Q.3 (a) Prove the following De Morgan's law:

$$\overline{A+B+C}=\overline{A}\cdot\overline{B}\cdot\overline{C}$$

(5 Marks)

Give your answer to Q.3(a) in the space below.

Proof. This can be done by using truth tables as follows:

A	B	C	$\overline{A+B+C}$	$\overline{A}\cdot\overline{B}\cdot\overline{C}$
0	0	0	1	1
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	0

The result is obvious.

Q.3 (b) Use the De Morgan's laws and any of first 17 rules of Boolean Algebra to prove the last rule, i.e., Rule 18:

$$A+\overline{A}\cdot B=A+B$$

(5 Marks)

Give your answer to Q.3(b) in the space below.

Proof.

$$A+\overline{A}\cdot B=\overline{\overline{A+\overline{A}\cdot B}}=\overline{\overline{A}\cdot\overline{\overline{A}\cdot B}}=\overline{\overline{A}\cdot(A+\overline{B})}=\overline{\overline{A}\cdot A+\overline{A}\cdot\overline{B}}=\overline{\overline{A}}\cdot\overline{\overline{B}}=A+B$$

or any other correct forms.

Q.3 (c) Consider a logical expression

$$W = (\overline{A+\overline{C}})\cdot(B+C)$$

(i) Construct a truth table for *W*.

(5 Marks)

(ii) Use the Karnaugh map technique to simply the logical expression.

(5 Marks)

(iii) Implement the logical expression obtained in (ii) using only 2 two-input NOR gates.

(5 Marks)

Give your answer to Q.3(c) in the space below. If necessary, continue on the next page.

Solution. (i) The truth table:

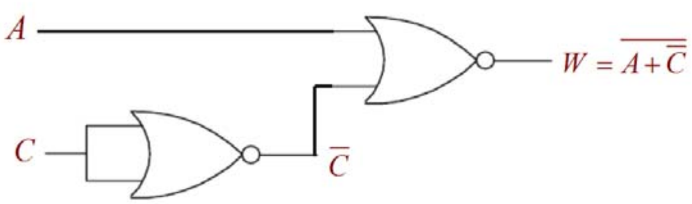
A	B	C	$A+\overline{C}$	$\overline{A+\overline{C}}$	$B+C$	W
0	0	0	1	0	0	0
0	0	1	0	1	1	1
0	1	0	1	0	1	0
0	1	1	0	1	1	1
1	0	0	1	0	0	0
1	0	1	1	0	1	0
1	1	0	1	0	1	0
1	1	1	1	0	1	0

(ii) The K-map:

	$A\cdot B$	$A\cdot\overline{B}$	$\overline{A}\cdot\overline{B}$	$\overline{A}\cdot B$
C	0	0	1	1
$\overline{C}$	0	0	0	0

Clearly, we have $W = \overline{A}\cdot C$.

(iii) Logic circuit implementation: $W = \overline{\overline{\overline{A}\cdot C}} = \overline{A+\overline{C}}$



Q.4 (a) Consider the transformer circuit shown in Figure Q4 (a) below. The transformer turns ratio is 1:2. The input to the circuit is the voltage source, $v_s(t)$. Determine the output voltage, $v_o(t)$, across the 2 H inductor.

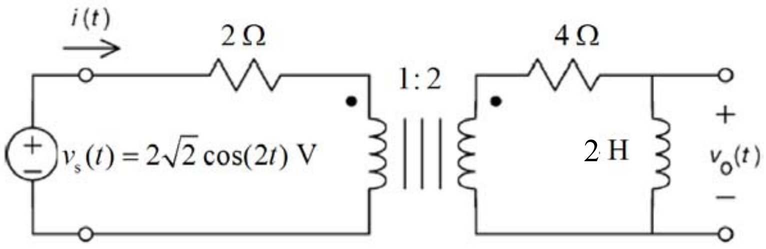


Figure Q4 (a)

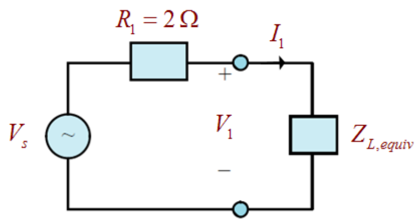
(10 Marks)

Give your answer to Q.4(a) in the space below. If necessary, continue on the next page.

Solution. $\omega = 2$. Thus, the total load impedance of the circuit is given by

$$Z_L = 4 + j2\times 2 = 4\times(1+j)$$

The circuit is equivalent to



with $Z_{L,equiv} = \dfrac{Z_L}{n^2} = \dfrac{4\times(1+j)}{2^2} = 1+j$ and $V_s = 2$. Thus, we have

$$I_1 = \frac{V_s}{2+(1+j)} = \frac{2}{3+j} = \frac{2(3-j)}{(3+j)(3-j)} = \frac{3-j}{5}$$

$$I_2 = \frac{I_1}{n} = \frac{3-j}{10}$$

$$V_o = I_2\cdot(j4) = \frac{3-j}{10}\cdot(j4) = \frac{2}{5}(1+j3) = \frac{2\sqrt{10}}{5}\angle\tan^{-1}3$$

Thus,

$$v_o(t) = \frac{2\sqrt{10}}{5}\sqrt{2}\cos(2t+\tan^{-1}3) = \frac{4\sqrt{5}}{5}\cos(2t+\tan^{-1}3) = 1.8\cos(2t+71.6°)$$

Q.4 (b) A DC power supply consists of a transformer feeding a half-wave rectifier together with a capacitor filter. It supplies a DC current of 1 A at 20 V DC to an electronic equipment. The AC input source is 230 V (rms) at 50 Hz. The filter capacitor has a capacitance of 60,000μF.

(i) Draw the circuit diagram of the supply arrangement.

(5 Marks)

(ii) Determine a suitable winding ratio for the transformer.

(5 Marks)

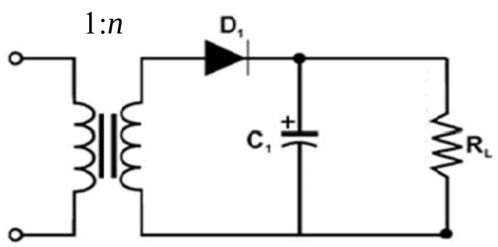
(iii) Determine the magnitude of the peak-to-peak ripple in the output voltage.

(5 Marks)

Give your answer to Q.4(b) in the space below. If necessary, continue on the next page.

Solution.

(i)



(ii) $V_{ave} \cong V_m = 20$.

$$n = \frac{V_2}{V_1} = \frac{V_m/\sqrt{2}}{230} = \frac{20}{230\sqrt{2}} = \frac{\sqrt{2}}{23} = 0.0615$$

(iii)

$$V_{p-p} = \frac{I_L T}{C_1} = \frac{I_L}{C_1 f} = \frac{1}{60000\times10^{-6}\times50} = \frac{1}{3}\text{ V}$$