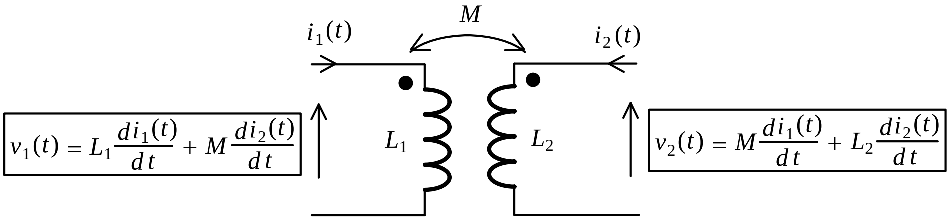
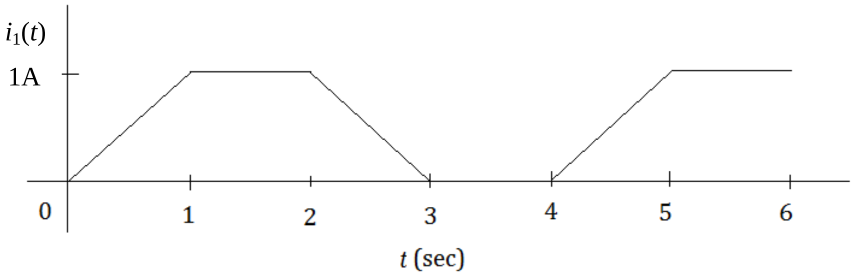
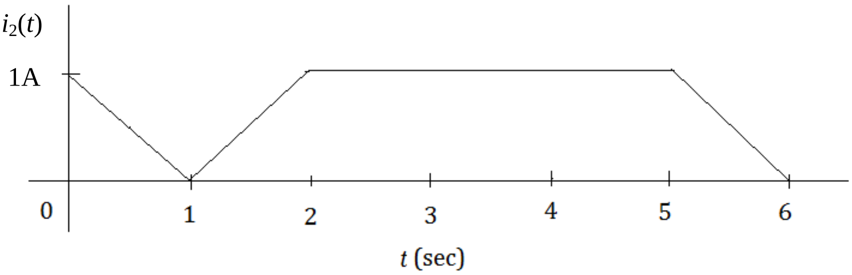
Q.3 Shown in the figure below is the expressions of the primary and secondary voltages of an ideal transformer. Assume that the transformer reluctance $\Re = 25$, the numbers of the turns of the primary and secondary windings are $N_1 = 10,\ N_2 = 5$, respectively.



$L_1 = \frac{N_1^2}{\Re}$, $L_2 = \frac{N_2^2}{\Re}$, $M^2 = L_1 L_2$ for no flux leakage and perfect coupling

(a) Given the waveforms of $i_1(t)$ and $i_2(t)$ as the figure below,





sketch the waveforms of the induced voltages, $v_1(t)$ and $v_2(t)$.

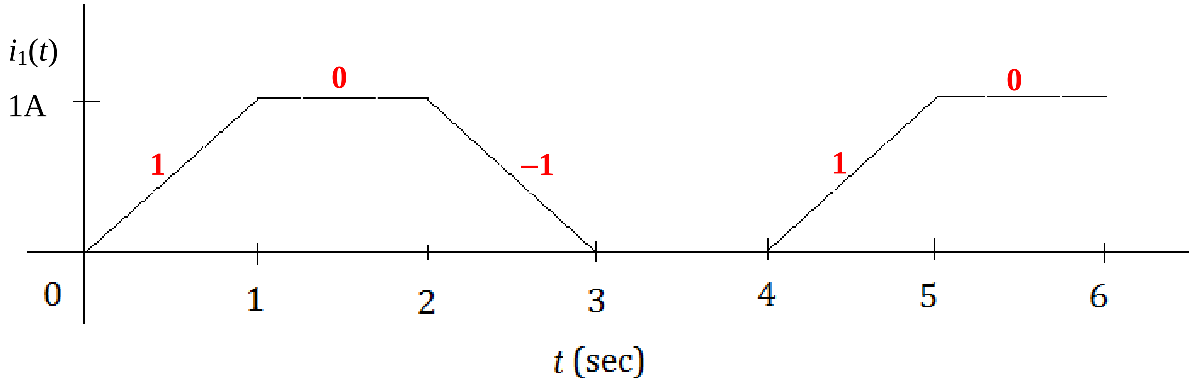
(10 Marks)

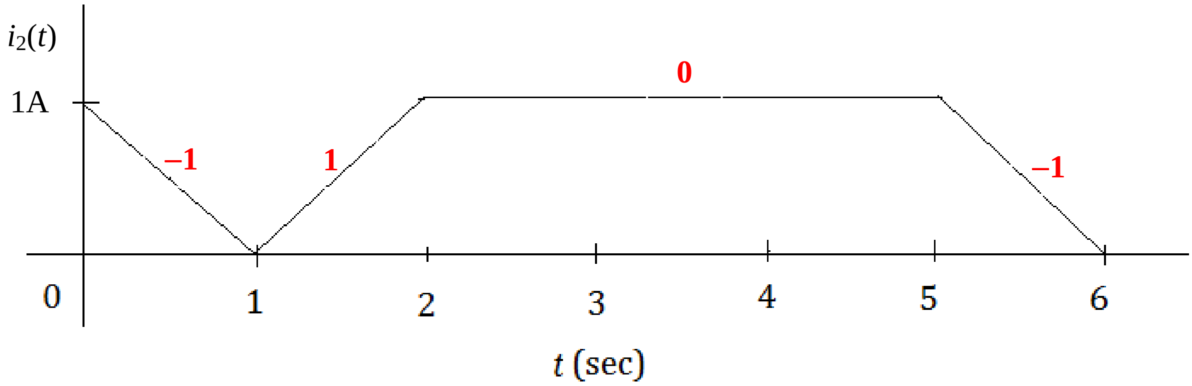
Solution:

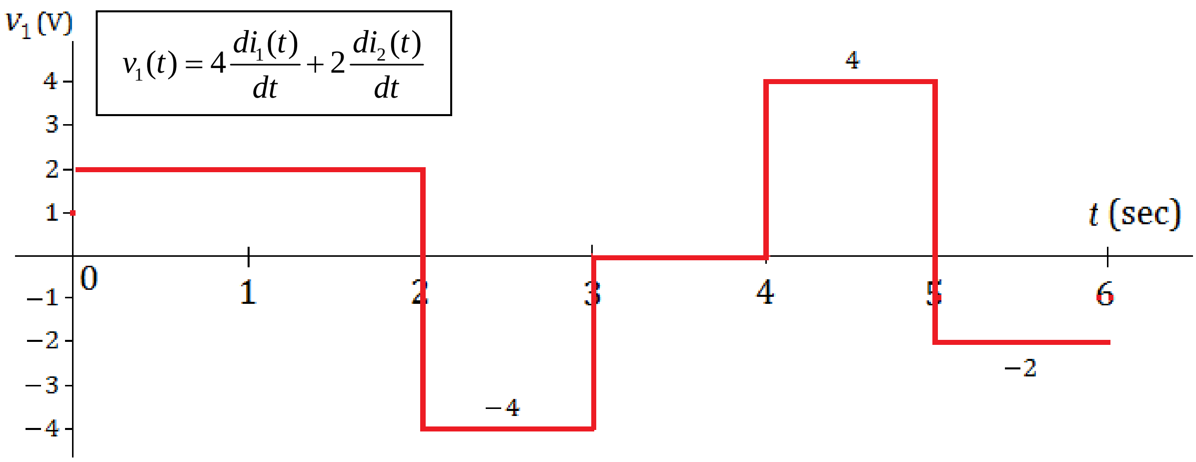
$$L_1 = \frac{N_1^2}{\Re} = \frac{100}{25} = 4, \quad L_2 = \frac{N_2^2}{\Re} = \frac{25}{25} = 1, \quad M = \sqrt{L_1 L_2} = 2$$

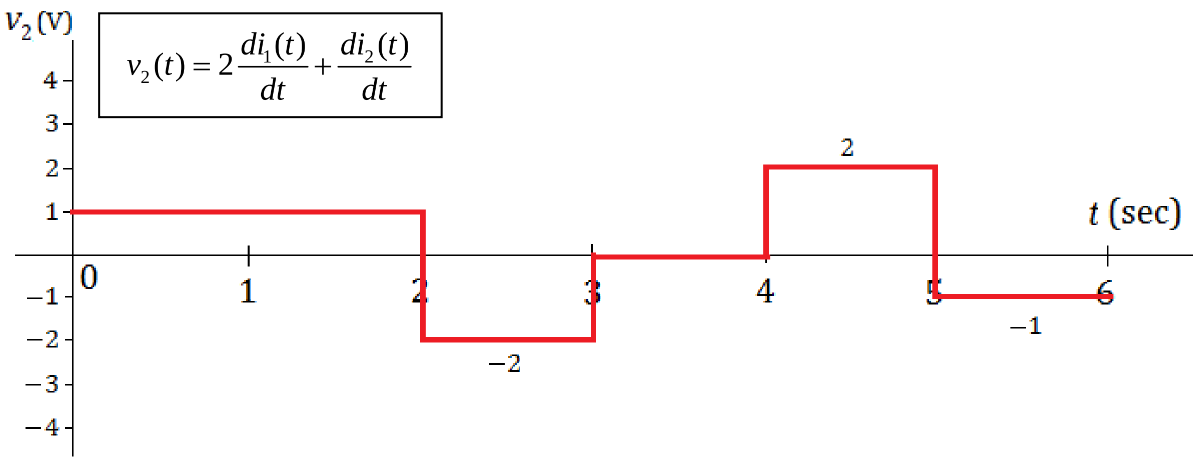
$$v_1(t) = 4\frac{di_1(t)}{dt} + 2\frac{di_2(t)}{dt}, \qquad v_2(t) = 2\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt}$$

Sketch the waveforms directly on the graphs below.

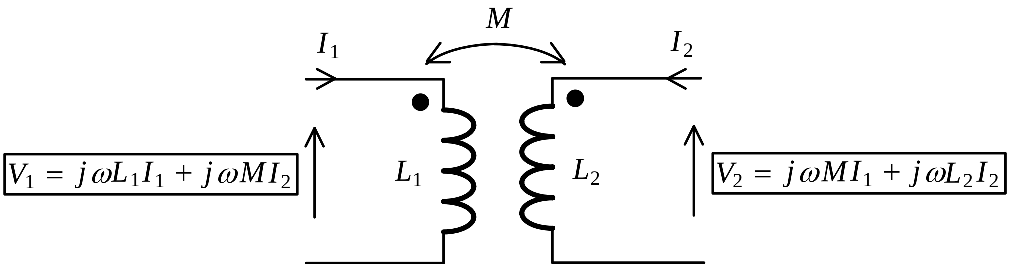








(b) For AC environments, show that the primary and secondary voltages in the phasor form are given as those in the figure below.



Hint: Assume $i_1(t) = \sqrt{2}\, r_1 \cos(\omega t + \theta_1)$ and $i_2(t) = \sqrt{2}\, r_2 \cos(\omega t + \theta_2)$.

(5 Marks)

Proof:

$$i_1(t) = \sqrt{2}\, r_1 \cos(\omega t + \theta_1) \quad \Rightarrow \quad I_1 = r_1 e^{j\theta_1}, \qquad i_2(t) = \sqrt{2}\, r_2 \cos(\omega t + \theta_2) \quad \Rightarrow \quad I_2 = r_2 e^{j\theta_2}$$

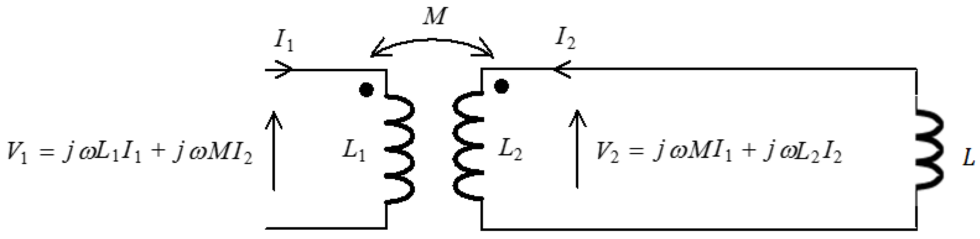
$$\begin{aligned} v_1(t) &= L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = -L_1 \sqrt{2} r_1 \omega \sin(\omega t + \theta_1) - M \sqrt{2} r_2 \omega \sin(\omega t + \theta_2) \\ &= L_1 \sqrt{2} r_1 \omega \cos(\omega t + \theta_1 + 90°) + M \sqrt{2} r_2 \omega \cos(\omega t + \theta_2 + 90°) \end{aligned}$$

$$V_1 = \omega L_1 r_1\, e^{j(\theta_1 + 90°)} + \omega M r_2 e^{j(\theta_2 + 90°)} = j\omega L_1 r_1\, e^{j\theta_1} + j\omega M r_2\, e^{j\theta_2} = j\omega L_1\, I_1 + j\omega M\, I_2$$

$$\begin{aligned} v_2(t) &= M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = -M \sqrt{2} r_1 \omega \sin(\omega t + \theta_1) - L_2 \sqrt{2} r_2 \omega \sin(\omega t + \theta_2) \\ &= M \sqrt{2} r_1 \omega \cos(\omega t + \theta_1 + 90°) + L_2 \sqrt{2} r_2 \omega \cos(\omega t + \theta_2 + 90°) \end{aligned}$$

$$V_2 = \omega M r_1\, e^{j(\theta_1 + 90°)} + \omega L_2 r_2 e^{j(\theta_2 + 90°)} = j\omega M r_1\, e^{j\theta_1} + j\omega L_2 r_2\, e^{j\theta_2} = j\omega M\, I_1 + j\omega L_2\, I_2$$

(c) The transformer is connected to a load inductor with inductance L = 2 H, as shown in the figure below. Recall that the transformer reluctance $\Re = 25$, the numbers of the turns of the primary and secondary windings are $N_1 = 10$, $N_2 = 5$, respectively. Determine the ratio of the primary and secondary currents (in phasor).



Why is the usual transformer property below no longer valid in such a situation?

$$\frac{I_1}{I_2} = -\frac{N_2}{N_1}$$

(10 Marks)

Solution:

$$Z_L = j\omega L = j2\omega$$

$$V_2 = j2\omega I_1 + j\omega I_2$$

By KVL,

$$V_2 + I_2 Z_L = j2\omega I_1 + j\omega I_2 + j2\omega I_2 = j2\omega I_1 + j3\omega I_2 = 0$$

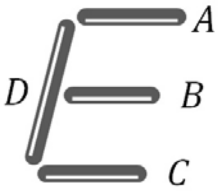
$$\Downarrow$$

$$2I_1 + 3I_2 = 0$$

$$\frac{I_1}{I_2} = -\frac{3}{2} \neq -\frac{N_2}{N_1} = -\frac{1}{2}$$

This is because $|Z_L| = |j2\omega| = 2\omega > |j\omega L_2| = |j\omega| = \omega$. The condition $|j\omega L_2| >> |Z_L|$ is invalid.

Q.4 An LED display panel shown in the figure below has four LED light bars labeled ***A***, ***B***, ***C*** and ***D***, respectively. You are required to design an appropriate digital logic circuit to display Letters ***F*** and ***L*** using the panel.



(a) Construct a truth table for your design.

(6 Marks)

(b) Obtain logical expressions for ***F*** and ***L***, respectively.

(6 Marks)

(c) Draw logic circuit implementations for ***F*** and ***L*** using only 2-input NOR gates (i.e., each NOR gate only has two input channels). For each letter, you cannot use more than 8 NOR gates for the implementation.

(10 Marks)

(d) What other English letters can be displayed by the panel?

(3 Marks)

<u>Solution to Q4 (a):</u>

	A	***B***	***C***	***D***	***F***	***L***
0	0	0	0	0	0	0
1	0	0	0	1	0	0
2	0	0	1	0	0	0
3	0	0	1	1	0	**1**
4	0	1	0	0	0	0
5	0	1	0	1	0	0
6	0	1	1	0	0	0
7	0	1	1	1	0	0
8	1	0	0	0	0	0
9	1	0	0	1	0	0
10	1	0	1	0	0	0
11	1	0	1	1	0	0
12	1	1	0	0	0	0
13	1	1	0	1	**1**	0
14	1	1	1	0	0	0
15	1	1	1	1	0	0

(6 Marks)

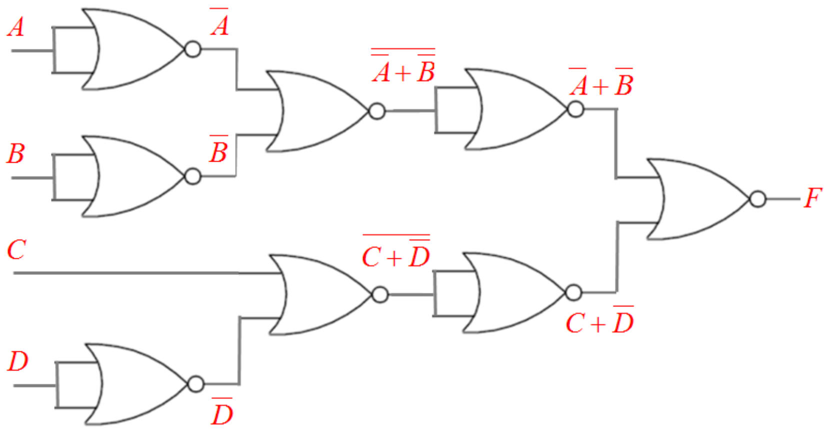
Solution to Q4 (b):

$$F = A \cdot B \cdot \overline{C} \cdot D, \qquad L = \overline{A} \cdot \overline{B} \cdot C \cdot D$$

(6 Marks)

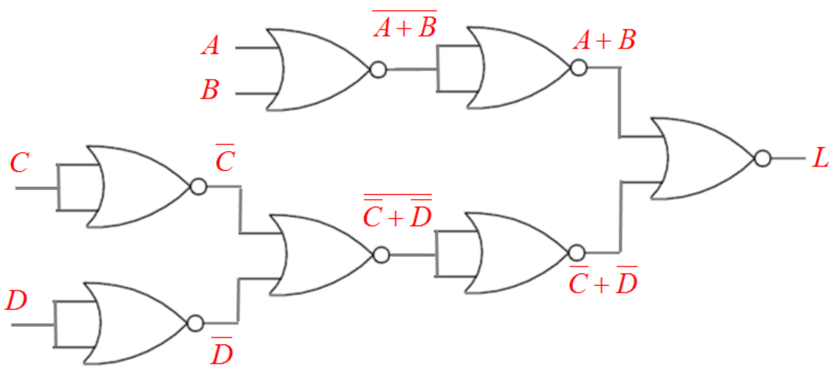
Solution to Q4 (c):

$$F = A \cdot B \cdot \overline{C} \cdot D = \overline{\overline{A \cdot B \cdot \overline{C} \cdot D}} = \overline{\left(\overline{A}+\overline{B}\right)+\left(C+\overline{D}\right)} = \overline{\overline{\overline{\overline{A}+\overline{B}}}+\overline{\overline{C+\overline{D}}}}$$



(5 Marks)

$$L = \overline{A} \cdot \overline{B} \cdot C \cdot D = \overline{\overline{\overline{A} \cdot \overline{B} \cdot C \cdot D}} = \overline{\left(A+B\right)+\left(\overline{C}+\overline{D}\right)} = \overline{\overline{\overline{A+B}}+\overline{\overline{\overline{C}+\overline{D}}}}$$



(5 Marks)

Solution to Q4 (d):

Letters, ***C***, ***E*** and ***I***.

(3 Marks)