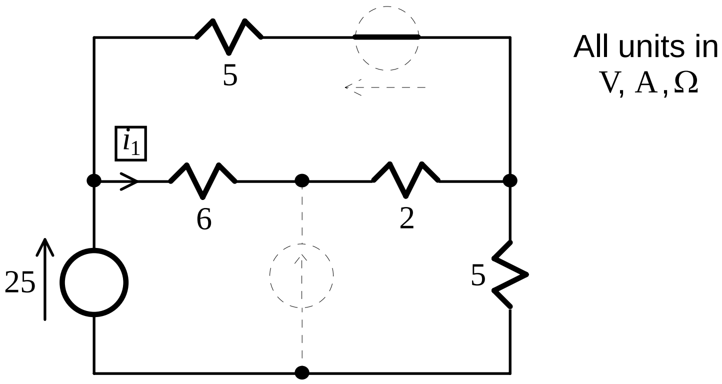
$$i_3 = 34.48 \times \left(\frac{0.2||0.1}{10 + 0.2||0.1} \right) = 34.48 \times \left(\frac{\frac{1}{5+10}}{10 + \frac{1}{5+10}} \right) = 34.48 \times \left(\frac{0.067}{10.07} \right) = 0.2294\,\text{A}$$

Actual load current

$$i = i_1 + i_2 + i_3 = 0.0847 + 0.1423 + 0.2294 = 0.4564\,\text{A}$$

Q.2

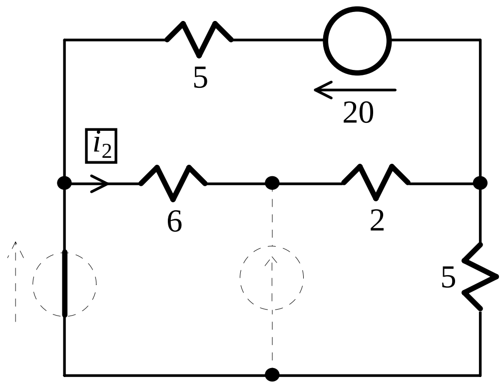
Current due to 25V *voltage source*



$$\text{Current from source} = \frac{25}{5 + 5||(6+2)} = \frac{25}{5 + \frac{1}{\frac{1}{5} + \frac{1}{8}}} = \frac{25}{5 + 3.08} = 3.09$$

$$i_1 = 3.09\left(\frac{5}{5+8} \right) = 1.19\,\text{A}$$

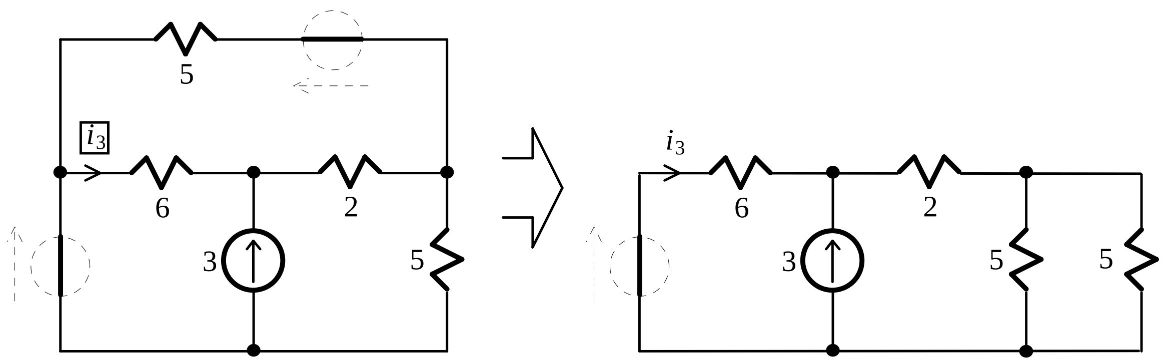
Current due to $20\,\mathrm{V}$ *voltage source*



Current from source $= \dfrac{20}{5+5\|(6+2)} = \dfrac{20}{5+\dfrac{1}{\frac{1}{5}+\frac{1}{8}}} = \dfrac{20}{5+3.08} = 2.48$

$$i_2 = 2.48\left(\frac{5}{5+8}\right) = 0.954\,\mathrm{A}$$

Current due to current source



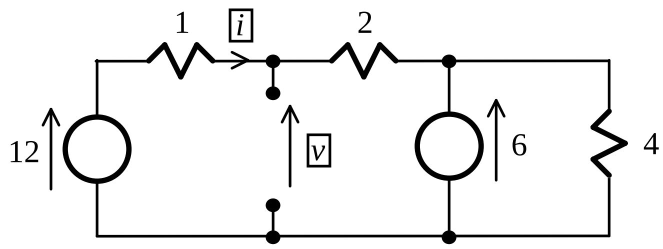
$$i_3 = -3\left[\frac{2+(5\|5)}{6+2+(5\|5)}\right] = -3\left[\frac{2+\dfrac{1}{0.2+0.2}}{8+\dfrac{1}{0.2+0.2}}\right] = -1.286\mathrm{A}$$

Actual current

$$i = i_1 + i_2 + i_3 = 1.19 + 0.954 - 1.286 = 0.858\,\mathrm{A}$$

Q.3

Open circuit voltage



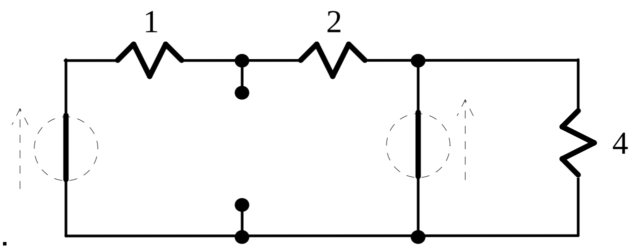
All units in V, A, Ω

.

$$i = \frac{12-6}{1+2} = 2$$

$$v = 12 - i = 10\,\mathrm{V}$$

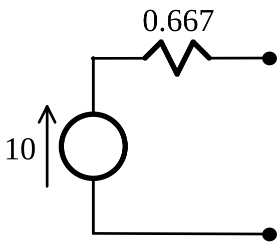
Equivalent resistance



.

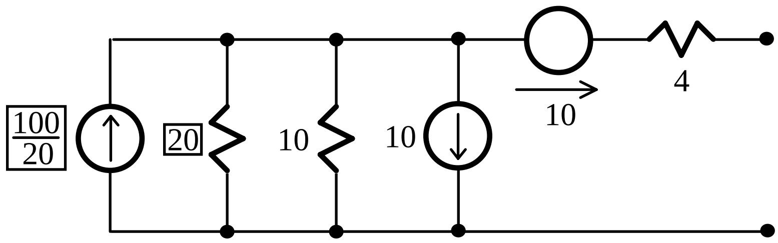
Resistance across terminals $= 1 \| 2 = \dfrac{1}{1+\dfrac{1}{2}} = 0.667\,\Omega$

Thevenin's equivalent circuit and maximum power



Maximum power transferable (with a $0.667\,\Omega$ load) $= \left(\dfrac{10}{2 \times 0.667}\right)^2 0.667 = 37.5\,\mathrm{W}$

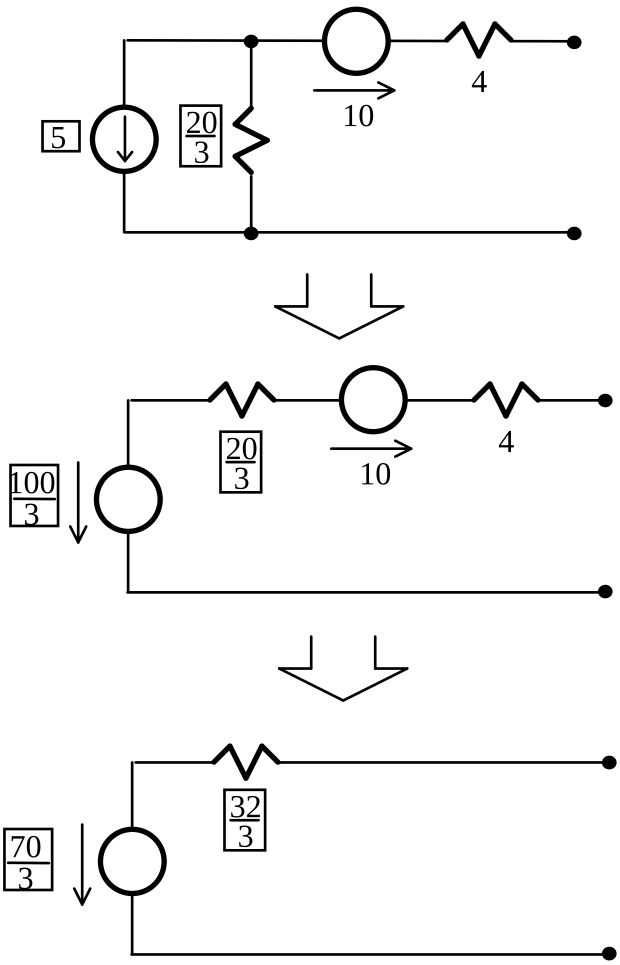
Q.4



All units in
V, A, Ω

Combined current of current sources $= 10 - \dfrac{100}{20} = 5$

Equivalent parallel resistance $= 20 \| 10 = \dfrac{1}{\dfrac{1}{20} + \dfrac{1}{10}} = \dfrac{20}{3}$



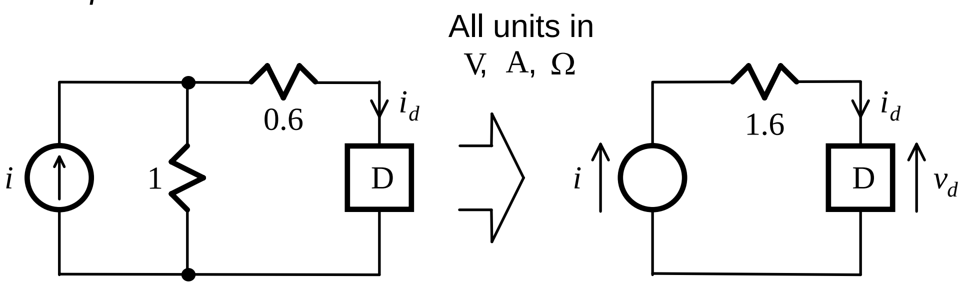
Resistor that draws $2\,\text{A} = \dfrac{70}{3 \times 2} - \dfrac{32}{3} = 1\Omega$

Resistor that absorbs the maximum power $= \dfrac{32}{3}\Omega$

Maximum power that can be transferred $= \left(\dfrac{70}{3} \Big/ \dfrac{32 \times 2}{3} \right)^2 \dfrac{32}{3} = \dfrac{1225}{96}\,\text{W}$

Q.5

Thevenin's equivalent circuit

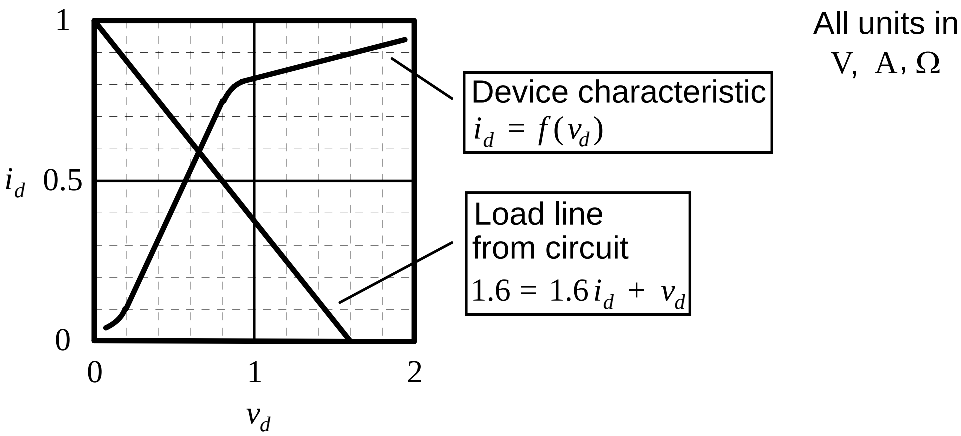


Device current given $i = 1.6\,\mathrm{A}$

Applying KVL:

$$i = 1.6 i_d + v_d = 1.6$$

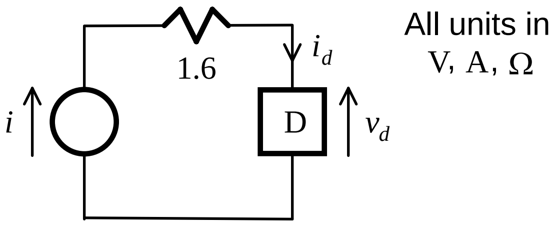
v_d and i_d can be found from solving this (which gives rise to the load line) and the relationship $i_d = f(v_d)$ given by the characteristic curve. Specifically, when $i_d = 0$, $v_d = 1.6$. Also, when $v_d = 0$, $i_d = 1$.



The point of intersection gives

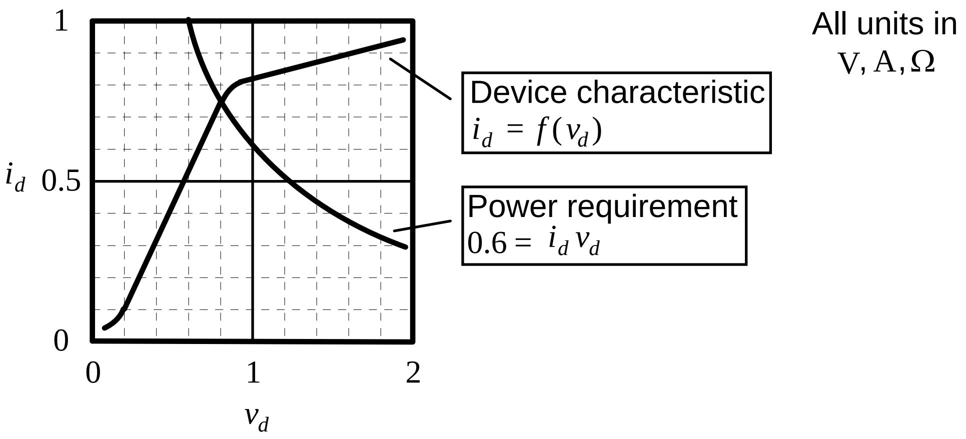
$$i_d = 0.6\,\mathrm{A}$$

Source current for power dissipated in D *to be* $0.6\,\mathrm{W}$



Power dissipated in $\mathrm{D} = i_d v_d = 0.6$

The device voltage and current can be found from solving this and the relationship $i_d = f(v_d)$ given by the characteristic curve:



The point of intersection gives

$$v_d \approx 0.8\,\mathrm{V}$$

$$i_d \approx 0.75\,\mathrm{A}$$

From KVL:

$$i = 1.6 i_d + v_d \approx 1.6(0.75) + 0.8 = 2\,\mathrm{A}$$

Q.6

Voltage gain

Applying KCL to the second half of the circuit:

$$v_2 = -20(50 i_1) = -1000 i_1$$

Applying KVL to the first half of the circuit:

$$v_1 = 2 i_1 + \frac{v_2}{5000}$$

Eliminating i_1:

$$v_1 = -2\left(\frac{v_2}{1000}\right) + \frac{v_2}{5000} = \frac{-1.8 v_2}{1000} \quad \Rightarrow \quad v_2 = -\frac{1000}{1.8} v_1$$

The voltage gain is

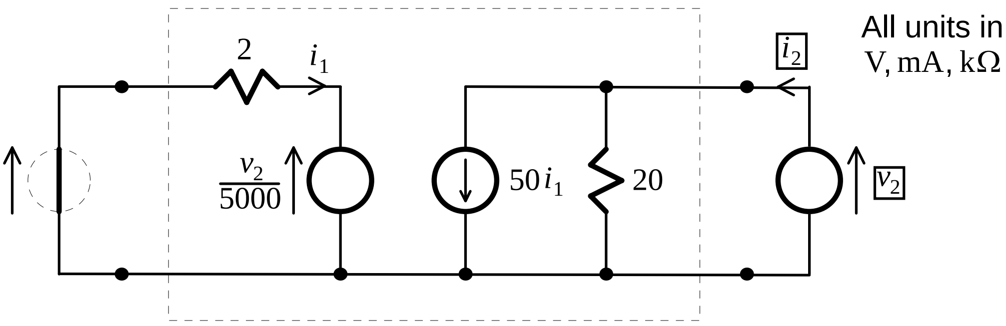
$$\frac{v_2}{v_1} = -\frac{1000}{1.8}$$

The gain in voltage magnitudes is

$$\left|\frac{v_2}{v_1}\right| = \frac{1000}{1.8} = 20\log\left(\frac{1000}{1.8}\right)\text{dB} = 55\text{dB}$$

Equivalent resistance

To determine the equivalent resistance as seen from the output terminals, all independent sources have to be replaced by their internal resistances and a voltage source has to be applied to these two terminals:



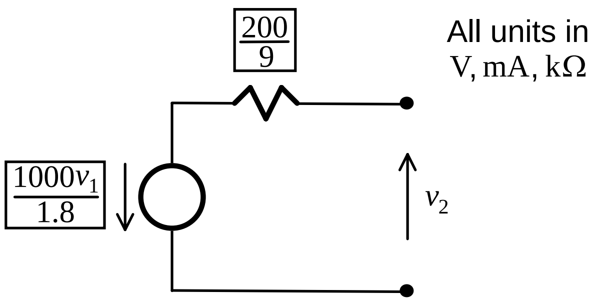
$$-2i_1 = \frac{v_2}{5000}$$

$$i_2 = 50i_1 + \frac{v_2}{20} = -50\left(\frac{v_2}{10000}\right) + \frac{v_2}{20} = \frac{9v_2}{200}$$

Equivalent resistance $= \dfrac{v_2}{i_2} = \dfrac{200}{9}\,\text{k}\Omega$

Note that in calculating this resistance or in using superposition, dependent sources must not be replaced by their internal resistances.

Thevenin's equivalent circuit



F.5 AC Circuit Analysis I

Q.1

	(a)	(b)
AC waveform	$5\sqrt{2}\sin(\omega t)$ $=5\sqrt{2}\cos\left(\omega t-\frac{\pi}{2}\right)$ $=\mathrm{Re}\left[\left(5e^{-j\pi/2}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$	$5\sqrt{2}\cos(\omega t)$ $=\mathrm{Re}\left[\left(5e^{j0}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$
Peak value	$5\sqrt{2}$	$5\sqrt{2}$
Frequency	$\omega\,\mathrm{rad/s}=\frac{\omega}{2\pi}\,\mathrm{Hz}$	$\omega\,\mathrm{rad/s}=\frac{\omega}{2\pi}\,\mathrm{Hz}$
RMS value	5	5
Phase	$-\frac{\pi}{2}=-90^0$	0
Phasor	$5\ e^{-j\pi/2}=5\ \underline{/-90^o}$	$5\ e^{j0}=5\ \underline{/0^o}=5$

	(c)	(d)
AC waveform	$10\sqrt{2}\sin\left(20t+30^0\right)$ $=10\sqrt{2}\cos\left(20t-60^0\right)$ $=\mathrm{Re}\left[\left(10e^{-j\pi/3}\right)\left(\sqrt{2}e^{j20t}\right)\right]$	$120\sqrt{2}\cos\left(314t-45^0\right)$ $=\mathrm{Re}\left[\left(120e^{-j\pi/4}\right)\left(\sqrt{2}e^{j314t}\right)\right]$
Peak value	$10\sqrt{2}$	$120\sqrt{2}$
Frequency	$20\,\mathrm{rad/s}=3.18\,\mathrm{Hz}$	$314\,\mathrm{rad/s}=50\,\mathrm{Hz}$
RMS value	10	120
Phase	$-\frac{\pi}{3}=-60^0$	$-\frac{\pi}{4}=-45^0$
Phasor	$10\,e^{-j\pi/3}=10\underline{/-60^o}$	$120e^{-j\pi/4}=120\underline{/-45^o}$

	(e)	(f)
AC waveform	$-50\sin\left(4t-\frac{\pi}{3}\right)$ $=35.4\sqrt{2}\cos\left(4t-\frac{\pi}{3}-\frac{\pi}{2}+\pi\right)$ $=\mathrm{Re}\left[\left(35.4e^{j\pi/6}\right)\left(\sqrt{2}e^{j4t}\right)\right]$	$0.25\cos\left(2t+100^0\right)$ $=0.177\sqrt{2}\cos\left(2t+1.75\right)$ $=\mathrm{Re}\left[\left(0.177e^{j1.75}\right)\left(\sqrt{2}e^{j2t}\right)\right]$
Peak value	50	0.25
Frequency	$4\text{ rad/s}=0.637\text{ Hz}$	$2\text{ rad/s}=0.318\text{ Hz}$
RMS value	35.4	0.177
Phase	$\frac{\pi}{6}=30^0$	$1.75=100^0$
Phasor	$35.4e^{j\pi/6}=35.4\angle 30^o$	$0.177e^{j1.75}=0.177\angle 100^o$

Q.2

(a)	$\frac{100}{\sqrt{2}}e^{j30^0}\text{ V}$	$\mathrm{Re}\left[\left(\frac{100}{\sqrt{2}}e^{j\pi/6}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=100\cos\left(314t+\frac{\pi}{6}\right)\text{V}$
(b)	$115e^{j\pi/3}\text{ V}$	$\mathrm{Re}\left[\left(115e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=115\sqrt{2}\cos\left(314t+\frac{\pi}{3}\right)\text{V}$
(c)	$-0.12e^{-j\pi/4}\text{ A}$	$\mathrm{Re}\left[\left(-0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=\mathrm{Re}\left[\left(e^{j\pi}0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $=0.12\sqrt{2}\cos\left(314t+\frac{3\pi}{4}\right)\text{A}$
(d)	$-0.69\angle 60^o\text{ A}$	$\mathrm{Re}\left[\left(-0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=\mathrm{Re}\left[\left(e^{j\pi}0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $=0.69\sqrt{2}\cos\left(314t+\frac{4\pi}{3}\right)$ $=0.69\sqrt{2}\cos\left(314t-\frac{2\pi}{3}\right)\text{A}$

Q.3

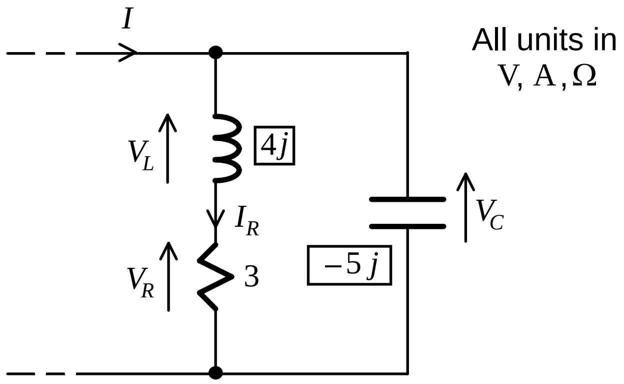
From $v_R(t) = 12\sqrt{2}\cos(2t)$ V

Frequency $= \omega = 2\,\text{rad/s}$

Impedance of capacitor $= \dfrac{1}{j\omega 0.1} = \dfrac{1}{j0.2} = -5j\,\Omega$

Impedance of inductor $= j\omega 2 = 4j\,\Omega$

$V_R = 12e^{j0} = 12$

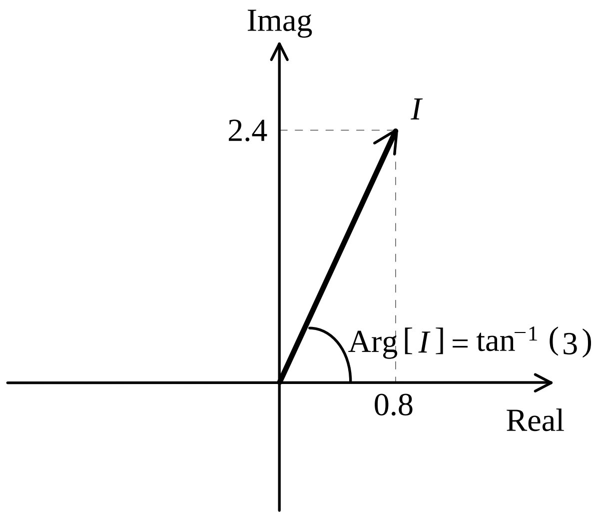


$$I_R = \frac{V_R}{3} = 4 \;\Rightarrow\; i_R(t) = 4\sqrt{2}\cos(2t)\,\text{A}$$

$$V_L = (4j)I_R = 16j\,\text{V} \;\Rightarrow\; v_L(t) = 16\sqrt{2}\cos\left(2t + \frac{\pi}{2}\right)\text{V}$$

$$V_C = V_R + V_L = 12 + 16j$$

$$I = I_R + \frac{V_C}{-5j} = 4 - \frac{12+16j}{5j} = 4 + 2.4j - 3.2 = 0.8 + 2.4j$$



$$|I| = |0.8 + 2.4j| = \sqrt{0.8^2 + 2.4^2} = 2.53$$

$$\text{Arg}[I] = \text{Arg}\left[0.8 + 2.4j\right] = \tan^{-1}\left(\frac{2.4}{0.8}\right) = 1.25$$

$$I = 0.8 + 2.4j = 2.53e^{j1.25} \;\Rightarrow\; 2.53\sqrt{2}\cos\left(2t + 1.25\right)\text{ A}$$

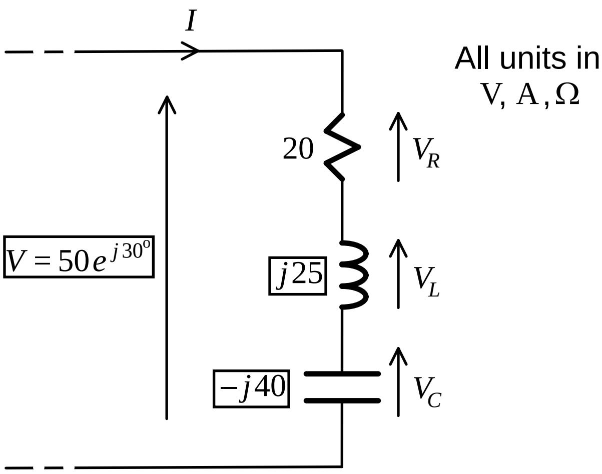
Q.4

Impedances and phasors

$v(t) = 50\sqrt{2}\cos\left(1250t + 30^0\right)\text{ V} \;\Rightarrow\; V = 50e^{j30^0}\text{ V}$.

Impedance of inductor $= j(1250)(0.02) = j25\Omega$

Impedance of capacitor $= \dfrac{1}{j(1250)(0.00002)} = -j40\Omega$



Total impedance $= 20 + j25 - j40 = 20 - j15 = \sqrt{20^2 + 15^2}\,e^{j\tan^{-1}\left(\frac{-15}{20}\right)} = 25e^{-36.9^0}\;\Omega$

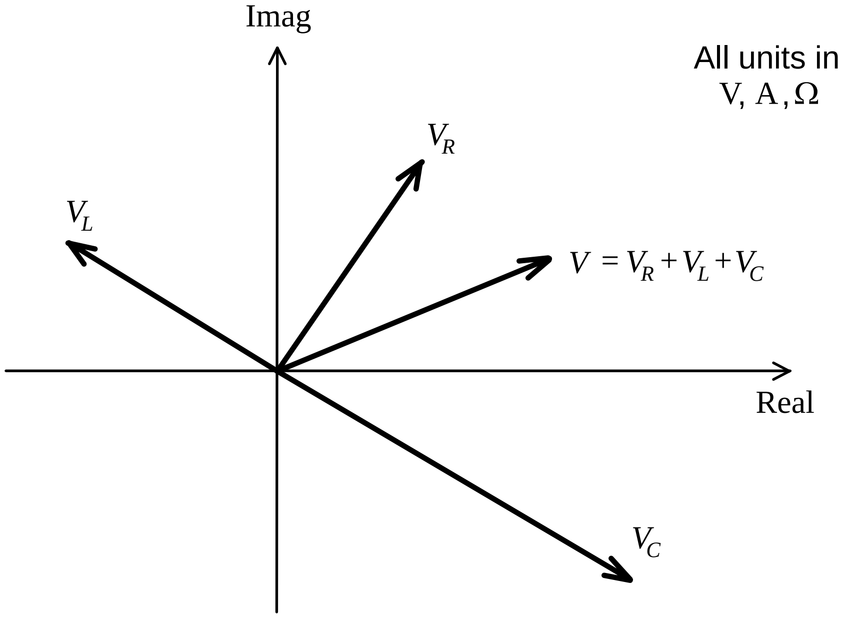
$$I = \frac{V}{20 + j25 - j40} = \frac{50e^{j30^0}}{25e^{j-36.9^0}} = 2e^{j66.9^0}\text{ A}$$

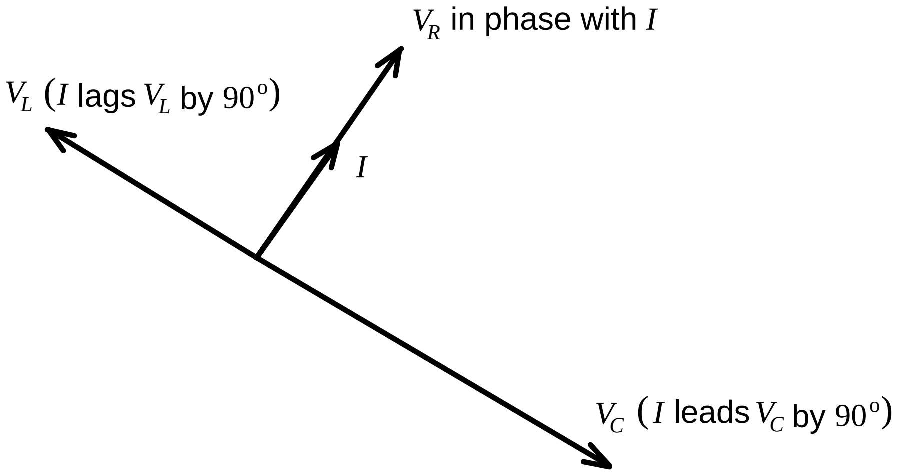
$$V_R = 20I = (20)\left(2e^{j66.9^0}\right) = 40e^{j66.9^0}\text{ V}$$

$$V_L = j25I = \left(25e^{j90^0}\right)\left(2e^{j66.9^0}\right) = 50e^{j156.9^0}\text{ V}$$

$$V_C = -j40I = \left(40e^{-j90^0}\right)\left(2e^{j66.9^0}\right) = 80e^{-j23.1^0}\text{ V}$$

Phasor diagram





Q.5

Components

The impedances of series RL, RC and LC circuits are

$$Z_{RL} = R + j\omega L = R + j100\pi L$$

$$Z_{RC} = R + \frac{1}{j\omega C} = R - \frac{j}{100\pi C}$$

$$Z_{LC} = j\omega L + \frac{1}{j\omega C} = j\left(100\pi L - \frac{1}{100\pi C}\right)$$

$20 + j30$ must correspond to a series RL circuit with components:

$$R_1 + j100\pi L_1 = 20 + j30 \;\Rightarrow\; R_1 = 20\Omega \text{ and } L_1 = \frac{30}{100\pi} = 0.0955\,\text{H}$$

$10 - j15$ must correspond to a series RC circuit with components:

$$R_2 - \frac{j}{100\pi C_2} = 10 - j15 \;\Rightarrow\; R_2 = 10\Omega \text{ and } C_2 = \frac{1}{100\pi(15)} = 212.3\,\mu\text{F}$$

Circuit admittance

$$\text{Circuit impedance} = Z = (20 + j30)\,\|\,(10 - j15) = \frac{1}{\dfrac{1}{20 + j30} + \dfrac{1}{10 - j15}}$$

$$\text{Circuit admittance} = \frac{1}{Z} = \frac{1}{20 + j30} + \frac{1}{10 - j15} = \frac{20 - j30}{20^2 + 30^2} + \frac{10 + j15}{10^2 + 15^2}$$

$$= 0.0154 - j0.0231 + 0.0308 + j0.0462 = (0.0462 + j0.0231)\,\Omega^{-1}$$

Power factor

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\text{Arg}(I) - \text{Arg}(V)] \\ \text{leading / lagging} = \begin{cases} \text{leading, } \text{Arg}(I) - \text{Arg}(V) > 0 \\ \text{lagging, } \text{Arg}(I) - \text{Arg}(V) < 0 \end{cases} \end{array} \right\rangle$$

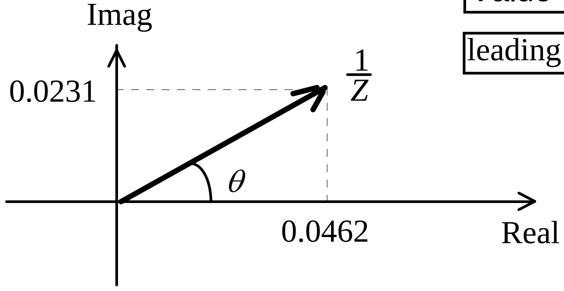
$$\text{Arg}(I) - \text{Arg}(V) = \text{Arg}\left(\frac{I}{V}\right) = \text{Arg}\left(\frac{1}{Z}\right) = -\text{Arg}(Z)$$

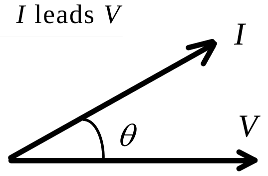
$$= \text{Arg}(0.0462 + j0.0231) = \tan^{-1}\left(\frac{0.0231}{0.0462}\right) = 0.464$$

$$\text{Power factor} \left\langle \begin{array}{c} \text{value} = \cos(0.464) = 0.894 \\ \text{leading / lagging} = \begin{cases} \text{leading, } 0.464 > 0 \\ \text{lagging, } 0.464 < 0 \end{cases} \end{array} \right\rangle = \left\langle \begin{array}{c} 0.894 \\ \text{leading} \end{array} \right\rangle$$

Value of p.f. $= \cos\theta$

leading p.f. as I leads V

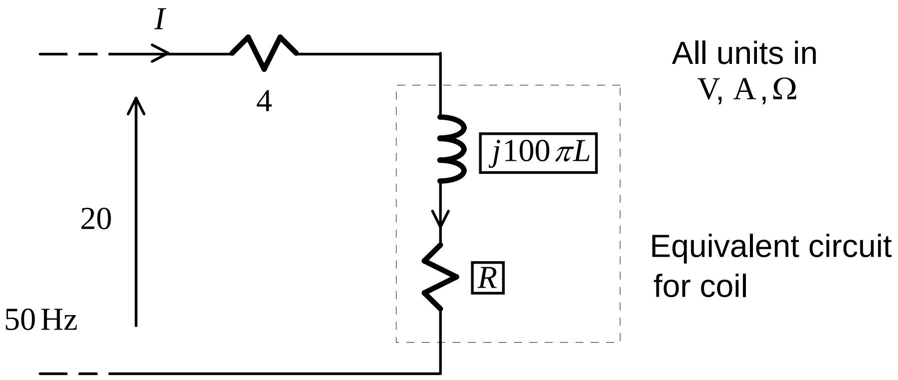




F.6 AC Circuit Analysis II

Q.1

Circuit diagram



Main equations

$$|\text{Voltage across } 4\Omega| = |4I| = 9 \Rightarrow |I| = \frac{9}{4}$$

$$|\text{Voltage across coil}| = |I(R + j100\pi L)| = 14$$

$$|R + j314L| = \frac{14}{|I|} = 14\left(\frac{4}{9}\right) = 6.222 \Rightarrow R^2 + (314L)^2 = 38.72$$

$$|\text{Supply}| = |I(4 + R + j100\pi L)| = 20$$

$$|4 + R + j314L| = \frac{20}{|I|} = 20\left(\frac{4}{9}\right) = 8.889 \Rightarrow (R+4)^2 + (314L)^2 = 79.01$$

Component values

$$\left[(R+4)^2 + (314L)^2\right] - \left[R^2 + (314L)^2\right] = 79.01 - 38.72$$

$$8R + 16 = 40.29 \Rightarrow R = \frac{40.29 - 16}{8} = 3.04\,\Omega$$

$$R^2 + (314L)^2 = 38.72 \Rightarrow (314L)^2 = 38.72 - 3.04^2 = 29.48 \Rightarrow L = \frac{\sqrt{29.48}}{314} = 0.0173\,\text{H}$$

Power and power factor

Power absorbed by coil $= |I|^2 R = \left(\frac{9}{4}\right)^2 (3.04) = 15.4\,\text{W}$

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\text{Arg}(\text{current}) - \text{Arg}(\text{voltage})] \\ \text{leading / lagging} = \begin{cases} \text{leading, Arg}(\text{current}) - \text{Arg}(\text{voltage}) > 0 \\ \text{lagging, Arg}(\text{current}) - \text{Arg}(\text{voltage}) < 0 \end{cases} \end{array} \right\rangle$$

$$\text{Arg}(\text{current}) - \text{Arg}(\text{voltage}) = \text{Arg}\left(\frac{\text{current}}{\text{voltage}}\right)$$

$$= \text{Arg}\left(\frac{1}{\text{Impedance}}\right) = -\text{Arg}(\text{Impedance})$$

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\text{Arg}(\text{impedance})] \\ \text{leading / lagging} = \begin{cases} \text{leading, Arg}(\text{impedance}) < 0 \\ \text{lagging, Arg}(\text{impedance}) > 0 \end{cases} \end{array} \right\rangle$$

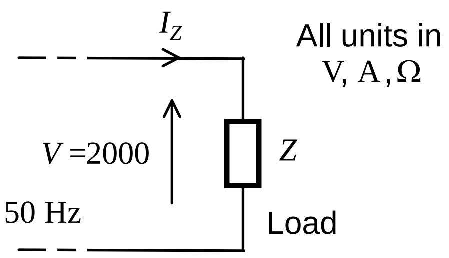
$$\text{Arg}(\text{impedance}) = \text{Arg}(R + 4 + j314L) = \text{Arg}(3.04 + 4 + j314 \times 0.0173)$$

$$= \text{Arg}(7.04 + j5.43) = \tan^{-1}\left(\frac{5.43}{7.04}\right) = 0.657$$

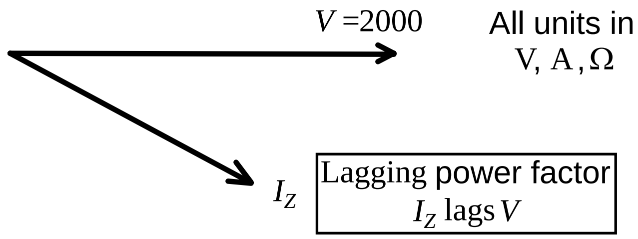
$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos(0.657) = 0.792 \\ \text{leading / lagging} = \begin{cases} \text{leading, } 0.657 < 0 \\ \text{lagging, } 0.657 > 0 \end{cases} \end{array} \right\rangle = \left\langle \begin{array}{l} 0.792 \\ \text{lagging} \end{array} \right\rangle$$

Q.2

Load current



$$\text{Load power factor} = \frac{\text{actual power}}{\text{apparent power}} \Rightarrow 0.5 = \frac{10000}{2000|I_Z|} \Rightarrow |I_Z| = 10\,\text{A}$$



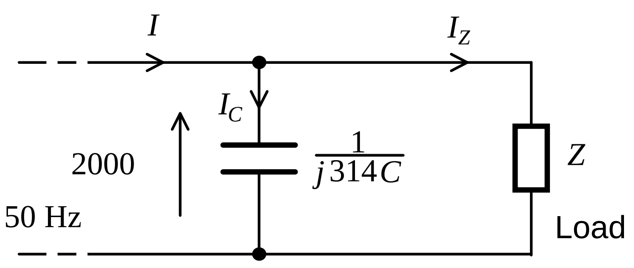
$$\begin{matrix}0.5\,\text{lagging} \\ \text{load p.f.}\end{matrix} \Rightarrow \left\langle \begin{matrix}\cos\left[\text{Arg}(I_Z) - \text{Arg}(V)\right] = 0.5 \\ \text{Arg}(I_Z) - \text{Arg}(V) < 0\end{matrix} \right\rangle$$

$$\Rightarrow \left\langle \begin{matrix}\text{Arg}(I_Z) = \pm\cos^{-1}(0.5) = \pm 1.05 \\ \text{Arg}(I_Z) < 0\end{matrix} \right\rangle$$

$$\Rightarrow \text{Arg}(I_Z) = -1.05$$

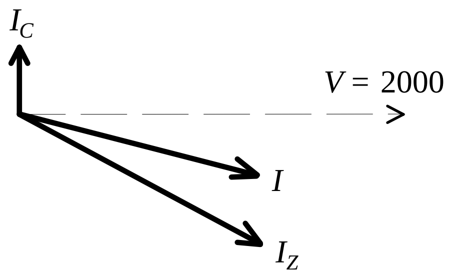
$$I_Z = |I_Z| e^{j\text{Arg}[I_Z]} = 10e^{-j1.05}\,\text{A}$$

Power factor improvement



$$I_C = (j314C)(2000) = j628000C$$

$$I = I_Z + I_C = 10e^{-j1.05} + j628000C = 5 + j(62800C - 8.66)$$



0.9 lagging overall p.f. $\Rightarrow \left\langle \begin{array}{l} \cos\left[\mathrm{Arg}(I) - \mathrm{Arg}(V)\right] = 0.9 \\ \mathrm{Arg}(I) - \mathrm{Arg}(V) < 0 \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{l} \mathrm{Arg}(I) = \pm 0.451 \\ \mathrm{Arg}(I) < 0 \end{array} \right\rangle$

$$\Rightarrow \mathrm{Arg}(I) = -0.451 \Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = -0.451$$

$$\Rightarrow C = \frac{8.66 + 5\tan(-0.451)}{628000} = 9.9\,\mu\,\mathrm{F}$$

unity overall p.f. $\Rightarrow \left\langle \begin{array}{l} \cos\left[\mathrm{Arg}(I) - \mathrm{Arg}(V)\right] = 1 \\ \mathrm{Arg}(I) - \mathrm{Arg}(V) = 0 \end{array} \right\rangle \Rightarrow \mathrm{Arg}(I) = 0$

$$\Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = 0 \Rightarrow C = \frac{8.66}{628000} = 0.0000138\,\mathrm{F} = 13.8\,\mu\,\mathrm{F}$$

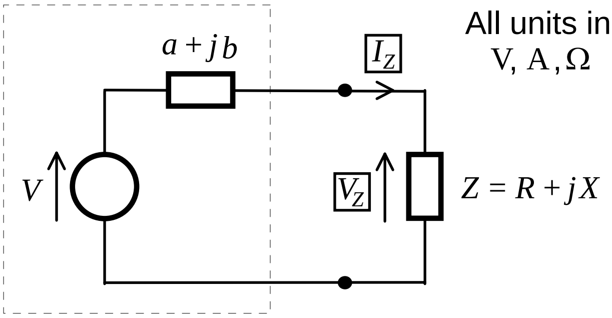
0.8 leading overall p.f. $\Rightarrow \left\langle \begin{array}{l} \cos\left[\mathrm{Arg}(I) - \mathrm{Arg}(V)\right] = 0.8 \\ \mathrm{Arg}(I) - \mathrm{Arg}(V) > 0 \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{l} \mathrm{Arg}(I) = \pm 0.644 \\ \mathrm{Arg}(I) > 0 \end{array} \right\rangle$

$$\Rightarrow \mathrm{Arg}(I) = 0.644 \Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = 0.644$$

$$\Rightarrow C = \frac{8.66 + 5\tan(0.644)}{628000} = 19.8\,\mu\mathrm{F}$$

Q.3

Power



Electrical system

$$I_Z = \frac{V}{(a+jb)+(R+jX)} = \frac{V}{(a+R)+j(b+X)}$$

$$V_Z = I_Z(R+jX)$$

Power absorbed $= p = \mathrm{Re}\left[I_Z^* V_Z\right] = \mathrm{Re}\left[\left|I_Z\right|^2 (R+jX)\right]$

$$= \left|I_Z\right|^2 \mathrm{Re}\left[R+jX\right] = R\left|I_Z\right|^2$$

$$= R\left|\frac{V}{(a+R)+j(b+X)}\right|^2 = \frac{R|V|^2}{(a+R)^2+(b+X)^2}$$

Maximum power transfer

For maximum p, the denominator should be as small as possible. As the numerator does not depend on X and the smallest value for $(b+X)$ is 0, maximum power will be absorbed if

$$X = -b$$

so that

$$p = \frac{|V|^2 R}{(a+R)^2}$$

Differentiating:

$$\frac{dp}{dR} = |V|^2\left[\frac{1}{(a+R)^2} - \frac{2R}{(a+R)^3}\right] = |V|^2\left[\frac{a-R}{(a+R)^3}\right]$$

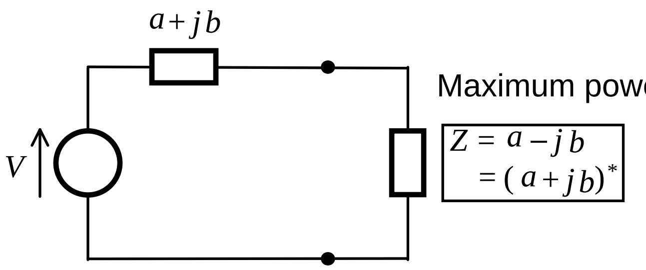
Thus, maximum p occurs when

$$R = a$$

and the maximum power transferable is

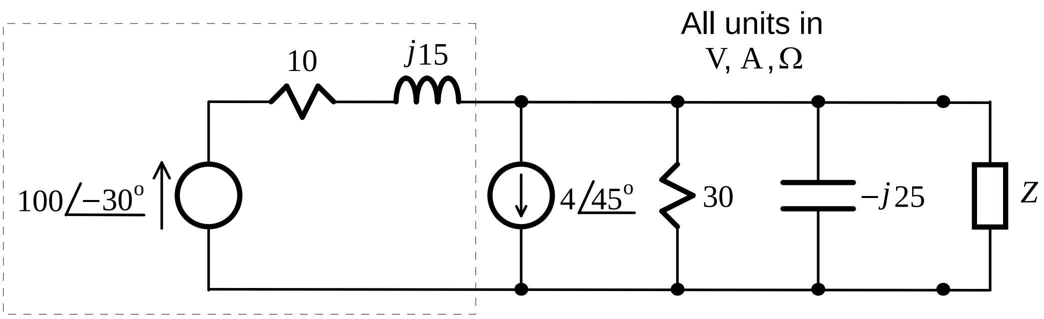
$$p = \frac{|V|^2 R}{(R+R)^2} = \frac{|V|^2}{4R} = \frac{|V|^2}{4a}\,\mathrm{W}$$

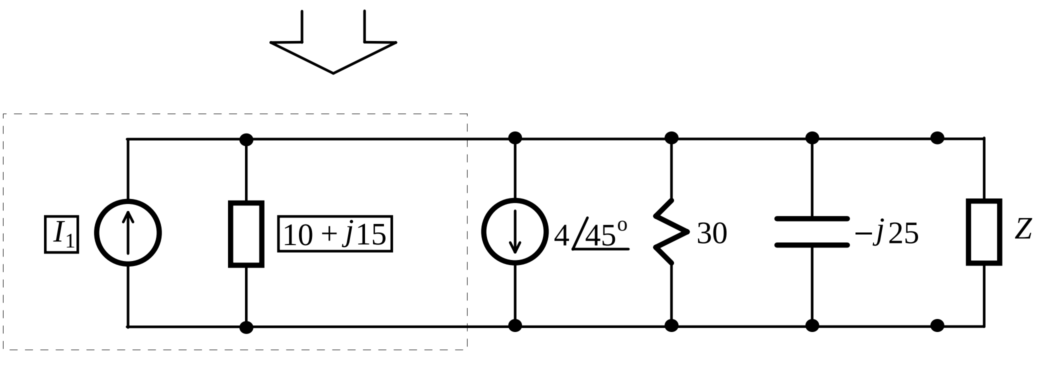
In general, maximum power transfer occurs when the load impedance is equal to the conjugate of the Thevenin's or Norton's impedance. When this occurs, the total impedance is purely resistive and the current and voltage in the circuit are in phase:



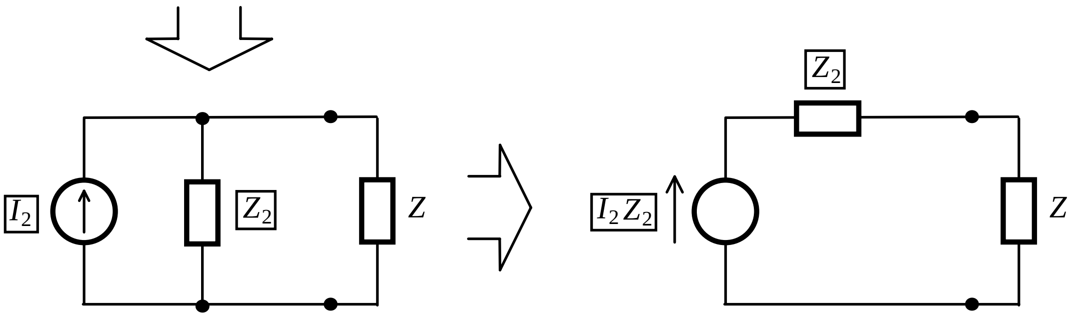
Q.4

Norton's and Thevenin's equivalent circuit





$$I_1 = \frac{100e^{-j30^0}}{10 + j15} = \frac{100e^{-j30^0}}{18e^{j56.3^0}} = 5.55e^{-j86.3^0} = 0.358 - j5.54$$



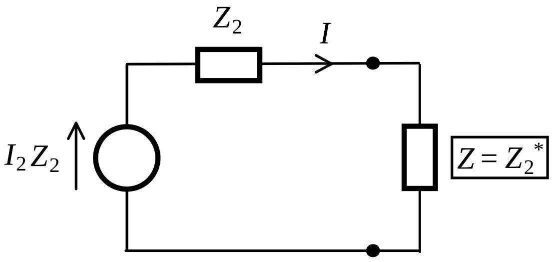
$$Z_2 = -j25 \,||\, 30 \,||\, (10 + j15) = \frac{1}{\frac{1}{-j25} + \frac{1}{30} + \frac{1}{10 + j15}} = \frac{1}{\frac{1}{-j25} + \frac{1}{30} + \frac{10 - j15}{10^2 + 15^2}}$$

$$= \frac{1}{0.0641 - j0.00615} = \frac{1}{0.0644 e^{-j5.48^0}} = 15.5 e^{j5.48^0}$$

$$I_2 = 0.359 - j5.55 - 4e^{j45^0} = -2.47 - j8.36 = 8.72 e^{-j106^0}$$

Maximum power transfer

From the previous problem, this occurs when



$$Z = Z_2^* = \left(15.5 e^{j5.48^0}\right)^* = 15.5 e^{-j5.48^0}$$

Total impedance $= Z + Z_2 = Z_2^* + Z_2 = 15.5 e^{j5.48^0} + 15.5 e^{-j5.48^0} = 2\left[15.5 \cos\left(5.48^0\right)\right]$

$$I = \frac{I_2 Z_2}{(Z + Z_2)} = \frac{8.72 e^{-j106^0}\, 15.5 e^{j5.48^0}}{2\left[15.5 \cos\left(5.48^0\right)\right]} = \frac{8.72 e^{-j101^0}}{2 \cos\left(5.48^0\right)}$$

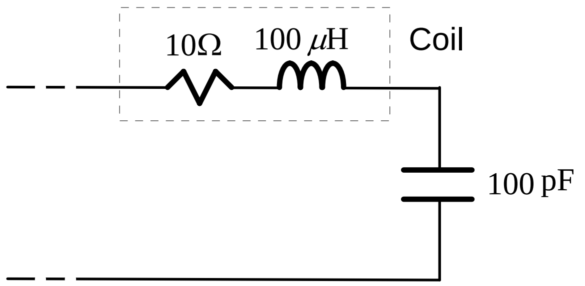
Thus, the maximum power transferable is

$$\mathrm{Re}\left[I^*(IZ)\right] = |I|^2 \,\mathrm{Re}\left[Z_2^*\right]$$

$$= \left|\frac{8.72 e^{-j101^0}}{2 \cos\left(5.48^0\right)}\right|^2 \mathrm{Re}\left[15.5 e^{-j5.48^0}\right] = \left|\frac{8.72}{2 \cos\left(5.48^0\right)}\right|^2 15.5 \cos\left(5.48^0\right)$$

$$= \frac{8.72^2 \times 15.5}{4 \cos\left(5.48^0\right)} = 297\,\mathrm{W}$$

Q.5



Resonant frequency $= \dfrac{1}{2\pi\sqrt{(100\times10^{-6})(100\times10^{-12})}} = 1.59\,\text{MHz}$

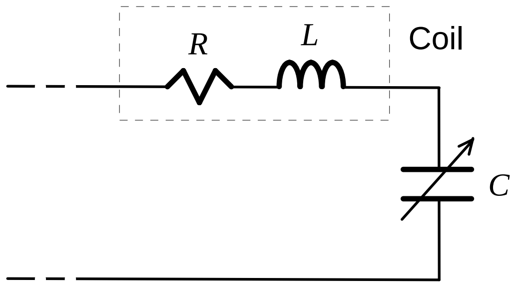
Q factor $= \dfrac{2\pi(1.59\times10^{6})(100\times10^{-6})}{10} = 100$

Since the Q factor is large, the circuit is bandpass in nature with

3dB cutoff frequencies $\approx 1.59\left(1\pm\dfrac{1}{200}\right)\text{MHz}$

Bandwidth $\approx \dfrac{1.59\times10^{6}}{100} = 15.9\,\text{kHz}$

Q.6



$C = 20 \cdots 500\,\text{pF} \;\Rightarrow$ resonant frequency $= \dfrac{1}{2\pi\sqrt{L(500\times10^{-12})}} \cdots \dfrac{1}{2\pi\sqrt{L(20\times10^{-12})}}$

$$= \frac{7.12}{\sqrt{L}} \cdots \frac{35.6}{\sqrt{L}}\,\text{kHz}$$

For the lowest tunable frequency to be $666\,\text{kHz}$:

$666 = \dfrac{7.12}{\sqrt{L}} \;\Rightarrow\; L = \left(\dfrac{7.12}{666}\right)^2 = 0.114\,\text{mH}$

The highest tunable frequency is then $\dfrac{35.6}{\sqrt{L}} = \dfrac{35.6}{\sqrt{0.114\times10^{-3}}} = 3.42\,\text{MHz}$