Almost every mechanical movement that we see around us is accomplished by an electric motor. Electric machines are a means of converting energy. Motors take electrical energy and produce mechanical energy. Electric motors are used to power hundreds of devices we use in everyday life. Motors come in various sizes. Huge motors that can take loads of 1000’s of Horsepower are typically used in the industry. Some examples of large motor applications include elevators, electric trains, hoists, and heavy metal rolling mills. Examples of small motor applications include motors used in automobiles, robots, hand power tools and food blenders. Micro-machines are electric machines with parts the size of red blood cells, and find many applications in medicine.

Electric motors are broadly classified into two different categories: DC (Direct Current) and AC (Alternating Current). Within these categories are numerous types, each offering unique abilities that suit them well for specific applications. In most cases, regardless of type, electric motors consist of a stator (stationary field) and a rotor (the rotating field or armature) and operate through the interaction of magnetic flux and electric current to produce rotational speed and torque. DC motors are distinguished by their ability to operate from direct current.

There are different kinds of D.C. motors, but they all work on the same principles. In this chapter, we will study their basic principle of operation and their characteristics. It’s important to understand motor characteristics so we can choose the right one for our application requirement. The learning objectives for this chapter are listed below.

**Learning Objectives:**

- Understand the basic principles of operation of a DC motor.
- Understand the operation and basic characteristics of simple DC motors.
- Compute electrical and mechanical quantities using the equivalent circuit.
- Use motor nameplate data.
- Study some applications of DC motors.

**Recommended text for this section of the course:**

(ii) Giorgio Rizzoni, *Principles and Applications of Electrical Engineering*, Chapter 17
4.1 Electromechanical Energy Conversion

An electromechanical energy conversion device is essentially a medium of transfer between an input side and an output side. Three electrical machines (DC, induction and synchronous) are used extensively for electromechanical energy conversion. Electromechanical energy conversion occurs when there is a change in magnetic flux linking a coil, associated with mechanical motion.

**Electric Motor**

The input is electrical energy (from the supply source), and the output is mechanical energy (to the load).

![Figure 1: Electric Motor](image)

**Electric Generator**

The input is mechanical energy (from the prime mover), and the output is electrical energy.

![Figure 2: Electric Generator](image)

4.2 Construction

DC motors consist of one set of coils, called armature winding, inside another set of coils or a set of permanent magnets, called the stator. Applying a voltage to the coils produces a torque in the armature, resulting in motion.

**Stator**

- The *stator* is the stationary outside part of a motor.
- The stator of a permanent magnet dc motor is composed of two or more permanent magnet pole pieces.
- The magnetic field can alternatively be created by an *electromagnet*. In this case, a DC coil (field winding) is wound around a magnetic material that forms part of the stator.

**Rotor**

- The *rotor* is the inner part which rotates.
- The rotor is composed of windings (called armature windings) which are connected to the external circuit through a mechanical commutator.
- Both stator and rotor are made of ferromagnetic materials. The two are separated by air-gap.

**Winding**

A winding is made up of series or parallel connection of coils.

- Armature winding - The winding through which the voltage is applied or induced.
- Field winding - The winding through which a current is passed to produce flux (for the electromagnet)
- Windings are usually made of copper.
4.3. DC Motor Basic Principles

4.3.1 Energy Conversion

If electrical energy is supplied to a conductor lying perpendicular to a magnetic field, the interaction of current flowing in the conductor and the magnetic field will produce mechanical force (and therefore, mechanical energy).

4.3.2 Value of Mechanical Force

There are two conditions which are necessary to produce a force on the conductor. The conductor must be carrying current, and must be within a magnetic field. When these two conditions exist, a force will be applied to the conductor, which will attempt to move the conductor in a direction perpendicular to the magnetic field. This is the basic theory by which all DC motors operate.

The force exerted upon the conductor can be expressed as follows.

\[ F = B i l \]  \hspace{1cm} \text{Newton} \hspace{1cm} \text{(1)}

where \( B \) is the density of the magnetic field, \( l \) is the length of conductor, and \( i \) the value of current flowing in the conductor. The direction of motion can be found using Fleming’s Left Hand Rule.

![Fleming’s Left Hand Rule](image)

Figure 3: Fleming’s Left Hand Rule

The first finger points in the direction of the magnetic field (first - field), which goes from the North pole to the South pole. The second finger points in the direction of the current in the wire (second - current). The thumb then points in the direction the wire is thrust or pushed while in the magnetic field (thumb - torque or thrust).

How much force will be created on a wire that is parallel to the magnetic field?
4.3.3 Principle of operation

Consider a coil in a magnetic field of flux density $B$ (figure 4). When the two ends of the coil are connected across a DC voltage source, current $I$ flows through it. A force is exerted on the coil as a result of the interaction of magnetic field and electric current. The force on the two sides of the coil is such that the coil starts to move in the direction of force.

![Figure 4: Torque production in a DC motor](image)

In an actual DC motor, several such coils are wound on the rotor, all of which experience force, resulting in rotation. The greater the current in the wire, or the greater the magnetic field, the faster the wire moves because of the greater force created.

At the same time this torque is being produced, the conductors are moving in a magnetic field. At different positions, the flux linked with it changes, which causes an $emf$ to be induced ($e = d\phi/dt$) as shown in figure 5. This voltage is in opposition to the voltage that causes current flow through the conductor and is referred to as a counter-voltage or back $emf$.

![Figure 5: Induced voltage in the armature winding of DC motor](image)

The value of current flowing through the armature is dependent upon the difference between the applied voltage and this counter-voltage. The current due to this counter-voltage tends to oppose the very cause for its production according to Lenz’s law. It results in the rotor slowing down. Eventually, the rotor slows just
enough so that the force created by the magnetic field \( F = Bil \) equals the load force applied on the shaft. Then the system moves at constant velocity.

**4.3.4 Torque Developed**

The equation for torque developed in a DC motor can be derived as follows.

The force on one coil of wire \( F = i l \times B \) Newton

Note that \( l \) and \( B \) are vector quantities

Since \( B = \phi/A \) where \( A \) is the area of the coil,

Therefore the **torque** for a multi turn coil with an armature current of \( I_a \):\[
T = K \phi I_a
\] \hspace{1cm} (2)

Where \( \phi \) is the flux/pole in weber, \( K \) is a constant depending on coil geometry, and \( I_a \) is the current flowing in the armature winding.

**Note:** Torque \( T \) is a function of force and the distance, equation (2) lumps all the constant parameters (eg. length, area and distance) in constant \( K \).

The mechanical power generated is the product of the machine torque and the mechanical speed of rotation, \( \omega_m \)

Or, \( P_m = \omega_m T \)

\( = \omega_m K \phi I_a \) \hspace{1cm} (3)

It is interesting to note that the same DC machine can be used either as a motor or as a generator, by reversing the terminal connections.

**4.3.5 Induced Counter-voltage (Back emf):**

Due to the rotation of this coil in the magnetic field, the flux linked with it changes at different positions, which causes an *emf* to be induced (refer to figure 5).

The induced *emf* in a single coil, \( e = d\phi_c/dt \)

Since the flux linking the coil, \( \phi_c = \phi \sin \omega t \)

Induced voltage : \( e = \omega \phi \cos \omega t \) \hspace{1cm} (4)
Note that equation (4) gives the emf induced in one coil. As there are several coils wound all around the rotor, each with a different emf depending on the amount of flux change through it, the total emf can be obtained by summing up the individual emfs.

The total emf induced in the motor by several such coils wound on the rotor can be obtained by integrating equation (4), and expressed as:

$$E_b = K \phi \omega_m$$

(5)

where $K$ is an armature constant, and is related to the geometry and magnetic properties of the motor, and $\omega_m$ is the speed of rotation.

The electrical power generated by the machine is given by:

$$P_{dev} = E_b I_a = K \phi \omega_m I_a$$

(6)

### 4.3.6 DC Motor Equivalent circuit

The schematic diagram for a DC motor is shown below. A DC motor has two distinct circuits: Field circuit and armature circuit. The input is electrical power and the output is mechanical power. In this equivalent circuit, the field winding is supplied from a separate DC voltage source of voltage $V_f$. $R_f$ and $L_f$ represent the resistance and inductance of the field winding. The current $I_f$ produced in the winding establishes the magnetic field necessary for motor operation. In the armature (rotor) circuit, $V_T$ is the voltage applied across the motor terminals, $I_a$ is the current flowing in the armature circuit, $R_a$ is the resistance of the armature winding, and $E_b$ is the total voltage induced in the armature.

![Figure 7: DC Motor representation](image)

### 4.3.7 Voltage Equation

Applying KVL in the armature circuit of Figure 7:

$$V_T = E_b + I_a R_a$$

(7)

where $V_T$ is voltage applied to the armature terminals of the motor and $R_a$ is the resistance of the armature winding.

*Note:* The induced voltage is typically represented by symbol $e$ (or $E$) and the terminal voltage by $v$ (or $V$).

At standstill, the motor speed is zero, therefore back emf is also zero. The armature current at starting is thus very large.

Applying KVL in the field circuit of Figure 7:

$$V_f = R_f I_f$$

(8)
Where \( V_f \) is voltage applied to the field winding (to produce the magnetic field), \( R_f \) is the resistance of the field winding, and \( I_f \) is the current through the field winding.

**How would the inductance of the field winding affect the motor operation under steady-state?**

### 4.3.8 Power Transfer Equation

We have earlier obtained the following relationship for torque developed in the motor (from equation 2):

\[
T_{dev} = K \phi I_a
\]

The developed power is the power converted to mechanical form, and is given by (from equation 3):

\[
P_{dev} = \omega_m T_{dev}
\]

This is the power delivered to the induced armature voltage (counter-voltage) and given by:

\[
E_b I_a \text{ (electrical power)} = \omega_m T_{dev} \text{ (mechanical power developed)} \tag{9}
\]

Note: The speed in revolutions per minute, \( N \), is related to the angular speed \( \omega \) (in radians per second) by

\[
\omega = \frac{2 \pi N}{60}
\]

\( N \) can be written as r/min or rpm, both mean the same thing.

Noting that the flux in the machine is proportional to the current flowing in the field winding (i.e. \( \phi \propto I_f \)), we can compare induced voltages at two different speeds.

If the induced voltage at the first operating speed \( N_1 \), and field winding current \( I_{f1} \) is given by:

\[
E_{b1} = K (K_f I_{f1}) \left( \frac{2 \pi N_1}{60} \right)
\]

and the induced voltage at the first operating speed \( N_2 \), and field winding current \( I_{f2} \) is given by:

\[
E_{b2} = K (K_f I_{f2}) \left( \frac{2 \pi N_2}{60} \right)
\]

Then the induced voltages at these operating points can be compared as:

\[
\frac{E_{b1}}{E_{b2}} = \frac{I_{f1} N_1}{I_{f2} N_2}
\]

This equation is useful in determining the speed of the DC motor at different operating conditions.
4.4 DC Machine Classification

DC Machines can be classified according to the electrical connections of the armature winding and the field windings. The different ways in which these windings are connected lead to machines operating with different characteristics. The field winding can be either self-excited or separately-excited, that is, the terminals of the winding can be connected across the input voltage terminals or fed from a separate voltage source (as in the previous section). Further, in self-excited motors, the field winding can be connected either in series or in parallel with the armature winding. These different types of connections give rise to very different types of machines, as we will study in this section.

4.4.1 Separately excited machines

- The armature and field winding are electrically separate from each other.
- The field winding is excited by a separate DC source.

![Separately excited DC Motor](image)

The voltage and power equations for this machine are same as those derived in the previous section.

Note that the total input power $= V_f I_f + V_T I_a$

4.4.2 Self excited machines

In these machines, instead of a separate voltage source, the field winding is connected across the main voltage terminals.

**Shunt machine**

- The armature and field winding are connected in parallel.
- The armature voltage and field voltage are the same.

![Shunt DC Motor](image)

Notice that in this type of motor,
Total current drawn from the supply, $I_L = I_f + I_a$ 
Total input power = $V_T I_L$

Voltage, current and power equations are given in equations (7), (8) and (9).

**Series DC machine**

- The field winding and armature winding are connected in series.
- The field winding carries the same current as the armature winding.

A series wound motor is also called a *universal* motor. It is universal in the sense that it will run equally well using either an ac or a dc voltage source.

Reversing the polarity of both the stator and the rotor cancel out. Thus the motor will always rotate the same direction irregardless of the voltage polarity.

![Series DC Motor Diagram](image)

**Compound DC machine**

If both series and shunt field windings are used, the motor is said to be compounded. In a compound machine, the series field winding is connected in series with the armature, and the shunt field winding is connected in parallel. Two types of arrangements are possible in compound motors:

- **Cumulative compounding** - If the magnetic fluxes produced by both series and shunt field windings are in the same direction (i.e., additive), the machine is called cumulative compound.
- **Differential compounding** - If the two fluxes are in opposition, the machine is differential compound.

In both these types, the connection can be either short shunt or long shunt.

*Note: Compound motors will not be discussed in this course.*

### 4.5 Performance calculations

In most applications, DC motors are used for driving mechanical loads. Some applications require that the speed remain constant as the load on the motor changes. In some applications the speed is required to be controlled over a wide range. It is therefore important to study the relationship between torque and speed of the motor.

#### 4.5.1 Speed Regulation
The performance measure of interest is the speed regulation, defined as the change in speed as full load is applied to the motor. It can be expressed as:

\[
\text{Speed regulation (SR)} = \left( \frac{N_{\text{no-load}} - N_{\text{full-load}}}{N_{\text{full-load}}} \right) \times 100\% \tag{10}
\]

Where \(N_{\text{no-load}}\) is the speed at no load, and \(N_{\text{full-load}}\) is the speed when full load is applied.

4.5.2 Torque-Speed Characteristics:

In order to effectively use a D.C. motor for an application, it is necessary to understand its characteristic curves. For every motor, there is a specific Torque/Speed curve and Power curve. The relation between torque and speed is important in choosing a DC motor for a particular application.

**Separately excited DC motor**

A separately excited DC motor equivalent circuit is shown in Figure 8.

From equation (5) and equation (7), we have two expressions for the induced voltage. Comparing the two:

\[
E_b = K \phi \omega_m = V_T - I_a R_a \tag{11}
\]

The torque developed in the rotor (armature) is given by:

\[
T_{\text{dev}} = K \phi I_a \tag{12}
\]

From equation (12), the current in the armature winding can be found as:

\[
I_a = \frac{T_{\text{dev}}}{K \phi} \tag{13}
\]

Substituting for \(I_a\) in equation (11) and rearranging the terms:

\[
V_T - K \phi \omega_m = R_a \left( \frac{T_{\text{dev}}}{K \phi} \right) \tag{14}
\]

Therefore, the torque developed in the rotor can be expressed as:

\[
T_{\text{dev}} = \frac{K \phi}{R_a} (V_T - K \phi \omega_m) \tag{15}
\]

This equation shows the relationship between the torque and speed of a separately excited DC motor. If the terminal voltage \(V_T\) and flux \(\phi\) are kept constant, the torque-speed relationship is a straight drooping line.

**Figure 11: Torque-speed characteristics of separately excited DC motor**

The plotted points are for a normal operating range and stall torque. The no-load speed is \(V_T/k\phi\).
The graph above shows a torque/speed curve of a separately excited D.C. motor. Note that torque is inversely proportional to the speed of the output shaft. In other words, there is a tradeoff between how much torque a motor delivers, and how fast the output shaft spins. Motor characteristics are frequently given as two points on this graph:

- The stall torque, represents the point on the graph at which the torque is a maximum, but the shaft is not rotating.
- The no load speed, is the maximum output speed of the motor (when no torque is applied to the output shaft).

The motor load determines the final operating point on the torque curve. As illustrated in the figure below, when a motor is connected to drive a load, the interaction of torque demanded by the load and the torque produced by the motor determines the point of operation.

\[ T_{load} + T_{dev} = T_{load} \]

\[ \omega_m = \frac{V_f}{R_a} + \frac{E_b}{R_a} \]

**Figure 12: Interaction of the DC Motor and Mechanical Load**

The above graph shows the interaction of DC motor and mechanical load. The starting torque of the motor is higher than the load torque demanded by the load. The difference between these two torques forces the motor to rotate. As the motor starts to rotate and picks up speed, the developed torque decreases (why?). The motor finally comes to a stable operating point when the two torques balance each other.

**DC Shunt Motor**

The DC shunt motor has the same equations for torque as for the separately excited motor, and has the same torque-speed characteristics as in Figure 11.

**DC Series Motor**

The DC series motor characteristics can be analyzed in much the same way as the shunt motor discussed earlier.

In series motors, the series field winding is connected in series with the armature (refer to figure 10).

The torque developed in the rotor is:

\[ T_{dev} = K \phi I_a \]

Assuming that the flux is directly proportional to field current (i.e. no magnetic saturation),
\( \phi \propto I_f \)

Since in a series motor, \( I_f = I_a \)

\( \phi = K_f I_a \)

where \( K_f \) is a constant that depends on the number of turns in the field winding, the geometry of the magnetic circuit and the \( B-H \) characteristics of iron.

Therefore, the torque developed in the rotor can be expressed as:

\[
T_{\text{dev}} = (K_f I_a)(K I_a) = K' I_a^2
\]

(17)

Applying KVL to the equivalent circuit (and ignoring \( L_f \) under steady state conditions),

\[
V_T = R_f I_a + R_a I_a + E_b
\]

(18)

But induced voltage can be expressed as

\[
E_b = K \phi \omega_m = K (K_f I_a) \omega_m = K' I_a \omega_m
\]

Substituting this in equation (18), we get

\[
I_a = \frac{V_T}{R_a + R_f + K' \omega_m}
\]

(19)

The torque developed:

\[
T_{\text{dev}} = \frac{K' V_T^2}{(R_a + R_f + K' \omega_m)^2}
\]

(20)

From this equation, if the terminal voltage \( V_T \) is kept constant, the speed is almost inversely proportional to the square root of the torque (figure 13). A high torque is obtained at low speed and a low torque is obtained at high speed.

\[ Figure 10 \text{ (redrawn): DC Series motor equivalent circuit} \]

\[ Figure 13: \text{Torque-speed characteristics of a series motor} \]
4.5.3 Efficiency

As power flows from DC motor input terminals to the output (shaft), some losses take place. Figure 14 shows the flow of power in a separately excited DC Motor.

Efficiency of the motor can be calculated as the ratio of output power to the total input power.

Referring to figure 8, total electrical input power,

\[ P_{in} = V_I I_a + V_f I_f \]  \hspace{1cm} (21)

where \( I_a \) is the armature current drawn from the supply terminals.

Power absorbed by the field winding is in turn converted to heat and is given by:

\[ P_{field-loss} = I_f^2 R_f \]  \hspace{1cm} (22)

\[ = \left( \frac{V_f}{R_f} \right)^2 R_f = \frac{V_f^2}{R_f} \]  \hspace{1cm} (23)

Some power is lost in the resistance of the armature winding, and can be calculated as:

\[ P_{arm-loss} = I_a^2 R_a \]  \hspace{1cm} (24)

The total power loss taking place in the two windings (which are made of copper) is the total copper loss.

Total copper loss = Field loss + Armature loss

Power developed and converted into mechanical power,

\[ \text{Power developed} = \text{Input power} - \text{Total copper loss} \]

Also,

\[ P_{dev} = I_a E_b = \omega_m T_{dev} \]  \hspace{1cm} (25)

The output power and torque are less than the developed values because of rotational losses, which include friction, windage, eddy-current and hysteresis losses. Rotational power loss is approximately proportional to motor speed.

From figure 14, we can find the final output power as:

\[ \text{Power output} = \text{Power developed} - \text{rotational losses} \]

The efficiency of the DC motor can be calculated as:

\[ \text{Efficiency} \; \eta = \frac{P_{out}}{P_{in}} \times 100\% \]  \hspace{1cm} (26)

This equation can also be expressed in terms of power losses in the motor:

\[ \text{Efficiency} = \frac{P_{out}}{P_{out} + P_{arm-loss} + P_{field-loss} + \text{Rotational loss}} \]  \hspace{1cm} (27)
It is important to mention that the total input and output power can be calculated in many different ways using the power flow diagram, depending on the information given. Also note that the torque developed inside the rotor is different from the final (output) torque supplied to the load due to rotational losses.

**DC Shunt Motor Power Flow**

- The losses and efficiency in a DC shunt motor can be calculated in a similar manner to that shown above, except that in this case \( \text{Power input} = V_I I_L \) (where \( I_L = I_a + I_f \)).

**DC Series Motor Power Flow**

- The losses and efficiency in a DC series motor can be calculated in a similar manner to that for DC shunt motor using the equations derived earlier, except that in this case \( I_f = I_a = I_L \).

4.5.4 DC Motor Rating

DC Motors are typically rated in terms of:

**Rated voltage:**

the operating voltage on the input side of the motor

**Rated power:**

Power (in horsepower – hp or watts) that the motor is designed to deliver to the load (i.e., output power) for continuous operation.  
(Note that 1 hp = 746 W)

**Rated speed**

Speed (in revolutions per minute, denoted by \( r/min \) or \( rpm \)) for which the motor is designed to operate for continuous operation.

**Rated load**
The load which the motor is designed to carry for (theoretically) infinite period of time. “Full load” or “rated load” operating condition refers to the operation of motor when it is delivering rated power to the load.

Note: A motor may not always operate at its rated power and/or speed. Operation above these values is not advisable due to overloading.

Example 1

A 230 V, 10 hp d.c. shunt motor delivers power to a load at 1200 r/min. The armature current drawn by the motor is 200 A. The armature circuit resistance of the motor is 0.2 \( \Omega \) and the field resistance is 115 \( \Omega \). If the rotational losses are 500W, what is the value of the load torque?

Solution

The back emf induced in the armature is:

\[ E_b = V_T - I_a R_a \]

\[ = 230 - 0.2 \times 200 = 190 \text{ V} \]

Power developed (in the rotor), \( P_{dev} = E_b I_a = 190 \times 200 = 38000 \text{ W} \)

Power delivered to the load, \( P_{load} = P_{dev} - P_{rot} = P_{out} \)

\[ = 38000 - 500 = 37500 \text{ W} \]

Load torque, \( T_{load} = \frac{P_{load}}{\omega_m} \)

where \( \omega_m = \frac{2\pi N}{60} \)

where \( N \) is the speed in revolutions per minute (r/min). The torque supplied to the load can be calculated as:

\[ T_{load} = \frac{37500}{\left(\frac{2\pi}{60}\right) \times 1200} = 298.4 \text{ N.m} \]

Further investigation: What will be the efficiency of the motor in this example?

Example 2

A series-connected DC motor has an armature resistance of 0.5 \( \Omega \) and field winding resistance of 1.5 \( \Omega \). In driving a certain load at 1200 rpm, the current drawn by the motor is 20A from a voltage source of \( V_T = 220 \text{ V} \). The rotational loss is 150W. Find the output power and efficiency.

Solution

Total input power:

\[ P_{in} = V_T I_a = 220 \times 20 = 4400 \text{ W} \]

Induced voltage,

\[ E_b = V_T - (R_f + R_a)I_a = 180 \text{ V} \]

Power developed in the armature can be calculated as:

\[ P_{dev} = E_b I_a = 180 \times 20 = 3600 \text{ W} \]

Output power delivered to the load:
\[ P_{\text{out}} = P_{\text{dev}} - P_{\text{rot}} = 3600 - 150 = 3450 \text{ W} \]

Therefore, efficiency can be calculated as:
\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = 78.41\% \]

### 4.6 DC Motor Speed Control

Many applications require the speed of a motor to be varied over a wide range. One of the most attractive features of DC motors in comparison with AC motors is the ease with which their speed can be varied.

We know that the back emf for a separately excited DC motor:
\[ E_b = K \phi \omega_m = V_T - I_a R_a \]

Rearranging the terms,
\[ \text{Speed} \quad \omega_m = \frac{(V_T - I_a R_a)}{K \phi} \quad (28) \]

From this equation, it is evident that the speed can be varied by using any of the following methods:
- Armature voltage control (By varying \( V_T \))
- Field Control (By Varying \( \phi \))
- Armature resistance control (By varying \( R_a \))

#### 4.6.1 Armature voltage control

This method is usually applicable to separately excited DC motors. In this method of speed control, \( R_a \) and \( \phi \) are kept constant.

In normal operation, the drop across the armature resistance is small compared to \( E_b \) and therefore:
\[ E_b \approx V_T \]

Since, \( E_b = K \phi \omega_m \)
Angular speed can be expressed as:
\[ \omega_m \approx \frac{V_T}{K \phi} \quad (29) \]

From this equation,
- If flux is kept constant, the speed changes linearly with \( V_T \).
- As the terminal voltage is increased, the speed increases and vice versa.

The relationship between speed and applied voltage is shown in figure 15. This method provides smooth variation of speed control.
4.6.2 Field Control ($\phi$)

In this method of speed control, $R_a$ and $V_T$ remain fixed.

Therefore, from equation (28):

$$\omega_m \propto 1/\phi$$

Assuming magnetic linearity, $\phi \propto I_f$

Or,

$$\omega_m \propto 1/I_f$$

(30)

i.e., **Speed can be controlled by varying field current $I_f$.**

- The field current can be changed by varying an adjustable rheostat in the field circuit (as shown in figure 16).
- By increasing the value of total field resistance, field current can be reduced, and therefore speed can be increased.

The relationship between the field winding current and angular speed is shown in figure 17.
4.6.3 Armature Resistance Control

The voltage across the armature can be varied by inserting a variable resistance in series with the armature circuit.

\[
T_{\omega m} = \frac{K\phi}{R_a}(V_T - K\phi\omega_m)
\]

For a load of constant torque, if \( V_T \) and \( \phi \) are kept constant, as the armature resistance \( R_a \) is increased, speed decreases. As the actual resistance of the armature winding is fixed for a given motor, the overall resistance in the armature circuit can be increased by inserting an additional variable resistance in series with the armature. The variation if speed with respect to change in this external resistance is shown in figure 19. This method provides smooth control of speed.

\[\text{Figure 18: Armature resistance method for speed control}\]

\[\text{Figure 19: Variation of speed with external armature resistance}\]

**DC Shunt Motor speed control**

All three methods described above can be used for controlling the speed of DC Shunt Motors.

**Series Motor speed control**

The speed is usually controlled by changing an external resistance in series with the armature. The other two methods described above are not applicable to DC series motor speed control.

4.7 DC Motor Starting

If connected directly across the supply, the starting current is dangerously high.

\[I_a = \frac{(V_T - E_b)}{R_a}\]

At standstill, \( E_b = 0 \), therefore the starting current

\[I_{a,\text{start}} = \frac{V_T}{R_a}\]
The starting current can be reduced by:

Reducing $V_T$ during starting
- requires a variable voltage supply

Increasing resistance in the armature circuit
- an additional resistance $R_{\text{ext}}$ can be connected in the armature circuit

What happens when a DC motor is connected across an AC supply?