Q.1 Consider a cruise control system given in Figure 1 below, in which the mass of the passenger car $m = 500$ kg, the friction coefficient $b = 100$ N· s/m.

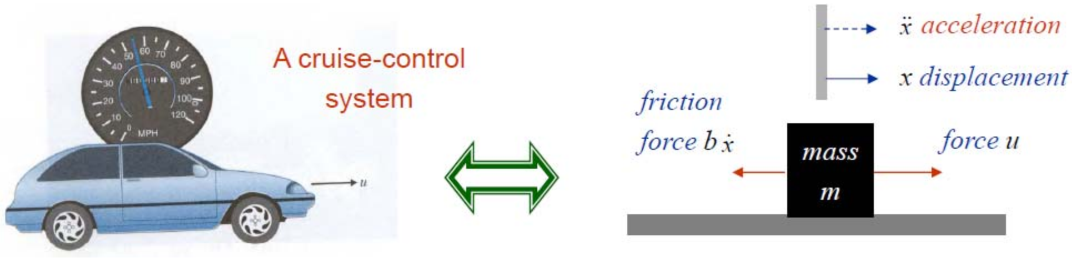

Figure 1

(a) Derive a time-domain model for the system in terms of the displacement, $x(t)$.

(5 marks)

<u>Solution:</u> By Newton's Law of motion,

$$f = ma \quad \Leftrightarrow \quad u - b\dot{x} = m\ddot{x} \quad \Leftrightarrow \quad m\ddot{x} + b\dot{x} = u$$

$$\Updownarrow$$

$$500\ddot{x} + 100\dot{x} = u \quad \Leftrightarrow \quad \ddot{x} + 0.2\dot{x} = 0.002u$$

(b) Derive a time-domain model for the system in terms of the car velocity, $v(t)$.

(5 marks)

<u>Solution:</u> Nothing $v = \dot{x}$, we have

$$\dot{v} + 0.2v = 0.002u$$

(c) Assume that the car is initially parked, i.e., $x(0) = 0$ and $v(0) = 0$, and the system input $u(t) = 500 \cdot 1(t)$, where $1(t)$ is a unit step function. Find an explicit expression for $v(t)$.

(5 marks)

Solution: With the given input, we have

$$\dot{v} + 0.2v = 1, \quad t \geq 0$$

Let the steady state solution $v_{ss} = k \quad \Rightarrow \quad \dot{v}_{ss} + 0.2v_{ss} = 0.2k = 1 \quad \Rightarrow \quad v_{ss} = 5$. For the transient response, we find the root of the characteristic polynomial

$$z + 0.2 = 0 \quad \Rightarrow \quad z = -0.2$$

We have $v_{tr}(t) = k_1 e^{-0.2t}$ and the complete solution $v(t) = 5 + k_1 e^{-0.2t}$. Then $v(0) = 0$ implies $v(0) = 5 + k_1 = 0 \quad \Rightarrow \quad k_1 = -5$ and

$$v(t) = 5 - 5e^{-0.2t}$$

(d) Is the system time-invariant? Why (5 marks)

Solution: Yes. It is characterized by an ODE with constant coefficients.

(e) Is the system is stable in terms of velocity. Why? (5 marks)

Solution: Yes. It has a system pole (the root of the characteristic polynomial) at – 0.2.

Q.2 Figure 2 below shows the Bode plot of a second order linear system.

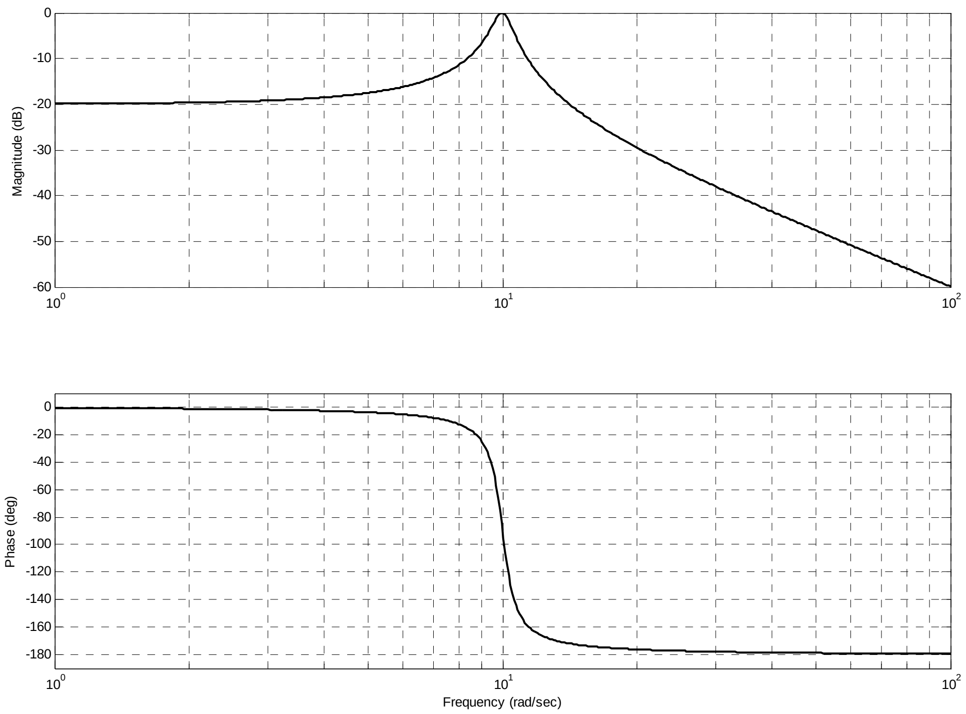

Figure 2

(a) Does the system have an integrator? Why? (4 marks)

Solution: It does not have an integrator as the Bode magnitude plot does not roll off 20 dB per decade at low frequencies.

(b) Does the system have a differentiator? Why? (4 marks)

Solution: It does not have an differentiator as the Bode magnitude plot does not roll up 20 dB per decade at low frequencies.

(c) Determine the DC gain of the system. (4 marks)

Solution: The system DC gain is −20 dB or 0.1.

(d) Determine the natural frequency and damping ratio of the system. (4 marks)

Solution: It is simple to observe from the plot that the system has a natural frequency $\omega_n = 10$ rad/sec and damping ratio $\zeta = 0.05$ as it has a peak response of 20 dB.

(e) Determine the transfer function of the system. (5 marks)

Solution:

$$H(s) = \frac{k_{\mathrm{DC}} \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{0.1 \times 100}{s^2 + 2 \times 0.05 \times 10s + 100} = \frac{10}{s^2 + s + 100}$$

(f) Determine its output signal when its input is $2\cos(10t + 13°)$. (4 marks)

Solution: At the frequency $\omega = \omega_n = 10$ rad/sec, it can be observed from the Bode plot that the magnitude response is 0 dB and phase response is – 90º. Thus, the output signal of the system is given by

$$2\cos(10t + 13° - 90°) = 2\cos(10t - 77°)$$