## Mid-term Test for EE3331C Feedback Control Systems – Part 2

LT4, 10–11:30am, 24 October 2017

*Answer all questions in the space provided.*

**Q.1** (a) Your Name ___________Key___________ Your Matric No: _________________ (2 marks)

(b) Who are the lecturers for this module (both Part 1 and Part 2)?

(2 marks)

Part 1 Lecturer: Arthur Tay

Part 2 Lecturer: Ben M. Chen

(c) What is the key property that a linear system should have?

(2 marks)

Superposition.

(d) What is a Bode plot?

(2 marks)

Bode plot is a plot of magnitude response and phase response of a transfer function, say $G(s)$, in the frequency domain with $s = j\,\omega$.

(e) Name one real control system example that the lecturer has mentioned in the class.

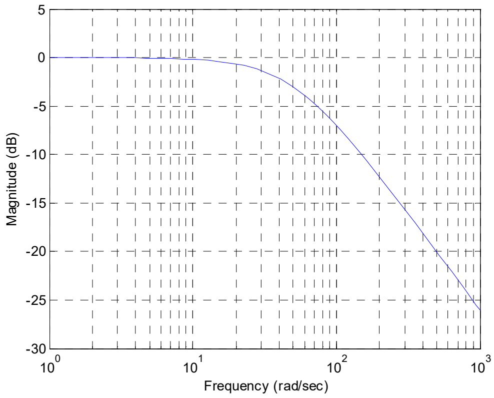
(2 marks)

Toilet water tank.

**Q.2** The magnitude responses for the following systems are shown in Figure Q.2 below.

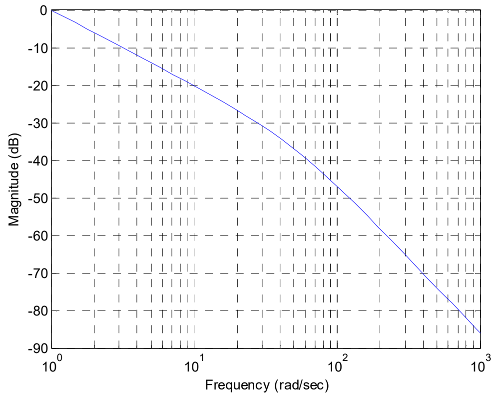
$$G_1(s)=\frac{50s}{s+50},\ G_2(s)=\frac{50}{s(s+50)},\ G_3(s)=\frac{50s}{s^2+50s+50},\ G_4(s)=\frac{50}{s+50}$$

Match the magnitude responses with the given transfer functions.



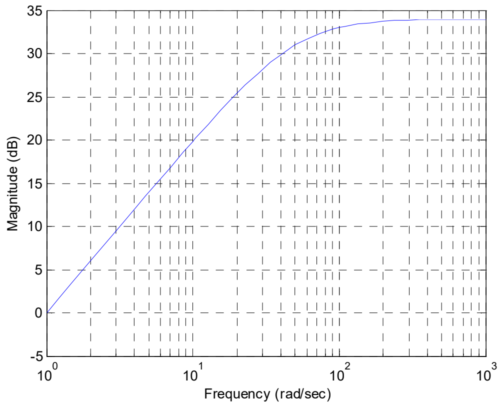

$$G_4(s)=\frac{50}{s+50}$$

(a)



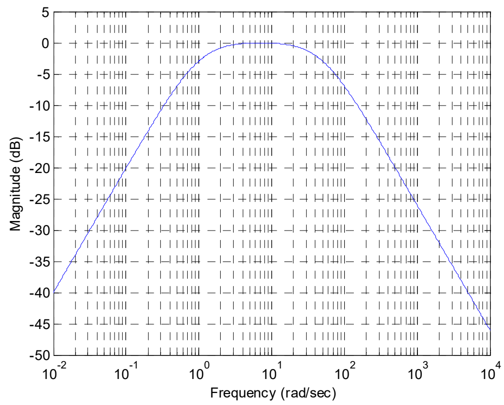

$$G_2(s)=\frac{50}{s(s+50)}$$

(b)




$$G_1(s)=\frac{50s}{s+50}$$

(c)




$$G_3(s)=\frac{50s}{s+50s+50}$$

(d)

Figure Q.2

Label the transfer functions directly on the graphics above.

(10 marks)

**Q.3** The magnitude response of a typical second order system characterized by

$$H(s) = \frac{Y(s)}{U(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

is given in Figure Q.3 below.

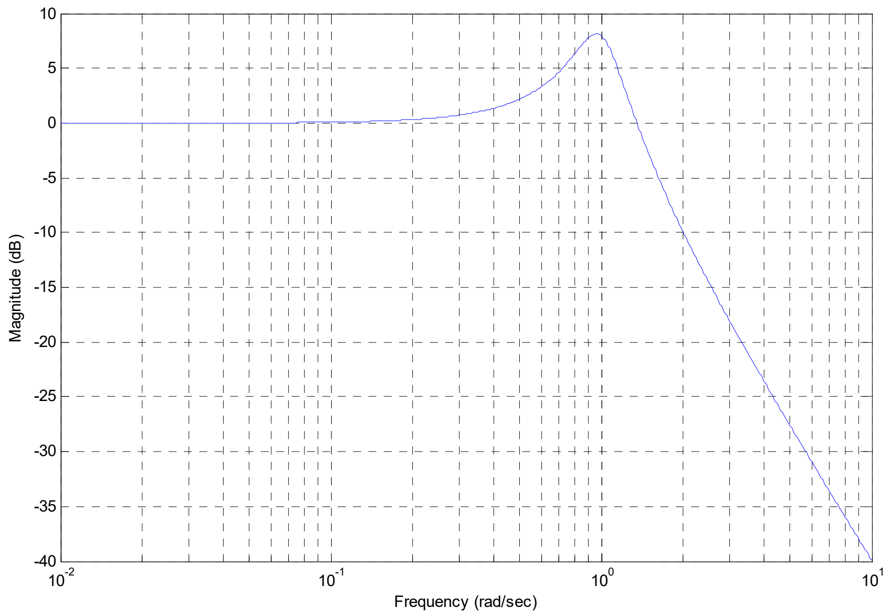



Figure Q.3

(a) Find the DC gain, $K$, the damping ratio, $\zeta$, and the natural frequency, $\omega_n$, of the given system.

(5 marks)

**Solution:** It is simple to observe from the magnitude response that the static or DC gain is unity, i.e., $K = 1$. The corner frequency, which is also the natural frequency, of the magnitude response is 1 rad/sec, i.e., $\omega_n = 1$ rad/sec. The peak at the corner frequency is about 8 dB, which is corresponding to a damping ratio $\zeta = 0.2$. Thus, the transfer function is given by

$$H(s) = \frac{Y(s)}{U(s)} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{s^2 + 0.4s + 1}$$

(b) Given an input signal, $u(t) = \cos t$, find its corresponding steady-state output, $y(t)$.

(5 marks)

**Solution:** For the given input, we have $\omega = 1$ rad/sec. Its corresponding frequency response is given by

$$H(j\omega)\big|_{\omega=1} = \frac{1}{j^2 + j0.4 + 1} = -j2.5 = 2.5\angle -90^\circ$$

Thus, the corresponding steady-state output is given by

$$y(t) = 2.5\cos(t - 90^\circ)$$

(c) Find the steady state error due to a unit step input.

(5 marks)

$$Y(s) = \frac{1}{s^2 + 0.4s + 1}U(s) = \frac{1}{s^2 + 0.4s + 1}\cdot\frac{1}{s}$$

$$\Rightarrow \quad y_{ss} = \lim_{s\to 0} s\cdot\frac{1}{s^2 + 0.4s + 1}\cdot\frac{1}{s} = 1 \quad \Rightarrow \quad e_{ss} = y_{ss} - u = 0$$

**Q.4** The transfer function of a television receiver has a frequency (magnitude) response as shown in Figure Q.4 below:

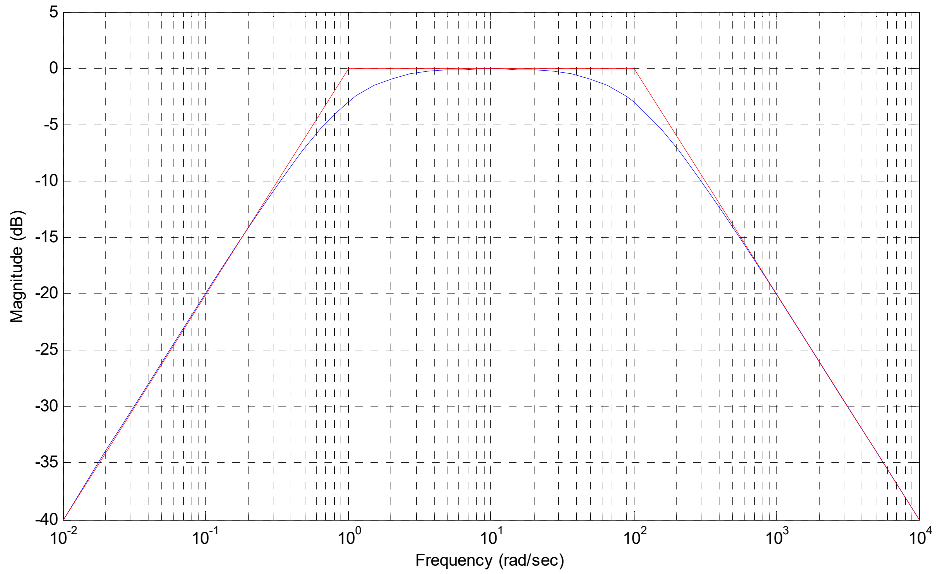



Figure 4

(a) Does the system have an integrator? Why?

(3 marks)

**Solution:** No. The magnitude response does not roll off at low frequency.

(b) Does the system have a differentiator? Why?

(3 marks)

**Solution:** Yes. The magnitude response does roll up 20 dB per decade at low frequency.

(c) Determine the transfer function of the system?

(3 marks)

**Solution:** From the asymptotes, we can obtain the transfer function

$$G(s) = \frac{s}{(1+s)(1+s/100)} = \frac{100s}{(s+1)(s+100)}$$

(d) Determine the magnitude of its output signal when its input is cos (1000*t* +13°)?

(3 marks)

**Solution:** From the given magnitude response, its gain = – 20 dB = 0.1 at $\omega$ = 1000 rad/s. Thus, the magnitude of the corresponding output signal is 0.1.

(e) What is the DC gain of the system?

(3 marks)

**Solution:** The DC gain is 0.