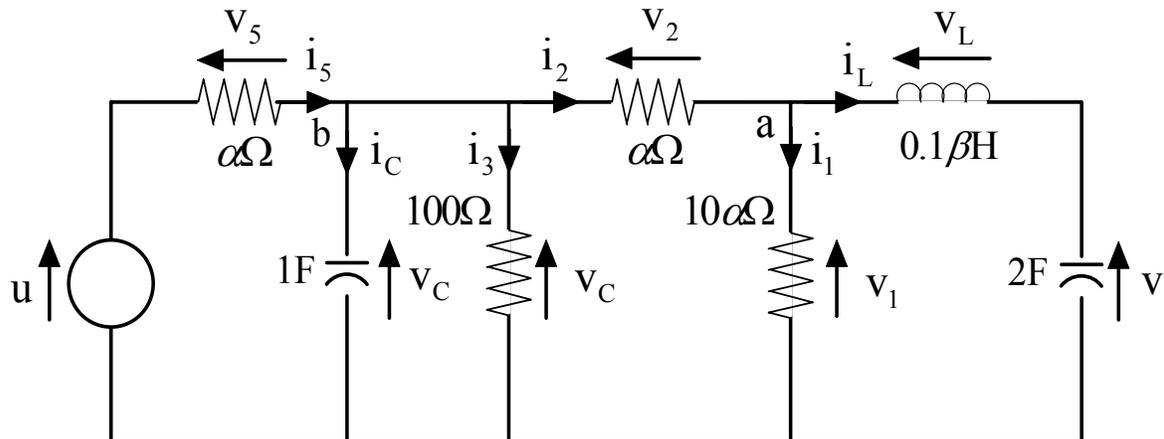

EE 2008
Circuits Take-Home Project

I have neither given nor received aid from others in this take-home project.

Seah Yong Huang, Lester (U017207J)
E21B



2. Identification of lucky parameter

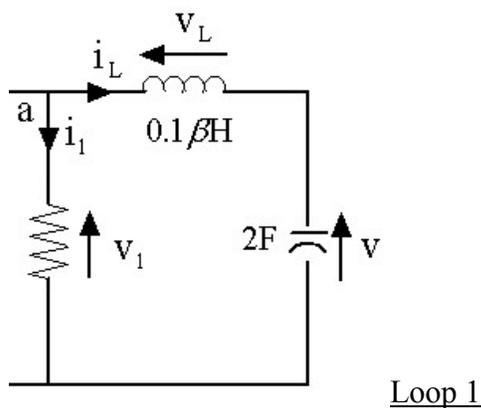
The lucky parameter taken according to the first letter of my family name and given name (Seah Yong Huang, Lester) are $\alpha = 19$ and $\beta = 25$.

3. Circuit Analysis

3 (a)

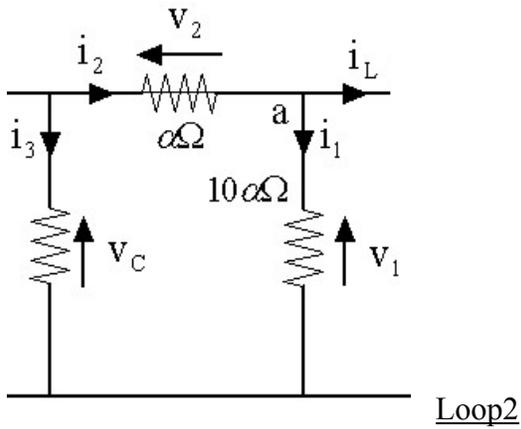
The first step to the analysis is to assign current variable to capacitors and voltage variable to inductors as in the diagram above. Next, we will identify the state variables.

They are $i_L = 2 \frac{dv}{dt}$, $i_c = \frac{dv_c}{dt}$ and $v_L = 0.1\beta \frac{di_L}{dt}$. The first state equation $\frac{dv}{dt} = 0.5i_L$ is easily obtained from the first state variable.



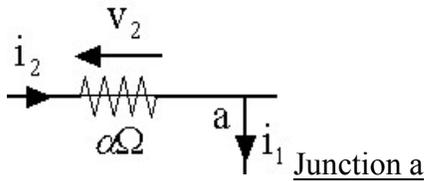
Applying KVL at loop 1, we have

$$\begin{aligned} v_1 &= v_L + v \\ &= 0.1\beta \frac{di_L}{dt} + v \end{aligned}$$



Applying KVL at loop 2, we get

$$\begin{aligned}
 v_c &= v_1 + v_2 \\
 &= 0.1\beta \frac{di_L}{dt} + v + v_2 \\
 &= 0.1\beta \frac{di_L}{dt} + v + i_2\alpha \quad \text{---(1)}
 \end{aligned}$$



Applying KCL at junction a, we get

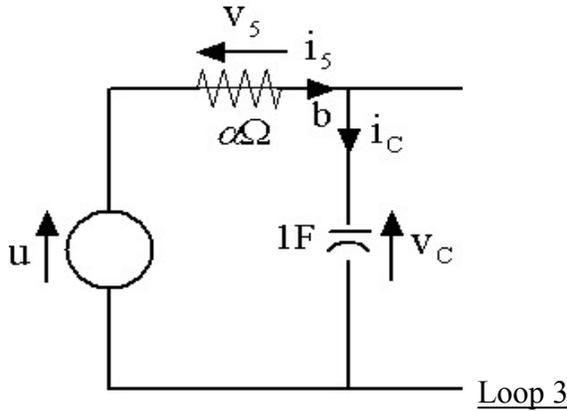
$$\begin{aligned}
 i_2 &= i_1 + i_L \\
 &= \frac{v_1}{10\alpha} + i_L \quad \text{---(2)}
 \end{aligned}$$

Substituting equation (2) into (1), we get

$$v_c = 0.1\beta \frac{di_L}{dt} + v + \alpha \left(\frac{1}{10\alpha} \left(0.1\beta \frac{di_L}{dt} + v \right) + i_L \right)$$

By rearranging the equation, we get the second state equation

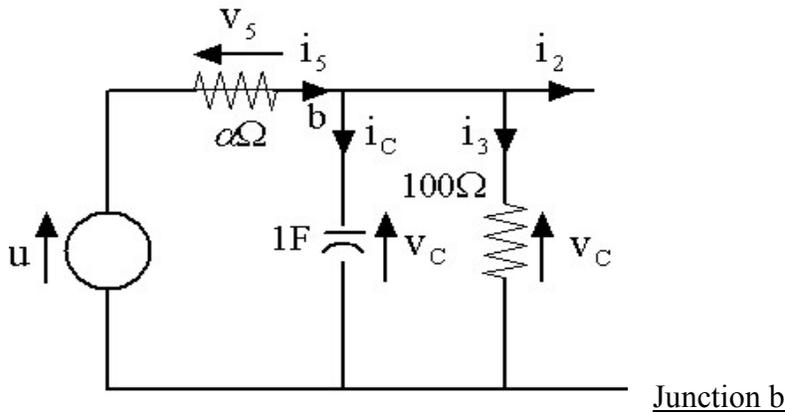
$$\frac{di_L}{dt} = \frac{100}{11\beta} v_c - \frac{100\alpha}{11\beta} i_L - \frac{10}{\beta} v$$



Applying KVL at loop 3, we get

$$v_c = u - i_5 \alpha$$

$$i_5 = \frac{u - v_c}{\alpha} \quad (3)$$



Applying KCL at junction b, we get

$$i_5 = i_c + i_3 + i_2$$

$$= \frac{dv_c}{dt} + \frac{v_c}{100} + \frac{v_1}{10\alpha} + i_L$$

$$= \frac{dv_c}{dt} + \frac{v_c}{100} + \frac{1}{10\alpha} \left[0.1\beta \frac{di_L}{dt} + v \right] + i_L \quad (4)$$

Substituting equation (3) and the state equation we obtain for $\frac{di_L}{dt}$ into equation (4), we get

$$\frac{u - v_c}{\alpha} = \frac{dv_c}{dt} + \frac{v_c}{100} + \frac{1}{10\alpha} \left[0.1\beta \left(\frac{100}{11\beta} \right) \left(v_c - \alpha i_L - \frac{11}{10} v \right) + v \right]$$

By rearranging the equation, we get the third state equation

$$\frac{dv_c}{dt} = - \left(\frac{12}{11\alpha} + \frac{1}{100} \right) v_c - \frac{10}{11} i_L + \frac{u}{\alpha}$$

Consolidating the three state equations, we get

$$\begin{aligned}\frac{dv}{dt} &= 0.5i_L \\ \frac{dv_c}{dt} &= -\left(\frac{12}{11\alpha} + \frac{1}{100}\right)v_c - \frac{10}{11}i_L + \frac{u}{\alpha} \\ \frac{di_L}{dt} &= \frac{100}{11\beta}v_c - \frac{100\alpha}{11\beta}i_L - \frac{10}{\beta}v\end{aligned}$$

Substituting the lucky numbers $\alpha = 19$ and $\beta = 25$ into the 3 states equation, we get

$$\begin{aligned}\frac{dv}{dt} &= 0.5i_L \\ \frac{dv_c}{dt} &= -0.0674v_c - 0.9091i_L + 0.0526u \\ \frac{di_L}{dt} &= -0.4v + 0.3636v_c - 6.9090i_L\end{aligned}$$

By letting $v' = \frac{dv}{dt}$, $v_c' = \frac{dv_c}{dt}$ and $i_L' = \frac{di_L}{dt}$, the state equations in compact matrix form is

$$\begin{pmatrix} v' \\ v_c' \\ i_L' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0.5 \\ 0 & -0.0674 & -0.9091 \\ -0.4 & 0.3636 & -6.9090 \end{pmatrix} \begin{pmatrix} v \\ v_c \\ i_L \end{pmatrix} + \begin{pmatrix} 0 \\ 0.0526 \\ 0 \end{pmatrix} u$$

and this corresponds to $x' = Ax + Bu$.

The output equation in compact matrix form is

$$v = (1 \quad 0 \quad 0) \begin{pmatrix} v \\ v_c \\ i_L \end{pmatrix} + 0u$$

and this corresponds to $y = Cx + Du$.

3(b)

To relieve us from the chores of manual calculation, we will use MATLAB to aid us in the calculation of the transfer function of the system. First, the variables A, B, C and D will be assigned to their corresponding matrices. Then enter the following exactly into the command window:

```
>> format short e
>> [NUM, DEN] = SS2TF(A, B, C, D, 1)
```

Finally, we obtain $NUM = 0 \quad -4.4409e-015 \quad -5.5511e-016 \quad 9.5692e-003$ and $DEN = 1.0000e+000 \quad 6.9764e+000 \quad 9.9636e-001 \quad 1.3483e-002$. Ignoring the s^2 and s terms as they are insignificant, we get the transfer function of the system to be

$$H(s) = \frac{0.0096}{s^3 + 6.9764s^2 + 0.9964s + 0.0135}$$

To compute the poles of the system, enter the following exactly into the command window:

```
>> roots(DEN)
```

and we obtain the poles of the system to be:

```
ans =
```

```
-6.8309
```

```
-0.1304
```

```
-0.0151
```

As all the poles are found in the left half complex plane, the system is considered stable. Using inverse laplace transform, we get the transfer function to be made up of the terms $e^{-6.8309t}$, $e^{-0.1304t}$ and $e^{-0.0151t}$. Thus, as time tends to infinity, the output response will reach a saturation level and this implies that the previous conclusion is correct.

2(c)

To derive the zero input response, simply replace variable B with x_0 when inputting into the command window.

```
>>[NUM1, DEN1] = SS2TF(A, x0, C, D, 1)
```

```
>>[numerator,denominator,direct] = residue(NUM1,DEN1)
```

$$\begin{aligned} C(sI - A)^{-1}x_0 &= \frac{-0.2s^2 + 0.1047 - 0.09451}{s^3 + 6.976s^2 + 0.9964s + 0.01348} \\ &= \frac{-0.2221}{s + 6.8309} + \frac{0.1444}{s + 0.1304} + \frac{-0.1223}{s + 0.0151} \end{aligned}$$

Then take inverse laplace transform of the above expression in partial fractions, we get the zero-input response to be

$$y_0(t) = -0.2221e^{-6.8309t} + 0.1444e^{-0.1304t} - 0.1223e^{-0.0151t}$$

As $u_0 = 1$, by taking laplace transform of u_0 , we get $U(s) = \frac{1}{s}$.

Multiplying $U(s)$ to the transfer function that we derived in part (b), we get

$$\begin{aligned} [C(sI - A)^{-1}B + D]U(s) &= \frac{0.0096}{s^3 + 6.9764s^2 + 0.9964s + 0.0135} \times \frac{1}{s} \\ &= \frac{-0.00003}{s + 6.8309} + \frac{0.0950}{s + 0.1304} + \frac{-0.8046}{s + 0.0151} + \frac{0.7097}{s} \end{aligned}$$

Taking inverse laplace transform of the above expression, we get the response to the 1V input signal

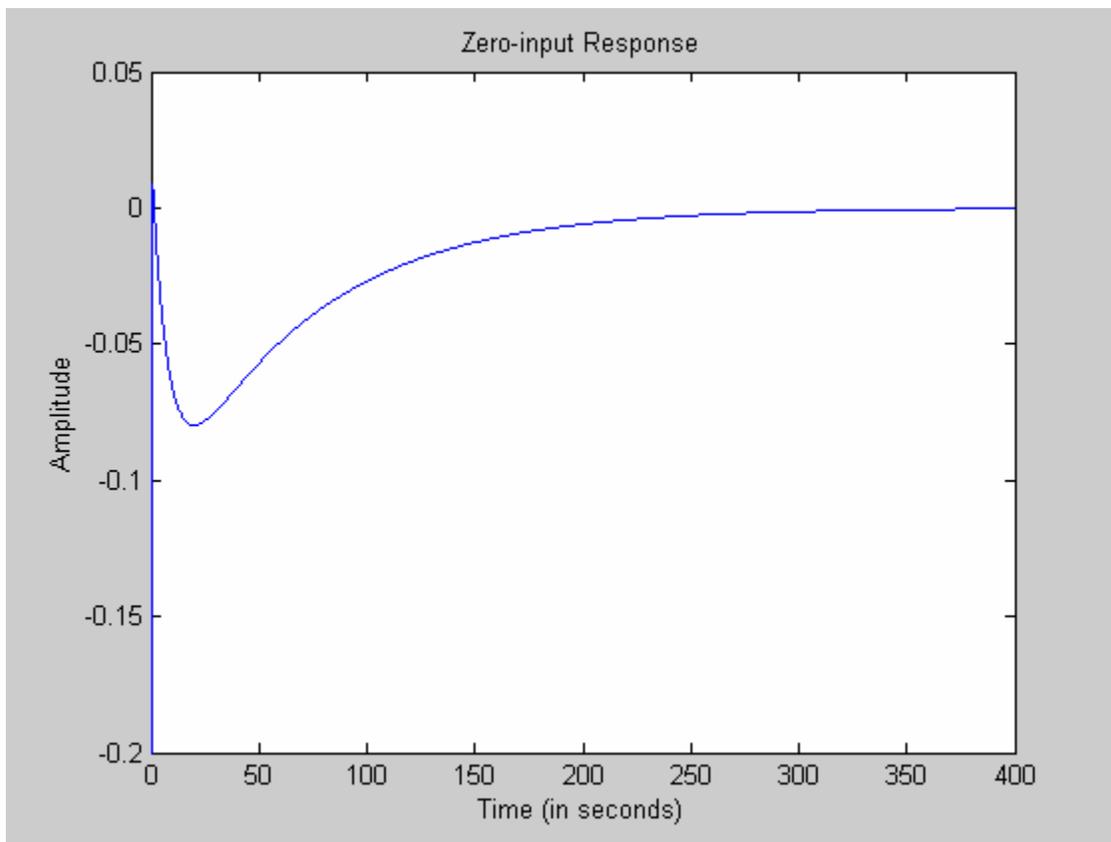
$$y_i(t) = -0.00003e^{-6.8309t} + 0.0950e^{-0.1304t} - 0.8046e^{-0.0151t} + 0.7097$$

Adding up, we get the complete response to be

$$\begin{aligned} y(t) &= y_i(t) + y_0(t) \\ &= -0.22213e^{-6.8309t} + 0.2394e^{-0.1304t} - 0.9269e^{-0.0151t} + 0.7097 \end{aligned}$$

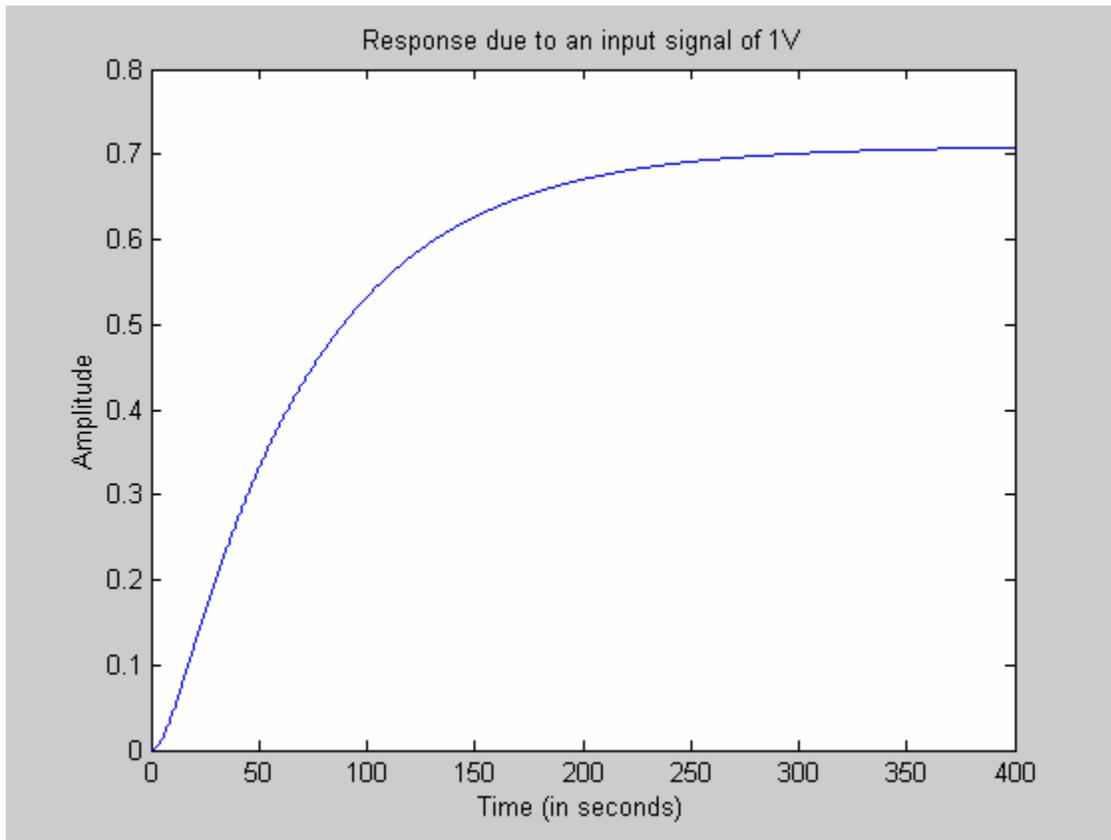
To plot the zero-input response graph, we need to assign a polynomial in MATLAB to hold the equation of $y_0(t)$ and assign a time variable t that hold values from 0 to 400 in step of 0.01 seconds.

```
>> t = 0:0.01:400;  
>> y0 = -0.2221*exp(-6.8309*t)+0.1444*exp(-0.1304*t)-0.1223*exp(-0.0151*t);  
>> plot(t,y0)
```



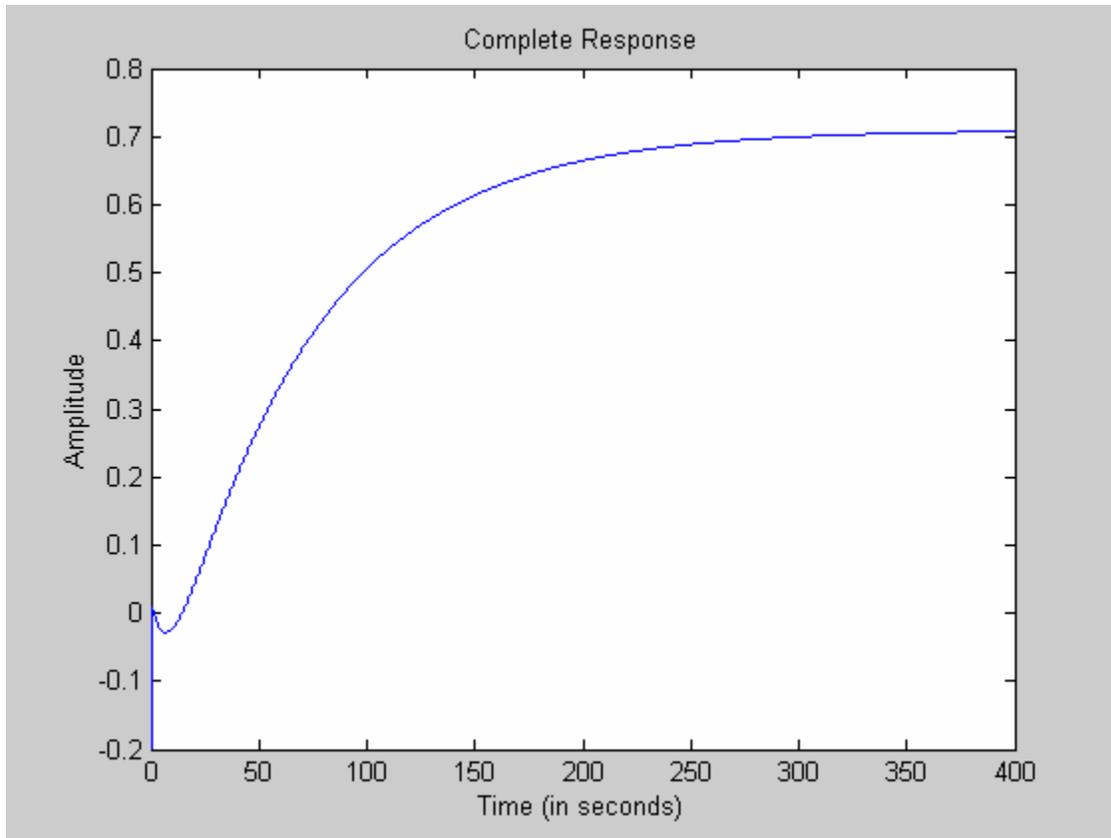
To plot the response to the 1V input signal, we assign $y_i(t)$. For the t variable, we can use the one assigned previously.

```
>> yt = -0.00003*exp(-6.8309*t)+0.0950*exp(-0.1304*t)-0.8046*exp(-0.0151*t)+0.7097  
>> plot(t,yt)
```



To plot the complete response graph, we need to assign $y(t)$. For the t variable, we can use the one assigned previously.

```
>> y = -0.2221*exp(-6.8309*t)+0.2394*exp(-0.1304*t)-0.9269*exp(-0.0151*t)+0.7097;  
>> plot(t,y)
```



It can be observed that the zero input response has a transient state of about 300s before reaching a steady state at 0V. The complete response curve is obtained by adding both the step response and the zero input curves. Thus, as zero input response goes to zero, the complete response curve follows closely to the step input curve during the transient state of about 300s and eventually reaches an output voltage approximately 0.7075 V. The complete response reaching a steady state at 0.7075V further verified that the system is stable.

Verification of the above results

To find zero input response and the complete response due to an input signal $u = 1V$, we will first set the time from $t = 0$ to $t = 400$.

```
>> T = 0:0.01:400;
```

Then, generate step response of the system corresponding to the specified time points given in T.

```
>> G1 = step(A,B,C,D,1,T);
```

To generate the zero input response, we must set the initial conditions of the system.
After setting the initial conditions of the system,

```
>> x0 = [-0.2;-0.2;3];
```

we then proceed on to use the initial function to generate the result.

```
>> G0 = initial(A,B,C,D,x0,T);
```

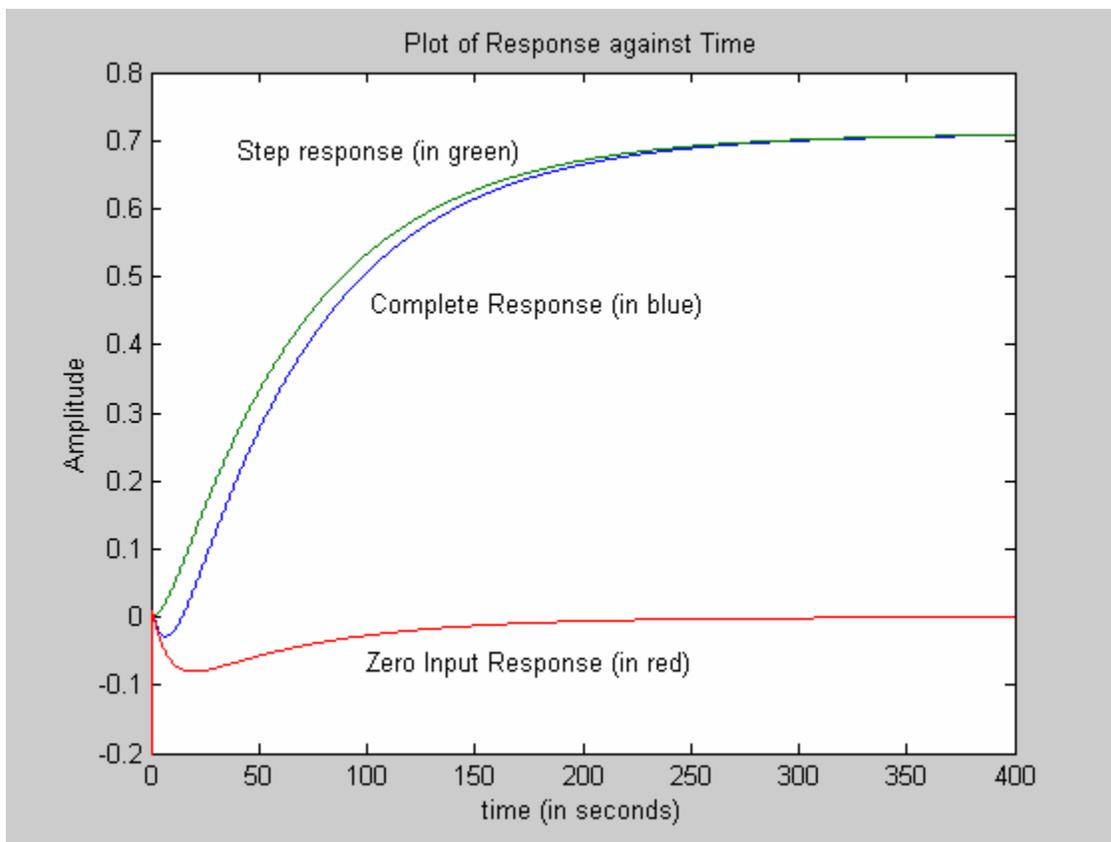
Adding up both the step response and zero input response, we get the complete response.

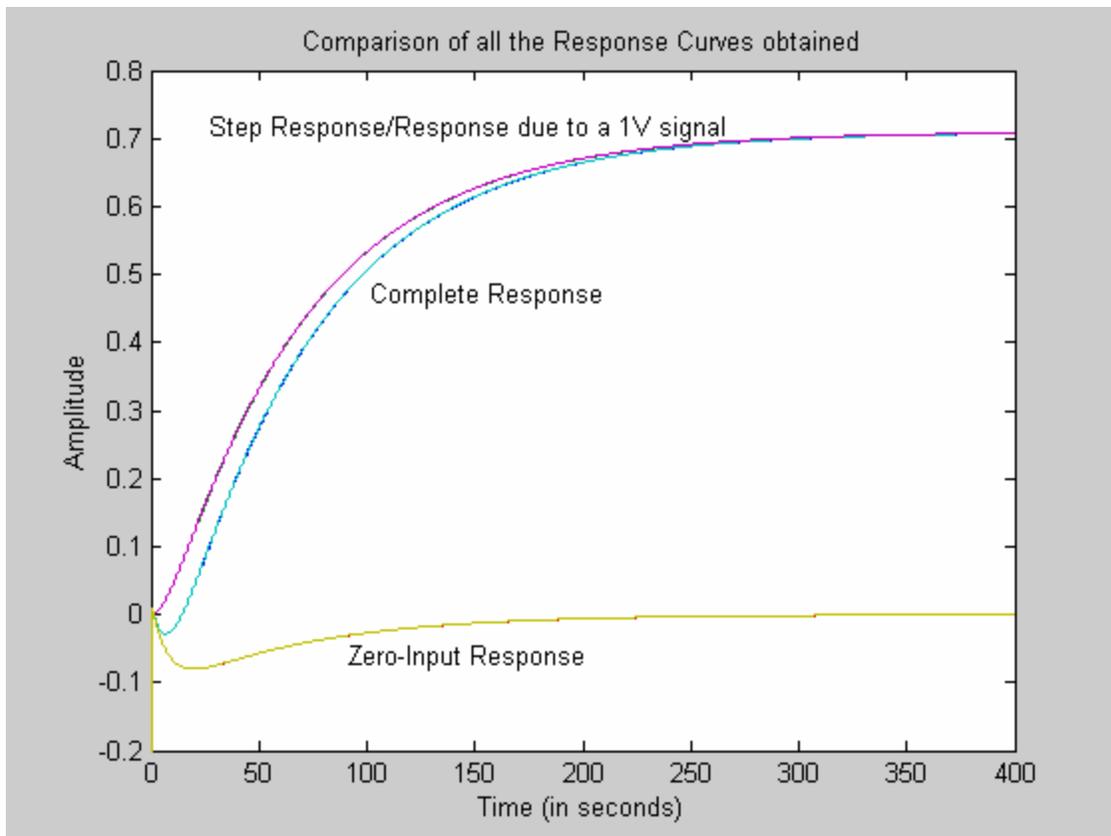
```
>> G = G0 + G1;
```

To plot all the responses onto a single graph, use the plot function.

```
>> plot(T,G,T,G1,T,G0)
```

The graph obtained is shown below.

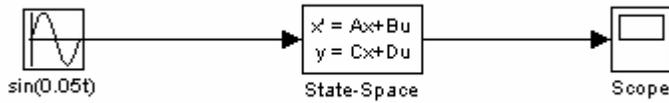




As it could be seen that the response curves coincide with each other almost perfectly, the zero-input response could be obtained by using the initial function, and the response due to the 1V signal can be easily obtained by using the step function. Hence, the response due to the 1V signal is verified by using results generated from the step response tabulated in the graph above.

3(d)

i) $u = \sin(0.05t)$ V

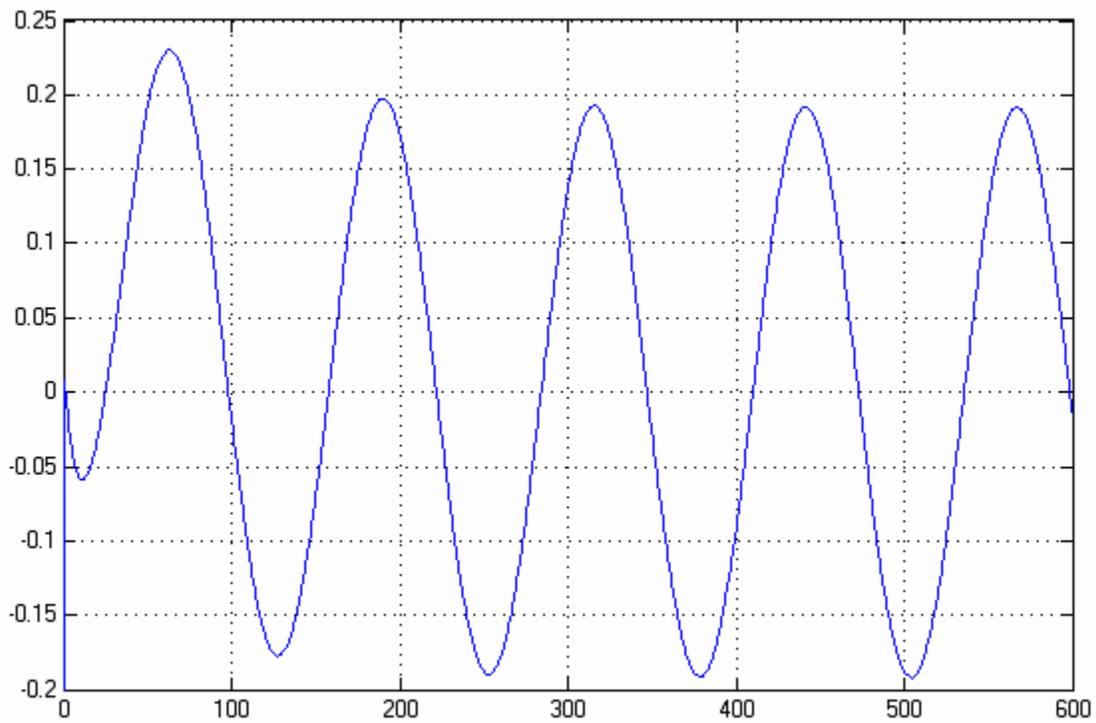


Block Parameters: sin(0.05t)

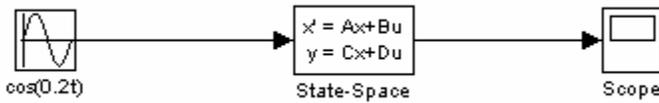
Sine Wave
 Output a sine wave where the sine type determines the computational technique used. The parameters in the two types are related through:
 Samples per period = $2\pi / (\text{Frequency} * \text{Sample time})$
 Number of offset samples = $\text{Phase} * \text{Samples per period} / (2\pi)$
 Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters:
 Sine type: Time-based
 Amplitude: 1
 Bias: 0
 Frequency (rad/sec): 0.05
 Phase (rad): 0
 Sample time: 0
 Interpret vector parameters as 1-D

OK Cancel Help Apply



ii) $u = \cos(0.2t)$ V

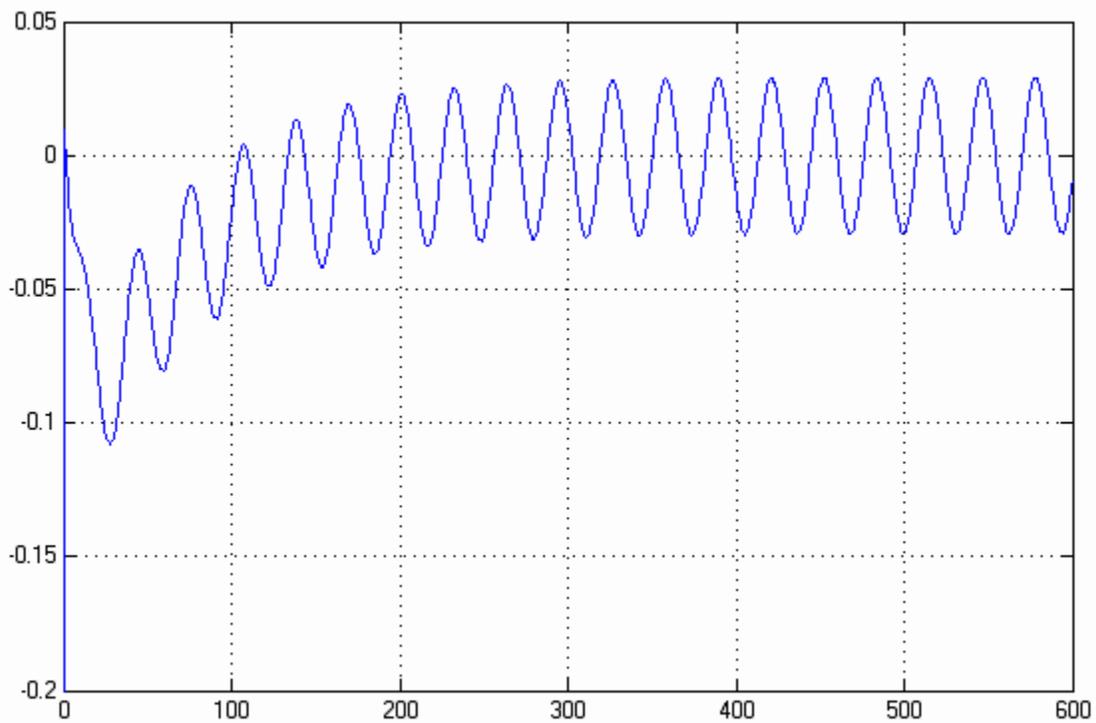


Block Parameters: cos(0.2t)

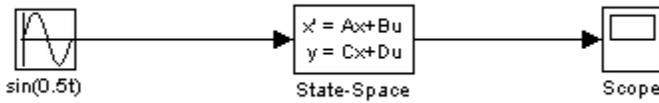
Sine Wave
 Output a sine wave where the sine type determines the computational technique used. The parameters in the two types are related through:
 Samples per period = $2\pi / (\text{Frequency} * \text{Sample time})$
 Number of offset samples = $\text{Phase} * \text{Samples per period} / (2\pi)$
 Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters
 Sine type: Time-based
 Amplitude: 1
 Bias: 0
 Frequency (rad/sec): 0.2
 Phase (rad): pi/2
 Sample time: 0
 Interpret vector parameters as 1-D

OK Cancel Help Apply



iii) $u = \sin(0.5t)$ V



Block Parameters: sin(0.5t)

Sine Wave

Output a sine wave where the sine type determines the computational technique used. The parameters in the two types are related through:

Samples per period = $2\pi / (\text{Frequency} * \text{Sample time})$

Number of offset samples = $\text{Phase} * \text{Samples per period} / (2\pi)$

Use the sample-based sine type if numerical problems due to running for large times (e.g. overflow in absolute time) occur.

Parameters

Sine type: Time-based

Amplitude: 1

Bias: 0

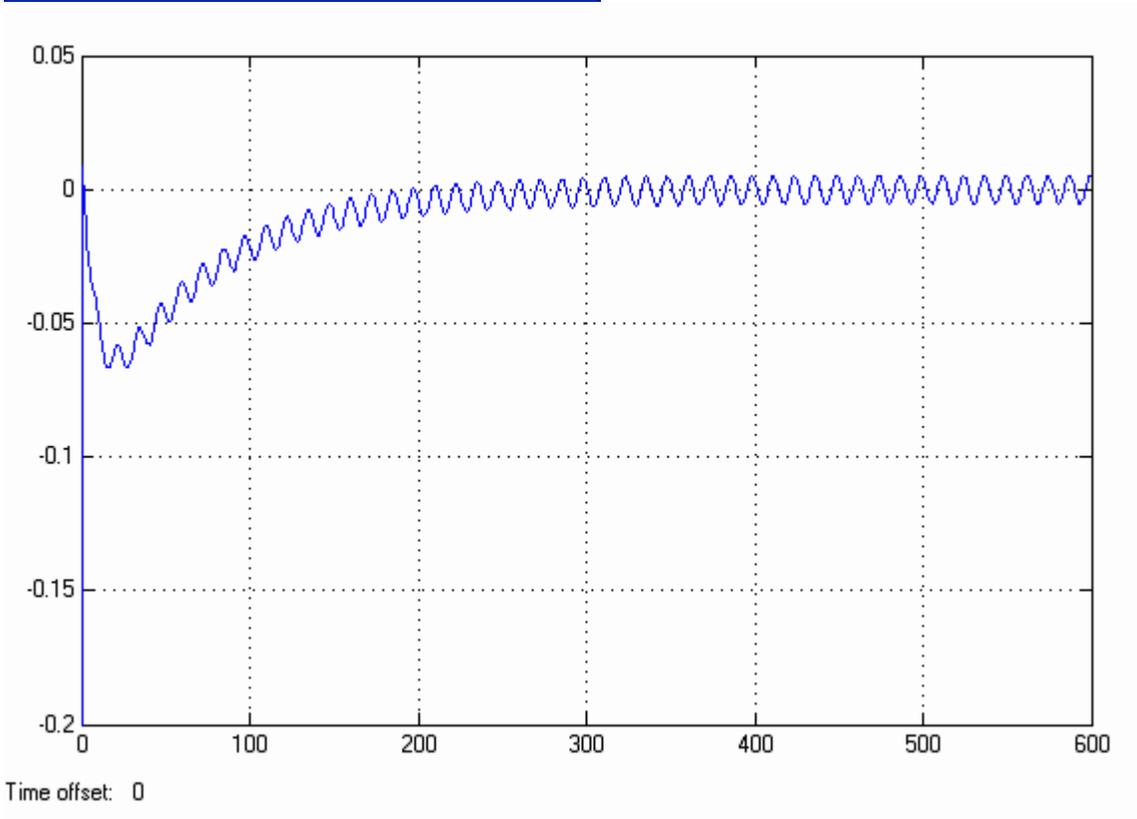
Frequency (rad/sec): 0.5

Phase (rad): 0

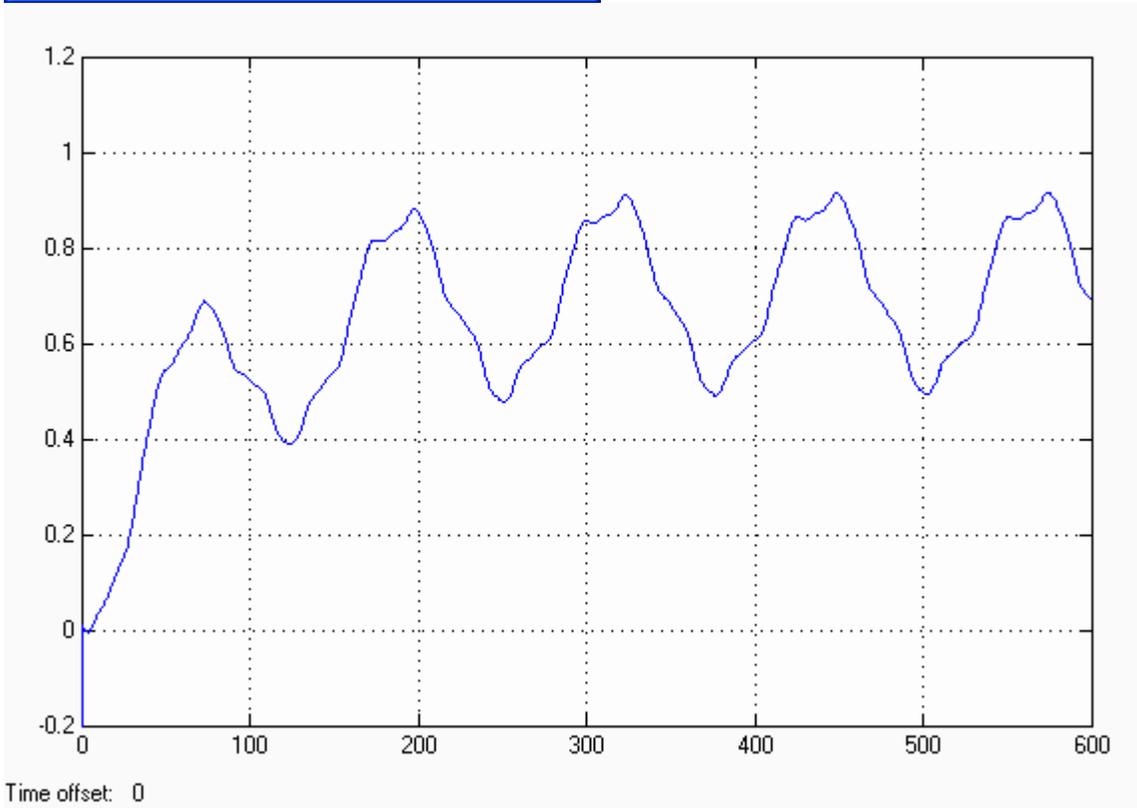
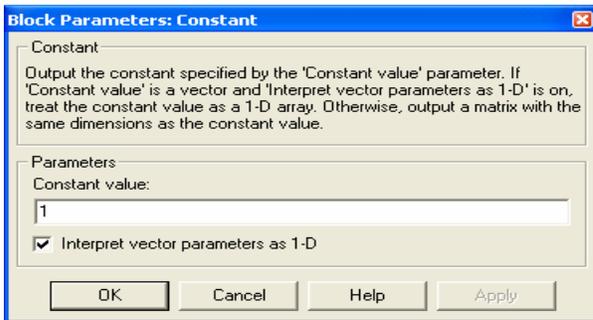
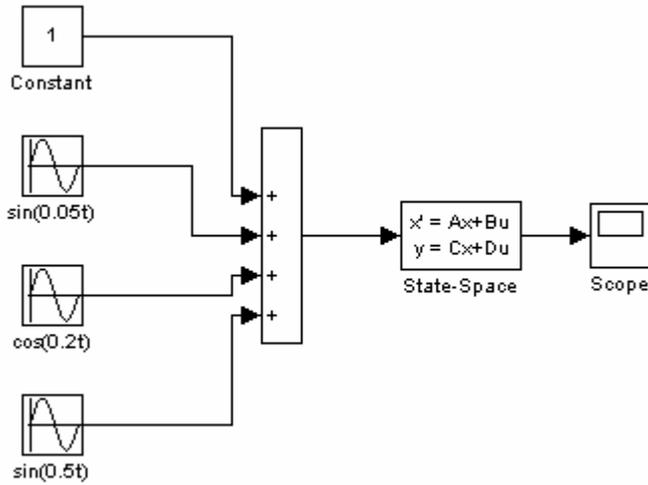
Sample time: 0

Interpret vector parameters as 1-D

OK Cancel Help Apply



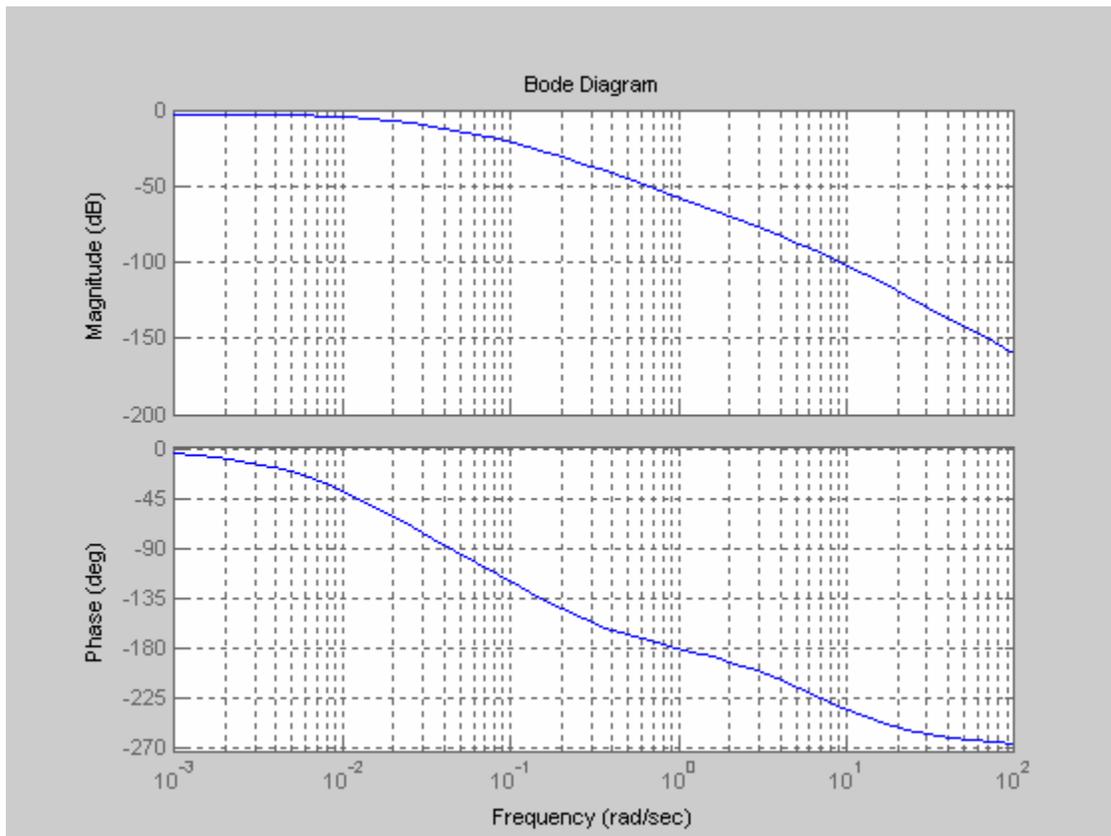
iv) $u = 1 + \sin(0.05t) + \cos(0.2t) + \sin(0.5t)$ V



Observation

1. For the four input signals into the system, transient response is first observed before the output settle down to a steady state response.
2. With further observation, it could be seen that frequency plays an important role in determining the type of response we get. Through comparison with the graphs, we discovered that the lower the input frequency, the higher the amplitude of the output responses would be. A phase shift of the output response could also be observed in some of the graphs.
3. In order to study the peculiarities that occur in some of the graph, I decided to go further to plot a Bode diagram using MATLAB.

```
>> sys = tf(NUM,DEN)
>> bode(sys)
>> grid on
```



4. From the bode diagram, it could be observed that the magnitude of the output signal decreases as the frequency of the input signal increases. The phase shift of the output signal decreases even more significantly with an increase of the frequency of the input signal.

5. For the case of the signal $u = \sin(0.05t)$, the frequency of the input signal is 0.05 rad/s and this is comparatively small compared to the other input frequencies. From the Bode diagram, we notice that there is small voltage drop when the input frequency of the input signal is small. This explains the much higher amplitude of the output response in case (i). As for case (ii) and (iii), their amplitudes are smaller considering the higher frequency of their input signals. Note that the amplitude of the output response in case (ii) is higher than that of case (iii) and smaller than that of case (i).
6. Similarly, the phase shift is also the smallest in case (i) than the case in (ii) and (iii) due to the lower frequency of its input signal. Based on the fact that frequency of a signal is inversely proportional to its period, the curve in case (i) also displays a larger time period compared to the cases in (ii) and (iii) as the frequency of response of the input signal in case (i) is smaller than that of those in case (ii) and (iii). Note that the phase shift in the output signal in (ii) is smaller than in case (iii) and the period of the curve in case (ii) is higher than case (iii) for similar reasons given above.
7. For the output curve in (i), it could be observed that the output signal actually undergoes a phase shift of about $\pi / 4$ radians and this could be easily verified by checking the Bode diagram. As for the other two curves in (ii) and (iii), it is not that easy to spot the phase shift though it could be done upon the magnification of the graph using MATLAB. However, the phase shift of that two output signals could still be easily obtained by checking against the Bode diagram.
8. As case (iv) is a case due to superposition of the various input signals in (i), (ii) and (iii), I will give a separate explanation from here onwards. Even though the input signal is made up of the superposition of the four other waveforms, transient response could still be observed before the output settles down to a steady response.
9. It could also be observed that the output waveform in (iv) resembles that in (i) upon careful observation. This implies the output signal is made up mostly by the waveform of lower frequency. The slight fluctuation of the output waveform is mainly due to the signals of lower frequencies as seen in the graphs in case (ii) and (iii) where the magnitude of their output signals are significantly reduced due to their high frequencies.
10. The reason why the output signal is always above positive is mainly due to the constant input signal of magnitude one.

Conclusion

The state variables analysis is useful in helping us to get the transfer function and the output response of the system regardless of the order and the number of charge storing elements in the system. Simulink is also a very useful tool in helping us to analyze the system without physically building it. The Bode diagram is also a very useful tool for us to analyze the frequency response of the system. As for the system, it could be said that this system given to us in this project is stable and useful in filtering out high frequency signals. In other words, this system can be used as a low pass filter to filter out unwanted high frequency disturbances in a low frequency signal.