Solution to Tutorial Set 1

Q.1

	(a)	(b)
AC waveform	$5\sqrt{2}\sin(\omega t)$ $= 5\sqrt{2}\cos\left(\omega t - \frac{\pi}{2}\right)$ $= \mathrm{Re}\left[\left(5e^{-j\pi/2}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$	$5\sqrt{2}\cos(\omega t)$ $= \mathrm{Re}\left[\left(5e^{j0}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$
Peak value	$5\sqrt{2}$	$5\sqrt{2}$
Frequency	$\omega\,\mathrm{rad/s} = \frac{\omega}{2\pi}\,\mathrm{Hz}$	$\omega\,\mathrm{rad/s} = \frac{\omega}{2\pi}\,\mathrm{Hz}$
RMS value	5	5
Phase	$-\frac{\pi}{2} = -90^0$	0
Phasor	$5\ e^{-j\pi/2} = 5\ \underline{/-90^o}$	$5\ e^{j0} = 5\ \underline{/0^o} = 5$

	(c)	(d)
AC waveform	$10\sqrt{2}\sin(20t + 30^0)$ $= 10\sqrt{2}\cos(20t - 60^0)$ $= \mathrm{Re}\left[\left(10e^{-j\pi/3}\right)\left(\sqrt{2}e^{j20t}\right)\right]$	$120\sqrt{2}\cos(314t - 45^0)$ $= \mathrm{Re}\left[\left(120e^{-j\pi/4}\right)\left(\sqrt{2}e^{j314t}\right)\right]$
Peak value	$10\sqrt{2}$	$120\sqrt{2}$
Frequency	$20\,\mathrm{rad/s} = 3.18\,\mathrm{Hz}$	$314\,\mathrm{rad/s} = 50\,\mathrm{Hz}$
RMS value	10	120
Phase	$-\frac{\pi}{3} = -60^0$	$-\frac{\pi}{4} = -45^0$
Phasor	$10\,e^{-j\pi/3} = 10\underline{/-60^o}$	$120e^{-j\pi/4} = 120\underline{/-45^o}$

	(e)	(f)
AC waveform	$-50\sin\left(4t-\frac{\pi}{3}\right)$ $=35.4\sqrt{2}\cos\left(4t-\frac{\pi}{3}-\frac{\pi}{2}+\pi\right)$ $=\mathrm{Re}\left[\left(35.4e^{j\pi/6}\right)\left(\sqrt{2}e^{j4t}\right)\right]$	$0.25\cos\left(2t+100^0\right)$ $=0.177\sqrt{2}\cos\left(2t+1.75\right)$ $=\mathrm{Re}\left[\left(0.177e^{j1.75}\right)\left(\sqrt{2}e^{j2t}\right)\right]$
Peak value	50	0.25
Frequency	$4\ \mathrm{rad/s}=0.637\ \mathrm{Hz}$	$2\ \mathrm{rad/s}=0.318\ \mathrm{Hz}$
RMS value	35.4	0.177
Phase	$\frac{\pi}{6}=30^0$	$1.75=100^0$
Phasor	$35.4e^{j\pi/6}=35.4\angle 30^{\circ}$	$0.177e^{j1.75}=0.177\angle 100^{\circ}$

Q.2

(a)	$\frac{100}{\sqrt{2}}e^{j30^0}\ \mathrm{V}$	$\mathrm{Re}\left[\left(\frac{100}{\sqrt{2}}e^{j\pi/6}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=100\cos\left(314t+\frac{\pi}{6}\right)\mathrm{V}$
(b)	$115e^{j\pi/3}\ \mathrm{V}$	$\mathrm{Re}\left[\left(115e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=115\sqrt{2}\cos\left(314t+\frac{\pi}{3}\right)\mathrm{V}$
(c)	$-0.12e^{-j\pi/4}\ \mathrm{A}$	$\mathrm{Re}\left[\left(-0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=\mathrm{Re}\left[\left(e^{j\pi}0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $=0.12\sqrt{2}\cos\left(314t+\frac{3\pi}{4}\right)\mathrm{A}$
(d)	$-0.69\angle 60^{\circ}\ \mathrm{A}$	$\mathrm{Re}\left[\left(-0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]=\mathrm{Re}\left[\left(e^{j\pi}0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $=0.69\sqrt{2}\cos\left(314t+\frac{4\pi}{3}\right)$ $=0.69\sqrt{2}\cos\left(314t-\frac{2\pi}{3}\right)\mathrm{A}$

Q.3

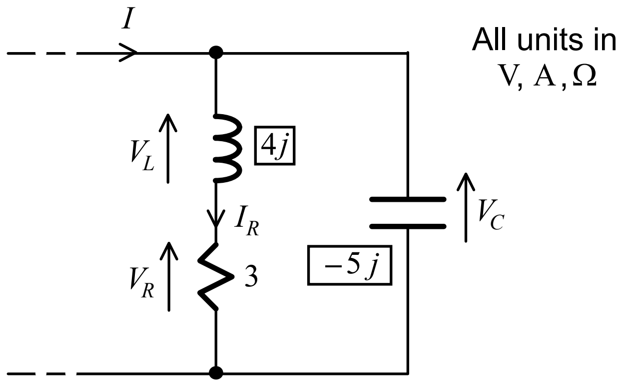
From $v_R(t) = 12\sqrt{2}\cos(2t)$ V

Frequency $= \omega = 2$ rad/s

Impedance of capacitor $= \dfrac{1}{j\omega 0.1} = \dfrac{1}{j0.2} = -5j\,\Omega$

Impedance of inductor $= j\omega 2 = 4j\,\Omega$

$V_R = 12e^{j0} = 12$

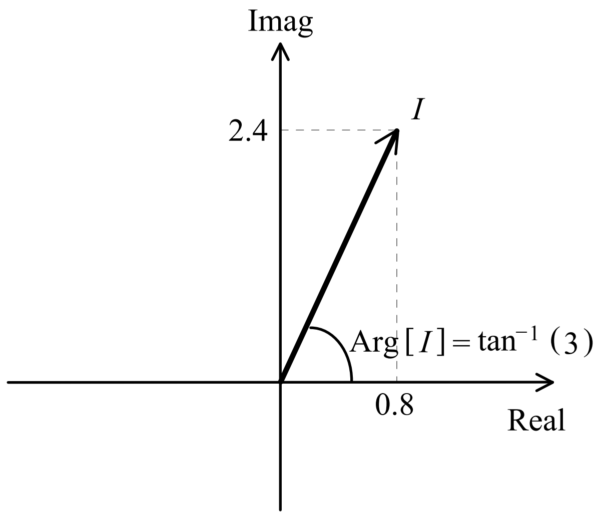


$$I_R = \frac{V_R}{3} = 4 \;\Rightarrow\; i_R(t) = 4\sqrt{2}\cos(2t)\text{ A}$$

$$V_L = (4j)I_R = 16j\text{ V} \;\Rightarrow\; v_L(t) = 16\sqrt{2}\cos\left(2t + \frac{\pi}{2}\right)\text{V}$$

$$V_C = V_R + V_L = 12 + 16j$$

$$I = I_R + \frac{V_C}{-5j} = 4 - \frac{12+16j}{5j} = 4 + 2.4j - 3.2 = 0.8 + 2.4j$$



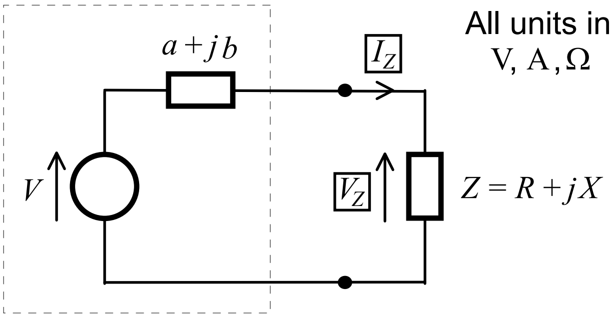
$$|I| = |0.8 + 2.4j| = \sqrt{0.8^2 + 2.4^2} = 2.53$$

$$\mathrm{Arg}[I] = \mathrm{Arg}[0.8 + 2.4j] = \tan^{-1}\left(\frac{2.4}{0.8}\right) = 1.25$$

$$I = 0.8 + 2.4j = 2.53e^{j1.25} \;\Rightarrow\; 2.53\sqrt{2}\cos(2t + 1.25)\ \mathrm{A}$$

Q.4

Power



Electrical system

$$I_Z = \frac{V}{(a + jb) + (R + jX)} = \frac{V}{(a + R) + j(b + X)}$$

$$V_Z = I_Z(R + jX)$$

Power absorbed $= p = \mathrm{Re}[I_Z^* V_Z] = \mathrm{Re}\left[|I_Z|^2(R + jX)\right]$

$$= |I_Z|^2\,\mathrm{Re}[R + jX] = R|I_Z|^2$$

$$= R\left|\frac{V}{(a + R) + j(b + X)}\right|^2 = \frac{R|V|^2}{(a + R)^2 + (b + X)^2}$$

Maximum power transfer

For maximum p, the denominator should be as small as possible. As the numerator does not depend on X and the smallest value for $(b + X)$ is 0, maximum power will be absorbed if

$$X = -b$$

so that

$$p = \frac{|V|^2 R}{(a + R)^2}$$

Differentiating:

$$\frac{dp}{dR} = |V|^2 \left[\frac{1}{(a+R)^2} - \frac{2R}{(a+R)^3} \right] = |V|^2 \left[\frac{a-R}{(a+R)^3} \right]$$

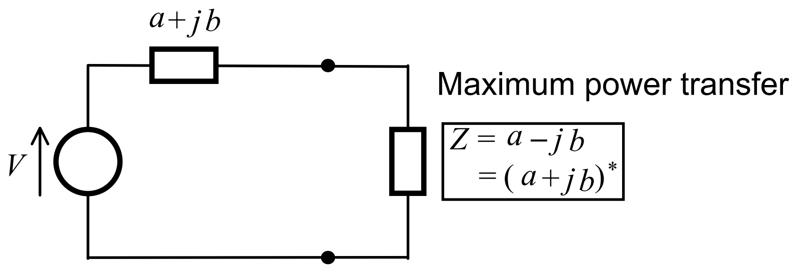
Thus, maximum p occurs when

$$R = a$$

and the maximum power transferable is

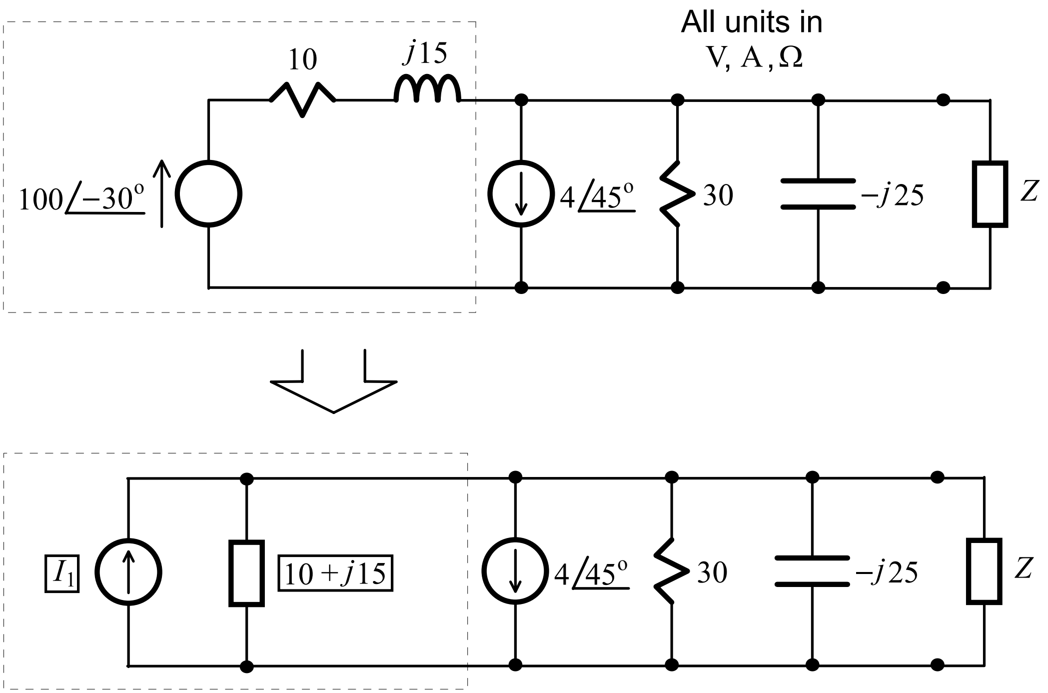
$$p = \frac{|V|^2 R}{(R+R)^2} = \frac{|V|^2}{4R} = \frac{|V|^2}{4a} \text{W}$$

In general, maximum power transfer occurs when the load impedance is equal to the conjugate of the Thevenin's or Norton's impedance. When this occurs, the total impedance is purely resistive and the current and voltage in the circuit are in phase:

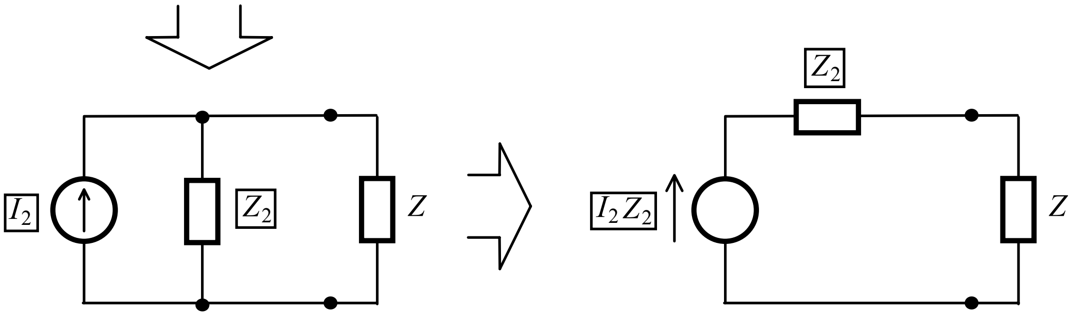


Q.5

Norton's and Thevenin's equivalent circuit



$$I_1 = \frac{100e^{-j30^0}}{10 + j15} = \frac{100e^{-j30^0}}{18e^{j56.3^0}} = 5.55e^{-j86.3^0} = 0.358 - j5.54$$



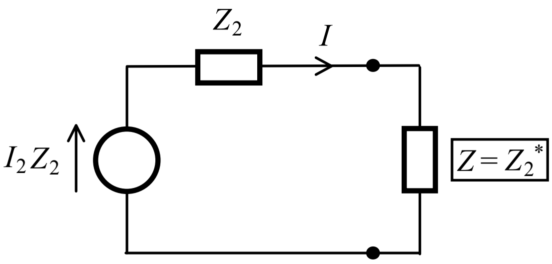
$$Z_2 = -j25 \,||\, 30 \,||\, (10 + j15) = \frac{1}{\dfrac{1}{-j25} + \dfrac{1}{30} + \dfrac{1}{10 + j15}} = \frac{1}{\dfrac{1}{-j25} + \dfrac{1}{30} + \dfrac{10 - j15}{10^2 + 15^2}}$$

$$= \frac{1}{0.0641 - j0.00615} = \frac{1}{0.0644e^{-j5.48^0}} = 15.5e^{j5.48^0}$$

$$I_2 = 0.359 - j5.55 - 4e^{j45^0} = -2.47 - j8.36 = 8.72e^{-j106^0}$$

Maximum power transfer

From the previous problem, this occurs when



$$Z = Z_2^* = \left(15.5e^{j5.48^0}\right)^* = 15.5e^{-j5.48^0}$$

Total impedance $= Z + Z_2 = Z_2^* + Z_2 = 15.5e^{j5.48^0} + 15.5e^{-j5.48^0} = 2\left[15.5\cos\left(5.48^0\right)\right]$

$$I = \frac{I_2 Z_2}{(Z + Z_2)} = \frac{8.72e^{-j106^0}\, 15.5e^{j5.48^0}}{2\left[15.5\cos\left(5.48^0\right)\right]} = \frac{8.72e^{-j101^0}}{2\cos\left(5.48^0\right)}$$

Thus, the maximum power transferable is

$$\mathrm{Re}\left[I^*(IZ)\right] = |I|^2\, \mathrm{Re}\left[Z_2^*\right]$$

$$= \left|\frac{8.72e^{-j101^0}}{2\cos\left(5.48^0\right)}\right|^2 \mathrm{Re}\left[15.5e^{-j5.48^0}\right] = \left|\frac{8.72}{2\cos\left(5.48^0\right)}\right|^2 15.5\cos\left(5.48^0\right)$$

$$= \frac{8.72^2 \times 15.5}{4\cos\left(5.48^0\right)} = 297\,\mathrm{W}$$