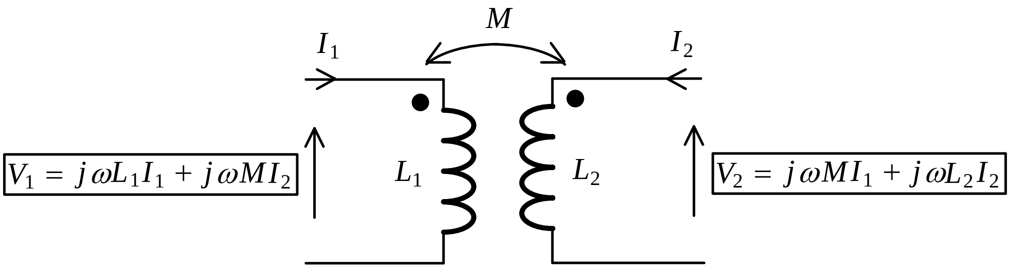
Q.3 (a) Shown in the figure below are the expressions of the primary and secondary voltages of an ideal transformer in the phasor form in AC environments.



The primary and secondary inductances of the transformer are respectively given by

$$L_1 = \frac{N_1^2}{\Re}, \quad L_2 = \frac{N_2^2}{\Re}, \quad M = \sqrt{L_1 L_2}$$

where $\Re$ is the reluctance of the transformer. Let $V_{1,\text{peak}}$ and $V_{2,\text{peak}}$ are respectively the peak values of the primary and secondary voltages of the transformer. Show that

$$\frac{V_{2,\text{peak}}}{V_{1,\text{peak}}} = \frac{N_2}{N_1}$$

(10 Marks)

Show: Consider

$$\frac{V_2}{V_1} = \frac{j\omega M I_1 + j\omega L_2 I_2}{j\omega L_1 I_1 + j\omega M I_2} = \frac{M I_1 + L_2 I_2}{L_1 I_1 + M I_2}$$

$$= \frac{\sqrt{L_1 L_2} I_1 + L_2 I_2}{L_1 I_1 + \sqrt{L_1 L_2} I_2}$$

$$= \frac{\sqrt{L_2}\left(\sqrt{L_1} I_1 + \sqrt{L_2} I_2\right)}{\sqrt{L_1}\left(\sqrt{L_1} I_1 + \sqrt{L_2} I_2\right)}$$

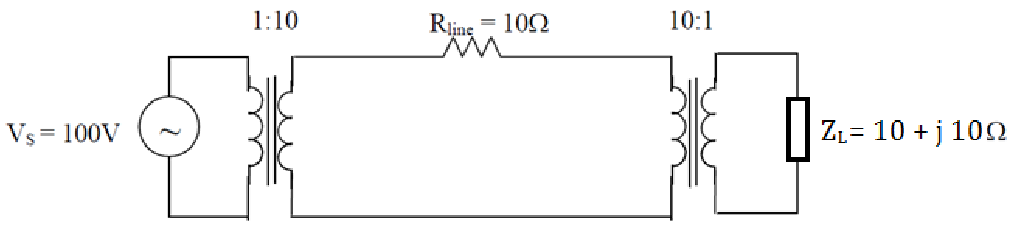
$$= \frac{\sqrt{L_2}}{\sqrt{L_1}} = \frac{\sqrt{N_2^2 / \Re}}{\sqrt{N_1^2 / \Re}}$$

$$= \frac{N_2}{N_1}$$

Thus,

$$\frac{|V_2|}{|V_1|} = \frac{N_2}{N_1} \quad \Rightarrow \quad \frac{V_{2,\text{peak}}}{V_{1,\text{peak}}} = \frac{\sqrt{2}|V_2|}{\sqrt{2}|V_1|} = \frac{N_2}{N_1}$$

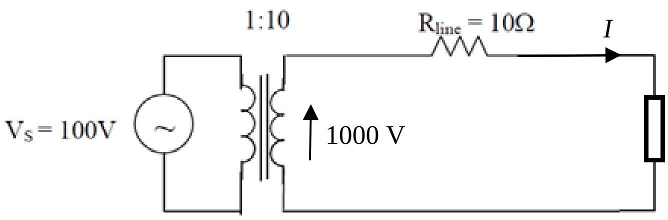
(b) The following transmission system is used to transmit electric power from the source to an AC device.



Determine the average power lost on the transmission line and the average power consumed by the load. What is the power efficiency?

(10 Marks)

Solution: Redraw the circuit as follows



$$Z_{\text{equiv}} = \frac{Z_L}{n^2} = \frac{10 + j10}{\left(1/10\right)^2}$$
$$= 1000 + j1000$$

$$I = \frac{1000}{10 + (1000 + j1000)} = \frac{1000}{1421.30e^{j\theta}} = 0.7036e^{-j\theta} \quad \left(\theta = 0.7804 \text{ rad} = 44.7149°\right)$$

$$P_{\text{lost}} = |I|^2 \cdot R_{\text{line}} = 0.7036^2 \times 10 = 4.95 \text{ W}$$

$$P_{\text{load}} = |I|^2 \cdot \text{Re}\left[Z_{\text{equiv}}\right] = 0.7036^2 \times 1000 = 495 \text{ W}$$

$$\text{power efficiency} = \frac{495}{495 + 4.95} = 99\%$$

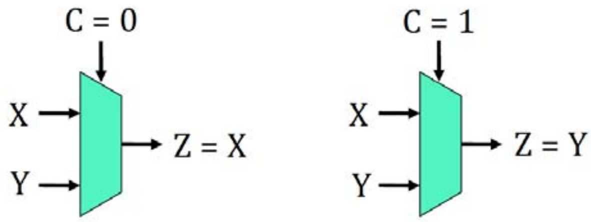
(c) Comment the fundamental difference between a rectifier circuit with and without a capacitor.

(5 Marks)

Solution: The fundamental difference is the resulting output voltages. The resulting output voltage of the rectifier circuit with a capacitor is much smoother.

This is because the capacitor is capable of storing energy. It discharges the energy stored to the load when the diode(s) are off.

Q.4 A multiplexer is a device setting its single output to the same value as one of its many inputs. Shown in the figure below is a multiplexer of two inputs, X and Y, in which X is selected when the control input C = 0 and Y is selected when C = 1.



(a) Assume that both X and Y are logical variables, construct a truth table for the above multiplexer.

(5 Marks)

(b) Obtain a logical expression for Z in the SOP form.

(5 Marks)

(c) Use Karnaugh map to simplify the expression of Z obtained in Part (b).

(5 Marks)

(d) Draw a logic circuit implementation for Z using no more than 7 two-input NOR gates (i.e., each NOR gate only has two input channels).

(10 Marks)

<u>Solution to Q4 (a):</u>

	X	Y	C	Z
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

(5 Marks)

<u>Solution to Q4 (b):</u>

$$Z = \overline{X} \cdot Y \cdot C + X \cdot \overline{Y} \cdot \overline{C} + X \cdot Y \cdot \overline{C} + X \cdot Y \cdot C$$

(5 Marks)

Solution to Q4 (c):

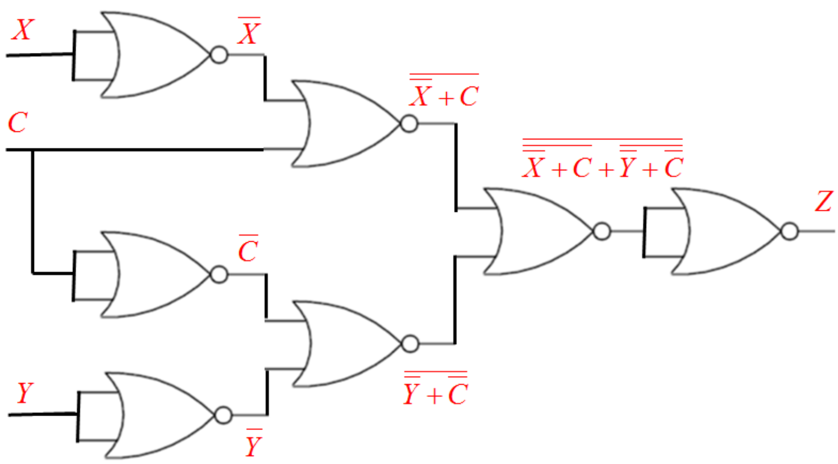
	$X \cdot Y$	$X \cdot \overline{Y}$	$\overline{X} \cdot \overline{Y}$	$\overline{X} \cdot Y$
C	1	0	0	1
$\overline{C}$	1	1	0	0

$$Z = X \cdot \overline{C} + Y \cdot C$$

(5 Marks)

Solution to Q4 (d):

$$Z = \overline{\overline{X \cdot \overline{C}}} + \overline{\overline{Y \cdot C}} = \overline{\overline{X} + C} + \overline{\overline{Y} + \overline{C}} = \overline{\overline{\overline{\overline{X} + C} + \overline{\overline{Y} + \overline{C}}}}$$



(10 Marks)