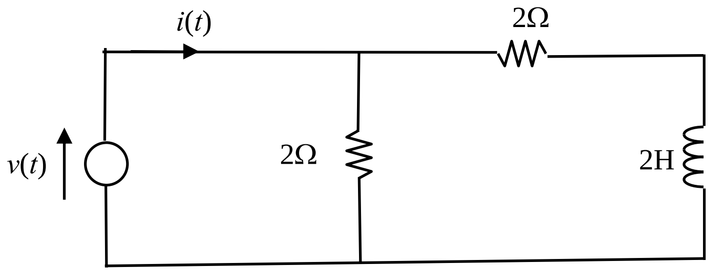
EE2010E Systems and Control Part 1 – Tutorial Set 1 (2 hours)

Q.1. In the following circuit (or electrical system), $v(t)$ is the system input and $i(t)$ is the system output.



(a) Derive a time-domain model for the circuit.

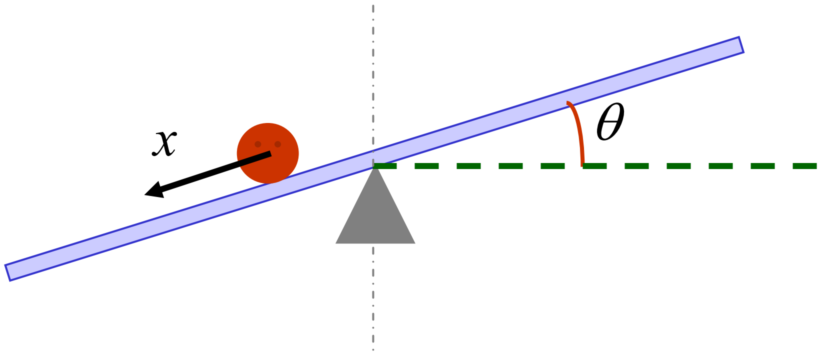
(b) Is the system is linear?

(c) Is the system is time invariant?

(d) Is that the system is causal?

(e) Is that the system is BIBO stable?

Q.2. Consider a ball and beam balancing mechanical system below. Let θ be the system input and let x, the displacement of the ball, be the system output. Assume that there is no friction on the surfaces.



(a) Derive a time-domain model for the mechanical system.

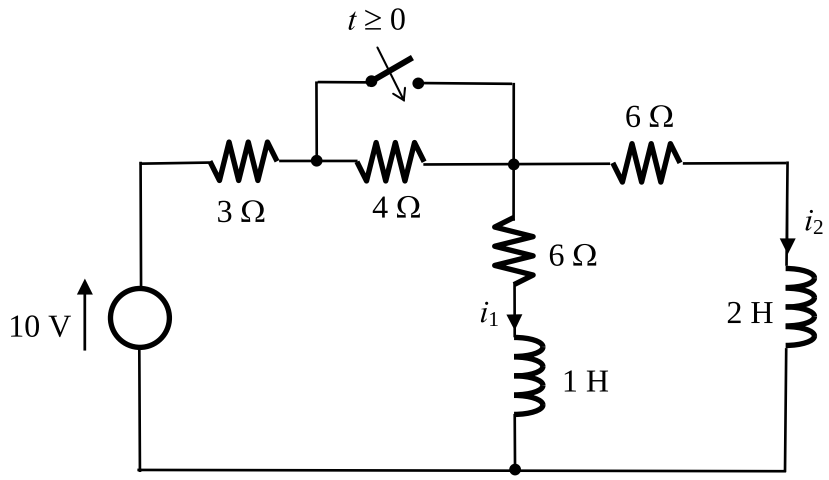
(b) Is the system is linear?

(c) Is the system is time invariant?

(d) Is that the system is causal?

(e) Is that the system is BIBO stable?

Q.3. In the electrical circuit given below, the switch has been in the position shown for a long time and is thrown to the other position for time $t \geq 0$.



(a) Determine the currents for both inductors for $t < 0$.

(b) Determine the currents and voltages for both inductors just right after the switch is closed.

(c) Derive the differential equation governing the circuit in terms of i_1.

(d) Compute the roots of its characteristic polynomial.

(e) Is the circuit over damped, under damped or critically damped?

Q.4. An input-output relationship of a thermometer can be modeled by the following differential equation:

$$5\frac{dy(t)}{dt} + y(t) = 0.99u(t)$$

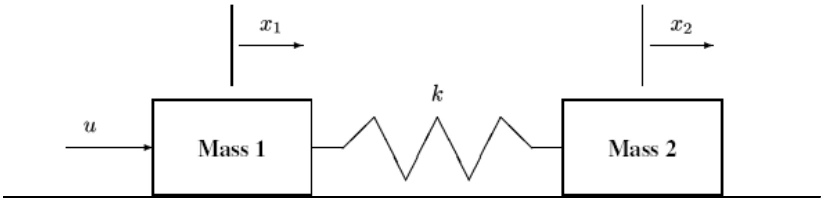
where $u(t)$ is the temperature of the environment in which the thermometer is placed, and $y(t)$ is the measured temperature.

The thermometer is inserted into a heat bath and the temperature reading is allowed to be stabilized before the temperature of the water in the heat bath is increased at a steady rate of 1°C/second. Assume that $t = 0$ at the instant when the hot bath temperature starts to increase.

(a) Suppose the measured temperature is 24.75°C when t = 0, i.e. $y(0) = 24.75$°C. What is the temperature of the heat bath?

(b) Write a mathematical expression to represent the temperature in the heat bath, $u(t)$. Then solve the differential equation to obtain the time-domain expression of the measured temperature, $y(t)$.

Q.5. Consider a two-mass-spring flexible mechanical system given below.



In the system, $u(t)$ is the input force, $k = 1$ is the spring constant, x_1 and x_2 are, respectively, the displacements of Mass 1 and Mass 2, which have masses of $m_1 = m_2 = 1$. Assume that there is no friction on the surfaces.

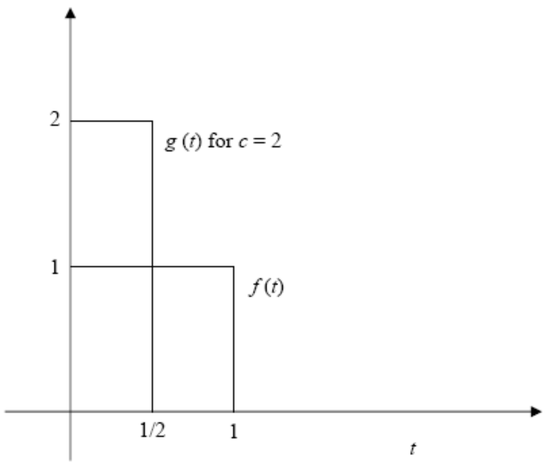
(a) Drive a differential equation of the mechanical system in terms of the displacement of Mass 2, i.e. x_2.

(b) Assuming that $u(t) = 1$ and the masses are initially stationary, show that $x_2(t) = 0.25t^2$ is a solution to the differential equation obtained in (a).

(c) Is the system BIBO stable?

EE2010E Systems and Control Part 1 – Tutorial Set 2 (2 hours)

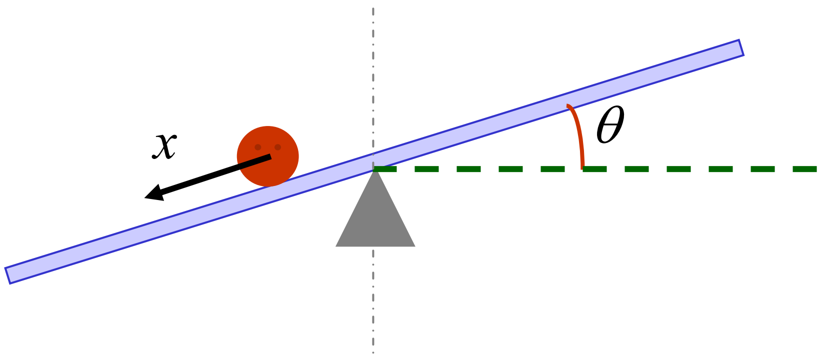
Q.1. Consider the square pulse $f(t)$ show in figure below. If we compress the pulse by a factor $c > 1$ and at the same time amplify its amplitude by the same factor c, we get a new function $g(t)$ as shown in the figure ($c = 2$ for the given figure).



(a) Find the Laplace transform of the function $g(t)$ from the transform of $f(t)$.

(b) Comment on what happens if c gets very large.

Q.2. Consider the ball and beam balancing mechanical system again as in Tutorial Set 1. Let θ be the system input and let x, the displacement of the ball, be the system output. Assume that θ is changing in a very small range, i.e. $\sin\theta \approx \theta$.



(a) Find the transfer function of the system from the input θ to the output x.

(b) Find the unit impulse response of the system.

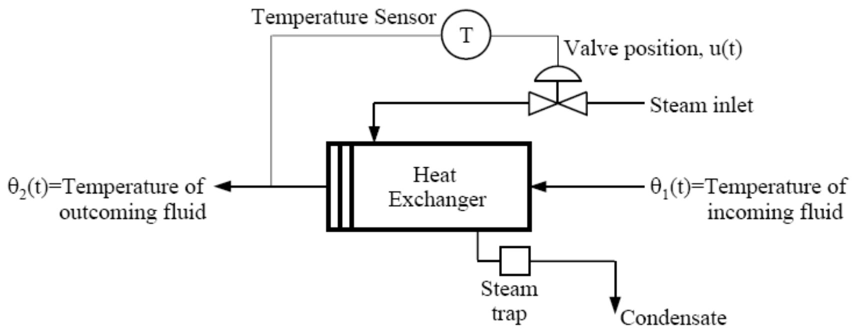
(c) Find the unit step response of the system.

Q.3. Use Laplace transform to solve the response $y(t)$ in the following integrodifferential equation:

$$\frac{dy(t)}{dt} + 5y(t) + 6\int_0^t y(\tau)d\tau = u(t), \quad y(0) = 2$$

Q.4. Figure below shows a heat exchanger (a device for transferring heat from one fluid to another, where the fluids are separated by a solid wall so that they never mix). The temperature of the outgoing fluid, $\theta_2(t)$, needs to be maintained at a desired value, $\theta_r(t)$. Factors which influence the exit temperature are:

- The valve position, $u(t)$, which adjusts the flow of steam into the system.
- unmeasurable disturbances in the temperature of the incoming fluid stream, $\theta_1(t)$.



The dynamic behavior of the heat exchanger may be modeled by the following equation:

$$\theta_2(s) = \frac{2}{(s+1)^2}U(s) + \frac{1}{s+1}\theta_1(s)$$

Let the valve position $u(t) = 2\,[\theta_r(t) - \theta_2(t)]$, i.e. it is proportional to the error of the desired value and the actual outgoing temperature.

(a) If $\theta_r(t)$ is a unit step function and $\theta_1(t) = 0$, determine the transfer function $\theta_2(s)/\,\theta_r(s)$ and then use it to calculate $\theta_2(t)$. Identify the transient and steady-state components in the step response.

(b) Given that $\theta_1(t)$ is a unit step function and $\theta_r(t) = 0$, find the transfer function $\theta_2(s)/\,\theta_1(s)$ and $\theta_2(t)$.

(c) Use superposition to obtain $\theta_2(t)$ given that both $\theta_r(t)$ and $\theta_1(t)$ are unit step functions. Find $\theta_2(\infty)$.

(d) Use the final value theorem instead to find $\theta_2(\infty)$ and compare it with the answer obtained in Part (c).

Q.5. Consider the first order system

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{\tau s + 1}$$

(a) Find the step response, $y_{\text{step}}(t)$.

(b) Find the impulse response, $y_{\text{impulse}}(t)$.

(c) Verify that

$$\dot{y}_{\text{step}}(t) = y_{\text{impulse}}(t) \qquad \text{and} \qquad \int_0^t y_{\text{impulse}}(\tau)d\tau = y_{\text{step}}(t)$$

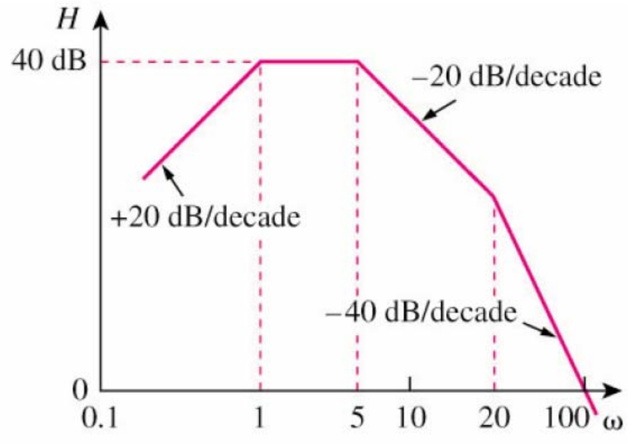
EE2010E Systems and Control Part 1 – Tutorial Set 3 (2 hours)

Q.1. Obtain the Bode plots for the following transfer function:

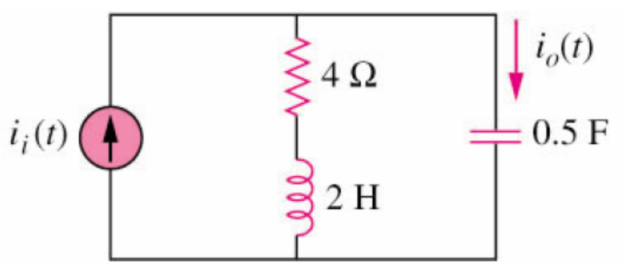
$$G(j\omega) = \frac{Y(j\omega)}{U(j\omega)} = \frac{10(j\omega + 10)}{j\omega(j\omega + 100)}$$

Given $u(t) = 5\cos(30t + 30°)$, find the corresponding output $y(t)$ using the Bode plots obtained above.

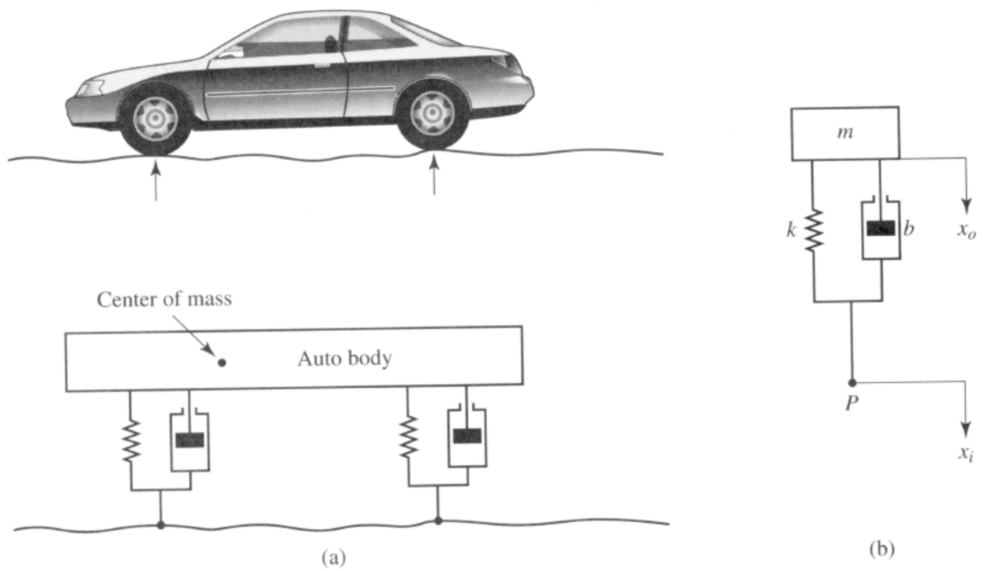
Q.2. A Bode plot of $H(j\omega)$ is given in the figure below. Obtain the transfer function $H(s)$.



Q.3. For the circuit below, obtain the transfer function $I_o(s)/I_i(s)$ and its poles and zeros.



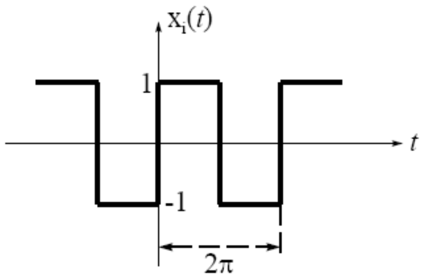
Q.4. A car suspension system and a very simplified version of the system are shown in Figures (a) and (b), respectively.



The transfer function of the simplified car suspension system is

$$G(s) = \frac{bs + k}{ms^2 + bs + k}$$

Suppose a toy car (m = 1 kg, k = 1 N/m and b = 1.414 N s / m) is traveling on a road that has speed reducing stripes and the input to the simplified car suspension system, x_i, may be modeled by the periodic square wave, of frequency ω = 1 rad/s, shown in Figure below.



Determine the steady-state displacement of the car body, $x_{o,ss}(t)$.

Hint : The Fourier Series representation of the periodic square wave shown in Figure above is

$$x_i(t) = \frac{4}{\pi}\left[\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \cdots\right]$$

Q.5. Consider the second order system

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

whose unit step response has a transient behavior described by the following parameters:

- Rise time, $t_r = 1.8/\omega_n$
- 2% settling time, $t_s = 4/(\zeta\omega_n)$
- Overshoot peak, $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

Sketch and shade the allowable region in the s-plane for the system poles if the step response requirements are

$$t_r < 0.9 \text{ seconds}, \quad t_s < 3 \text{ seconds}, \quad M_p < 10\%$$