Sample Solutions to Homework Assignment 2

National University of Singapore

Department of Electrical and Computer Engineering

EE3304 Digital Control Systems

Home Assignment II

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The plant for this assignment is given as

$$G(s) = \frac{6\times10^7}{s^2}$$

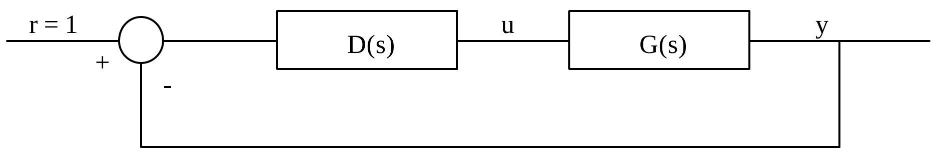

Fig.1: System in continuous time domain

Step One:

Determine the open-loop gain K to satisfy requirements on the steady state error.

$$G(s) = \frac{6\times10^7}{s^2}$$

Discretize the plant, we can get

$$G(z) = \frac{0.3z + 0.3}{z^2 - 2z + 1}$$

Hence,

$$G(z) = +\infty$$

Whatever the value K is, the steady state error will always be zero. This conclusion comes from the lecture notes section 2.3.

Step Two:

Find new open-loop crossover frequency from desired $w_n = w_{\max}$, the point the phase lead is added.

According to the design specification, the settling time is 8 milliseconds, and the maximum overshoot is less than 25%.

Read from the relationship diagram of Mp% vs. ?, we get

$$z \geq 0.4 \Rightarrow z = 0.6$$

Read from the relationship diagram of ? vs. phase margin

$$f_{desired} = 57^\circ$$

$$t_s \cong \frac{4.6}{z w_n}$$

$$\Rightarrow w_{\max} = w_n \cong \frac{4.6}{z t_s} = \frac{4.6}{0.6 \times 8 \times 10^{-3}} = 958$$

Step Three:

Evaluate the phase margin of the uncompensated system using the value of K obtained in Step One. K can be any value.

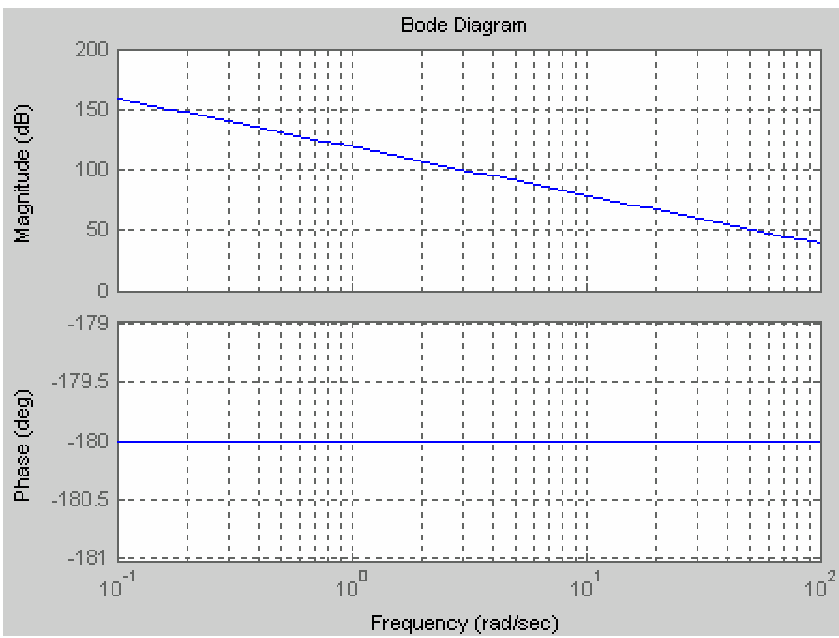

Fig. 2: Bode plot for the uncompensated system with D.C gain K

According to Fig. 2, the original phase margin is 0.

Step Four:

$$f_{\max} = f_{desired} - f_{orginal} + f_{allowance}$$
$$\Rightarrow f_{\max} = 57^\circ - 0^\circ + 5^\circ = 62^\circ$$

Step Five:

$$a = \frac{1 - \sin f_{\max}}{1 + \sin f_{\max}} = \frac{1 - \sin 62^{\circ}}{1 + \sin 62^{\circ}} = 0.0414$$

$$t = \frac{1}{w_{\max}\sqrt{a}} = \frac{1}{958 \times \sqrt{0.0414}} = 0.0042$$

Hence, the lead compensator in w-domain:

$$D(w) = K\frac{tw + 1}{a\,tw + 1} = K\left(\frac{0.0042\,w + 1}{0.00026\,w + 1}\right)$$

which can be converted back to z-domain using the inverse bilinear transformation.

$$D(z) = D(w)\Big|_{w = \frac{2(z-1)}{T(z+1)}} = K\frac{t\,\frac{2(z-1)}{T(z+1)} + 1}{a\,t\,\frac{2(z-1)}{T(z+1)} + 1} = K\left(\frac{13.65z - 13.33}{z - 0.6776}\right)$$

When $K = 0.015$,

Verification in w-domain

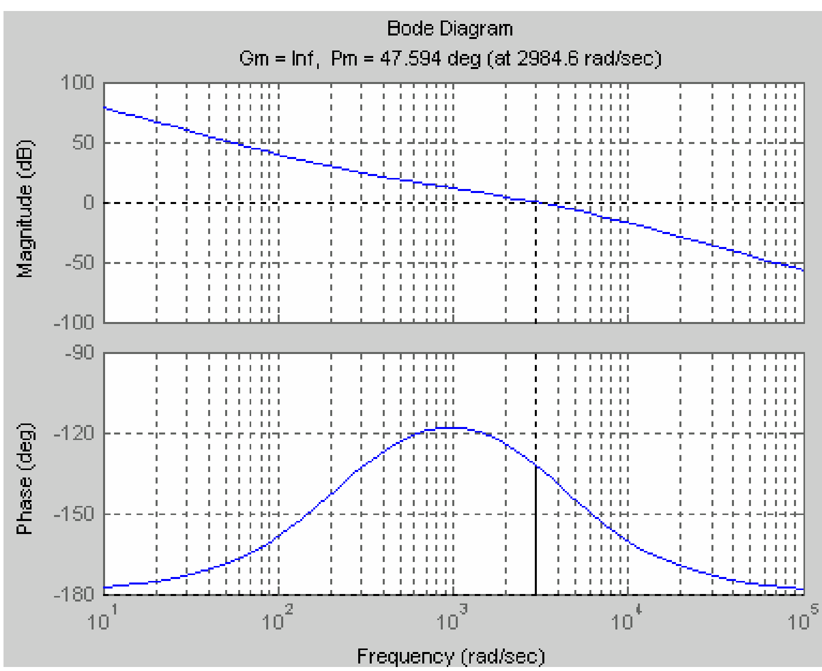

Fig. 3: Bode plot for the compensated system in s-domain

Verification in z-domain

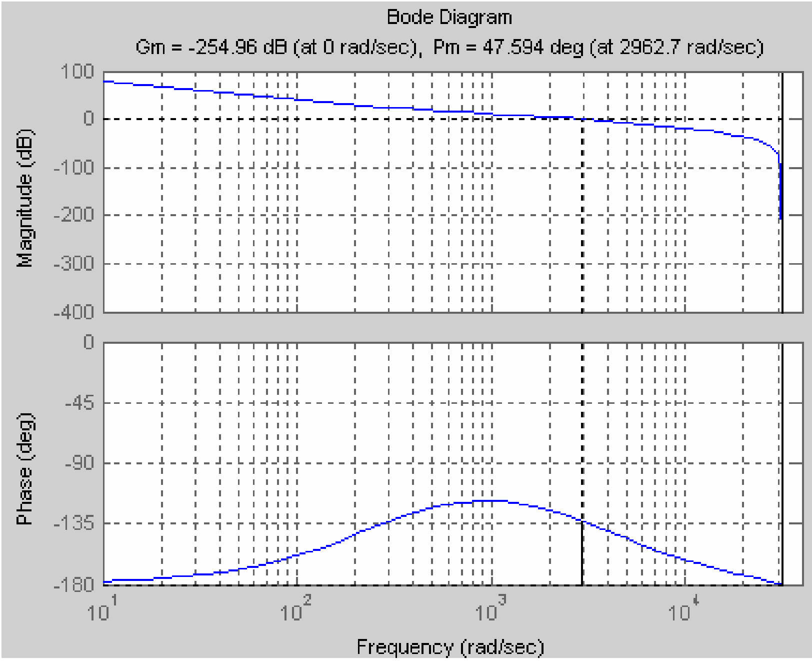

Fig. 4: Bode plot for the compensated system in z-domain

From the above two diagrams, we can see the phase margins in both s-domain and z-domain are the same 47^o, which is less than the desired one. But the difference is not very significant.

Verification in discrete-time-domain

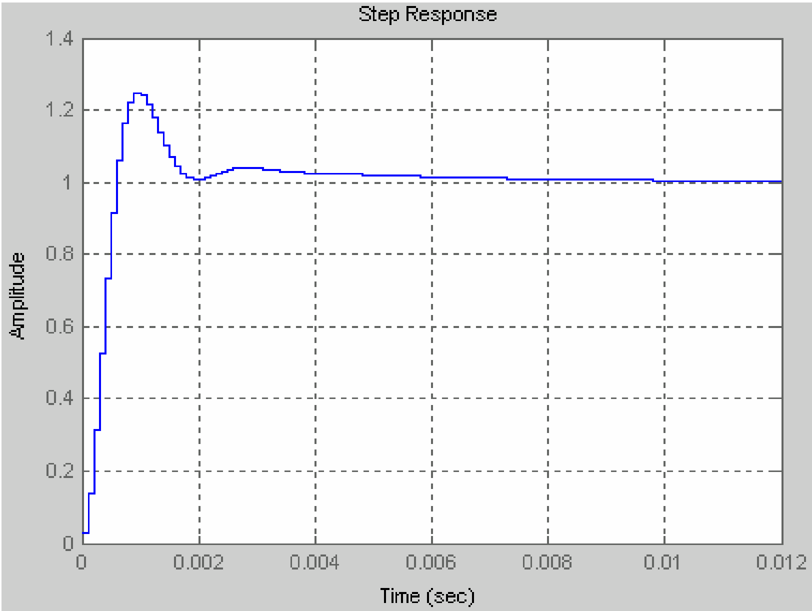

Fig. 5: Step response in the discrete-time-domain

Verification in continuous-time-domain

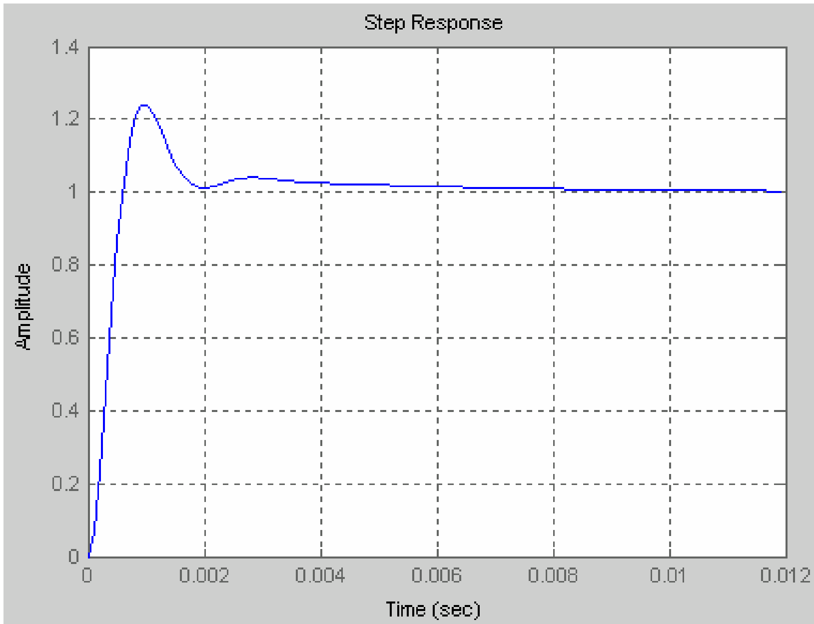

Fig. 6: Step response in the continuous-time-domain

From the above simulation with MatLab,

$$\begin{cases} M_p\% \cong 23\% < 25\% \\ t_s = 5ms < 8ms \end{cases}$$

The lead compensator $\dfrac{6.278\text{e-}005\,s + 0.015}{0.0002602\,s + 1}$ can meet the design specifications.

Appendix A: M-File

```
clear all;
close all;
%Initialize variables
Ts = 1e-4;                      %Sampling period
Tset = 8e-3;                    %Settling time
dRatio = 0.6;                   %Damping ratio
Wn = 4.6/(dRatio*Tset);         %Natural frequency
a = 0;
t = 0;
Pmax = 0;                       %Frequency ahould be compensated
Pdesired = 57;                  %Desired pahse margin
Porigin = 0;                    %Original phase margin
Pallowance = 5;
%Define plant
G_num = [1 0 0];
G_den = [6e7];
G = tf(G_den, G_num);
D_G = c2d(G, Ts, 'tustin');
%Define controller
K = 0.015;
temp = allmargin( K*G );
Porigin = temp.PhaseMargin;
clear temp;
Pmax = Pdesired - Porigin + Pallowance;
a = (1-sin((Pmax/180)*pi))/(1+sin((Pmax/180)*pi));
t = 1/(Wn*a^(1/2));
D_num = [a*t 1];
D_den = [t 1];
D = K * tf(D_den, D_num);
D_D = c2d(D, Ts, 'tustin');
%Define the feedback system
FSys = feedback(D*G, 1);
D_FSys = feedback(D_D*D_G, 1);
%Plot the diagram
figure;
bode(K*G);
grid;
figure;
margin(D*G);
grid;
figure;
margin(D_D*D_G);
grid;
figure;
step(FSys);
grid;
figure;
step(D_FSys);
grid;
```