Annex G.7. A Past Year Exam Paper

Appendix C.4 will be attached to this year's paper!

Q.1 (a) Using nodal analysis, derive (but DO NOT simplify or solve) the equations for determining the nodal voltages in the circuit of Fig. 1(a).

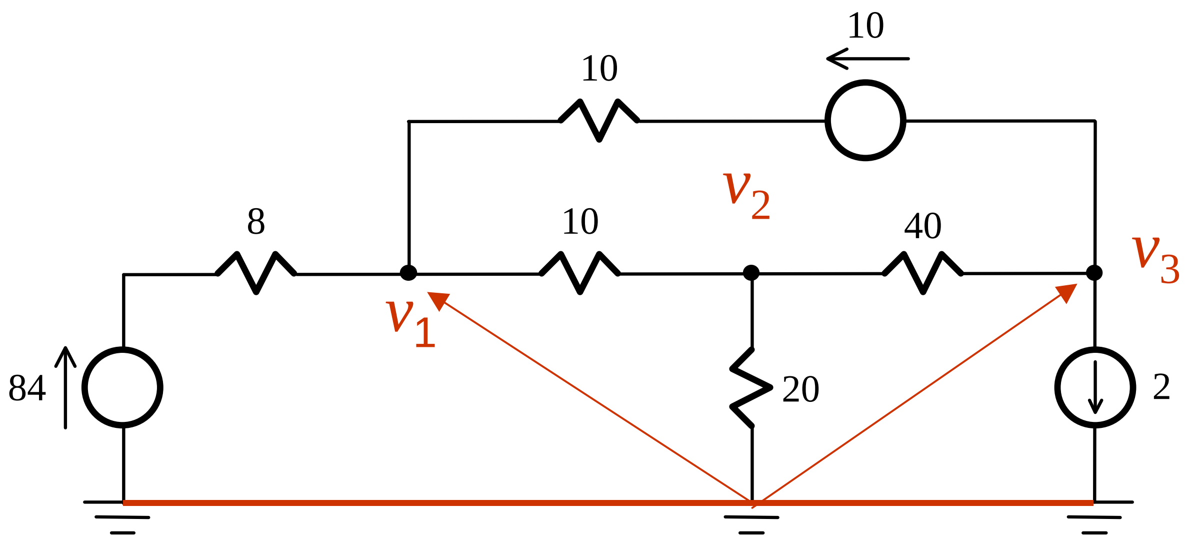


Fig. 1(a)

Numbering the nodes in the circuit by 1, 2 and 3 from left to right, and applying KCL:

$$\frac{v_1 - 84}{8} + \frac{v_1 - v_2}{10} + \frac{v_1 - v_3 - 10}{10} = 0 \quad \frac{v_2}{20} + \frac{v_2 - v_1}{10} + \frac{v_2 - v_3}{40} = 0 \quad \frac{v_3 - v_2}{40} + 2 + \frac{v_3 - v_1 + 10}{10} = 0$$

(b) Using mesh analysis, derive (but DO NOT solve) the matrix equation for determining the loop currents in the circuit of Fig. 1(b). Note that the circuit has a dependent source.

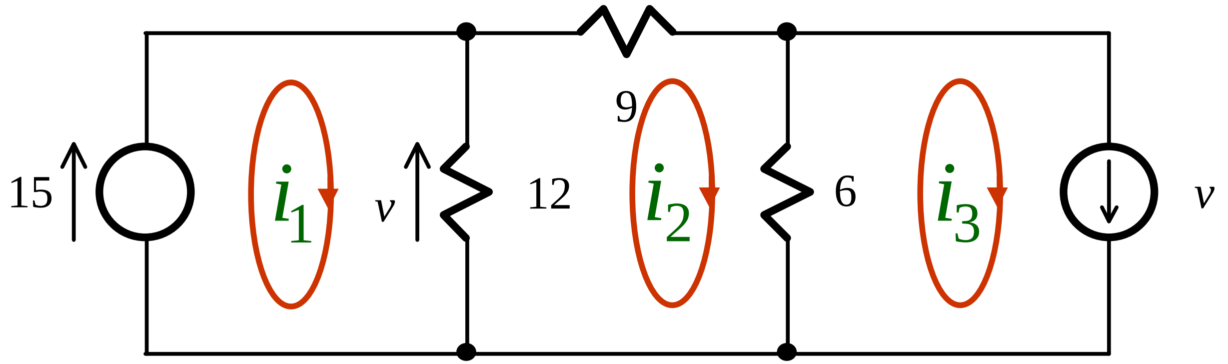


Fig. 1(b)

Relating loop to branch currents and applying KVL:

$$15 = v = 12(i_1 - i_2)$$

$$6(i_2 - i_3) + 9i_2 + 12(i_2 - i_1) = 0$$

$$\Rightarrow \quad -12i_1 + 27i_2 - 6i_3 = 0$$

$$i_3 = v$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & -12 & 27 & -6 \\ 1 & 0 & 0 & 0 \\ 0 & 12 & -12 & 0 \end{bmatrix} \begin{bmatrix} v \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 15 \\ 15 \end{bmatrix}$$

(c) Determine the Thevenin or Norton equivalent circuits as seen from terminals A and B of the network of Fig. 1(c). What is the maximum power that can be obtained from these two terminals?

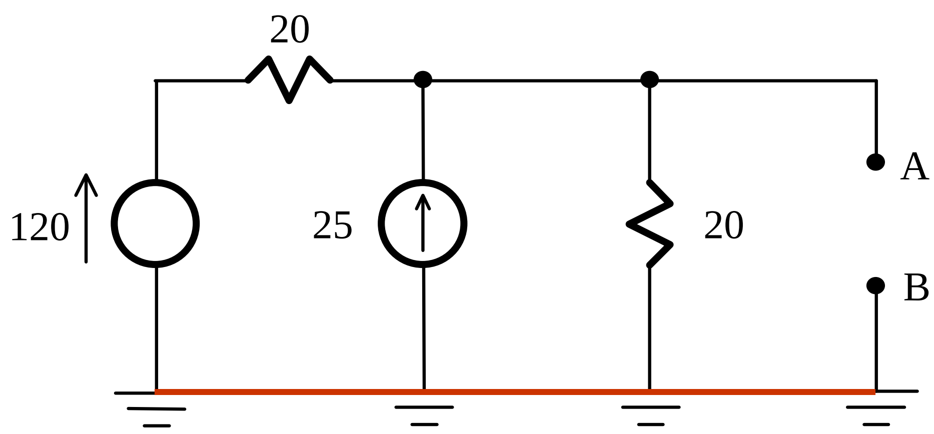


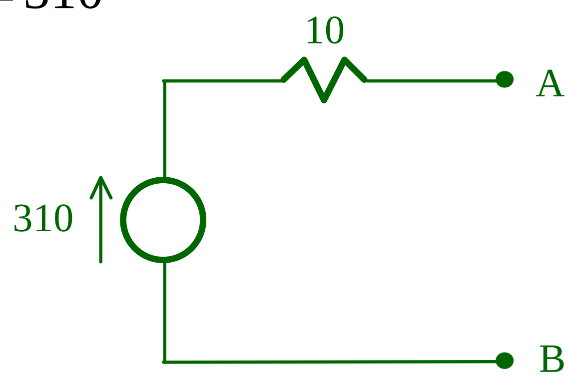
Fig. 1(c)

Replacing all independent sources with their internal resistances, the resistance across A and B is

$$R = 20 \parallel 20 = 10$$

Using superposition, the open circuit voltage across A and B is

$$v_{AB} = 120\left(\frac{20}{20+20}\right) + 25\left(\frac{20}{20+20}\right)20$$
$$= 310$$



The maximum power $p = \dfrac{310^2}{4(10)}$

Q.2 (a) A 5 kW electric motor is operating at a lagging power factor of 0.5. If the input voltage is

$$v(t) = 500\sin\left(wt + 10^0\right)$$

determine the apparent power, and find the phasor and sinusoidal expression for the input current.

Letting *V* and *I* to be the voltage and current phasors, the apparent power is

$$\frac{5000}{0.5} = 10000\text{VA} = |VI| = |V||I|$$

where $V = \frac{500}{\sqrt{2}} e^{j\left(10^0 - 90^0\right)} = \frac{500}{\sqrt{2}} e^{-j80^0}$ and $|I| = \frac{10000}{|V|} = \frac{10000\sqrt{2}}{500} = 20\sqrt{2}$

$$\arg(I) - \arg(V) = -\cos^{-1}(0.5)$$

$$i(t) = 20\sqrt{2}\sqrt{2}\cos\left(wt - 80^0 - \cos^{-1} 0.5\right)$$
$$= 40\cos\left(wt - 80^0 - \cos^{-1} 0.5\right)$$

⟸ $I = 20\sqrt{2}e^{j\left(-80^0 - \cos^{-1} 0.5\right)}$

(b) In the circuit of Fig. 2(b), the current $i(t)$ is the excitation and the voltage $v(t)$ is the response. Determine the frequency response of the circuit. Derive (but DO NOT solve) an equation for finding the "resonant" frequency at which the frequency response becomes purely real.

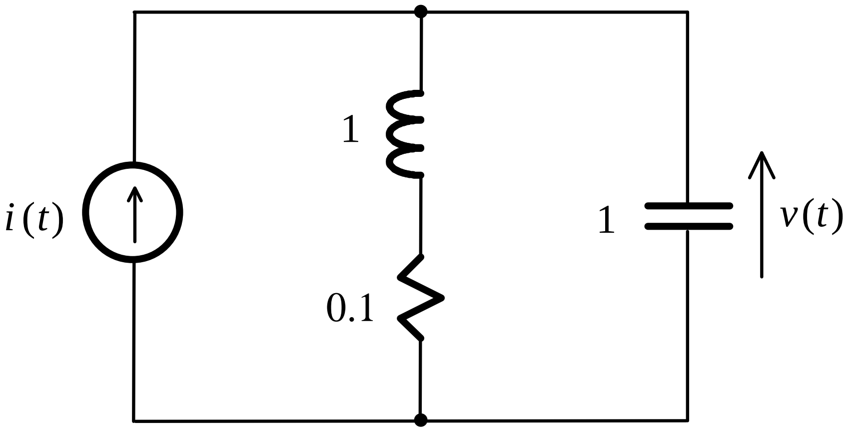


Fig. 2(b)

Using phasor analysis

$$H(f) = \frac{V}{I} = \frac{1}{jw}\left(\frac{0.1 + jw}{0.1 + jw + \frac{1}{jw}} \right)$$

$$= \frac{0.1 + jw}{1 + j0.1w - w^2}$$

The phase response is

$$\arg[H(f)] = \tan^{-1}\left(\frac{w}{0.1}\right) - \tan^{-1}\left(\frac{0.1w}{1 - w^2}\right)$$

The resonant frequency is therefore given by

$$\tan^{-1}(10w) = \tan^{-1}\left(\frac{0.1w}{1 - w^2}\right)$$

(c) A series RLC resonant circuit is to be designed for use in a communication receiver. Based on measurements using an oscilloscope, the coil that is available is found to have an inductance of 25.3mH and a resistance of 2 Ω. Determine the value of the capacitor that will give a resonant frequency of 1 kHz. If a *Q* factor of 100 is required, will the coil be good enough?

$$f_0 = \frac{1}{2p\sqrt{LC}} \quad \Rightarrow \quad C = \left(\frac{1}{2p\sqrt{L}\,f_0}\right)^2 = \left(\frac{1}{2p\sqrt{25.3\times10^{-3}}\,1000}\right)^2 = 1m\mathrm{F}$$

$$Q = \frac{w_0 L}{R} = \frac{2p\,1000(25.3)10^{-3}}{2} = 79.5$$

Since this is less than 100, the coil is not good enough.

Q.3 (a) In the circuit of Fig. 3(a), the switch has been in the position shown for a long time and is thrown to the other position for time $t \geq 0$. Determine the values of $i(t)$, $v_C(t)$, $v_R(t)$, $v_L(t)$, and $di(t)/dt$ just after the switch has been moved to the final position?

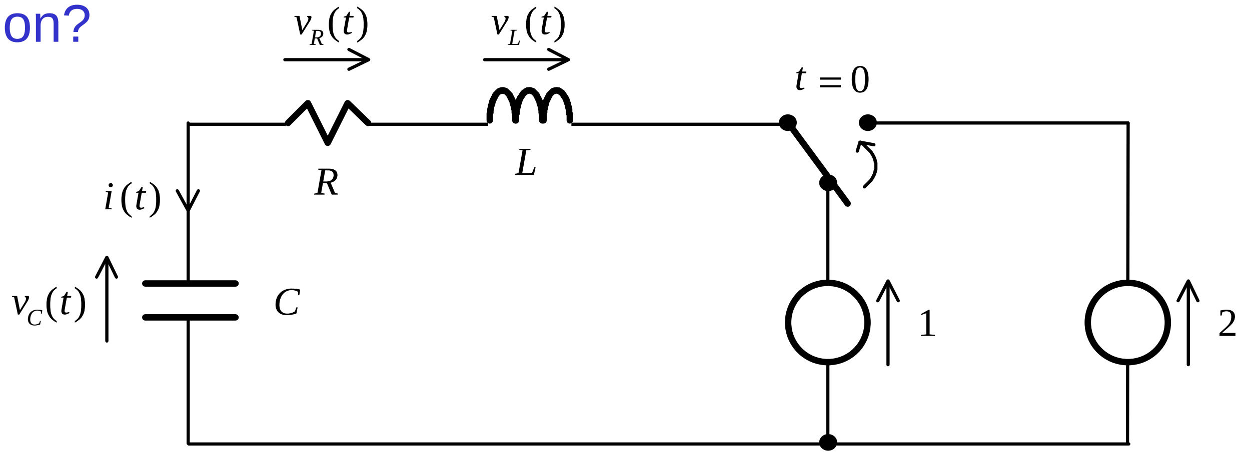


Fig. 3(a)

Taking all the voltages and currents to be constants for $t < 0$:

$$i(t) = C\frac{dv_C(t)}{dt} = 0$$

$$v_R(t) = Ri(t) = 0$$

$$v_L(t) = L\frac{di(t)}{dt} = 0$$

$$v_C(t) + v_R(t) + v_L(t) = 1 \Rightarrow v_C(t) = 1$$

Applying continuity for $i(t)$ and $v_C(t)$:

$$i(0) = 0 \quad v_R(0) = Ri(0) = 0 \quad v_C(0) = 1$$

$$v_C(0) + v_R(0) + v_L(0) = 2 \quad \Rightarrow \quad v_L(0) = 1$$

$$v_L(t) = L\frac{di(t)}{dt} \Rightarrow \left.\frac{di(t)}{dt}\right|_{t=0} = \frac{v_L(0)}{L} = \frac{1}{L}$$

(b) For $v_S(t) = \cos(t+1)$, derive (but DO NOT solve) the differential equation from which $i(t)$ can be found in the circuit of Fig. 3(b). Is this differential equation sufficient for $i(t)$ to be determined?

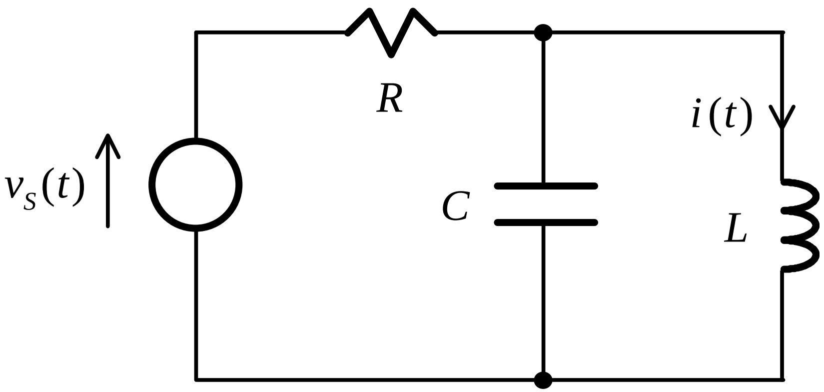


Fig. 3(b)

$$v_L(t) = L\frac{di(t)}{dt}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = C\frac{dv_L(t)}{dt} = CL\frac{d^2i(t)}{dt^2}$$

$$i_R(t) = i_C(t) + i(t) = CL\frac{d^2i(t)}{dt^2} + i(t)$$

$$v_R(t) = Ri_R(t) = RCL\frac{d^2i(t)}{dt^2} + Ri(t)$$

Applying KVL:

$$v_S(t) = v_R(t) + v_L(t)$$
$$= RCL\frac{d^2i(t)}{dt^2} + Ri(t) + L\frac{di(t)}{dt} = \cos(t+1)$$

This is not sufficient for $i(t)$ to be determined.

(c) The differential equation characterizing the current $i(t)$ in a certain RCL circuit is

$$\frac{d^2 i(t)}{dt^2} + \frac{1}{CR}\frac{di(t)}{dt} + \frac{i(t)}{CL} = e^{jt}$$

Determine the condition for R, L and C such that the circuit is critically damped.

The characteristic equation for the transient response is

$$z^2 + \frac{z}{CR} + \frac{1}{CL} = 0$$

$$z_{1,2} = \frac{-\frac{1}{CR} \pm \sqrt{\frac{1}{C^2R^2} - \frac{4}{CL}}}{2}$$

Thus, the circuit will be critically damped if

$$\frac{1}{C^2R^2} = \frac{4}{CL}$$

Q.4 (a) Determine the mean and rms values of the voltage waveform in Fig. 4(a). If this waveform is applied to a 20 Ω resistor, what is the power absorbed by the resistor?

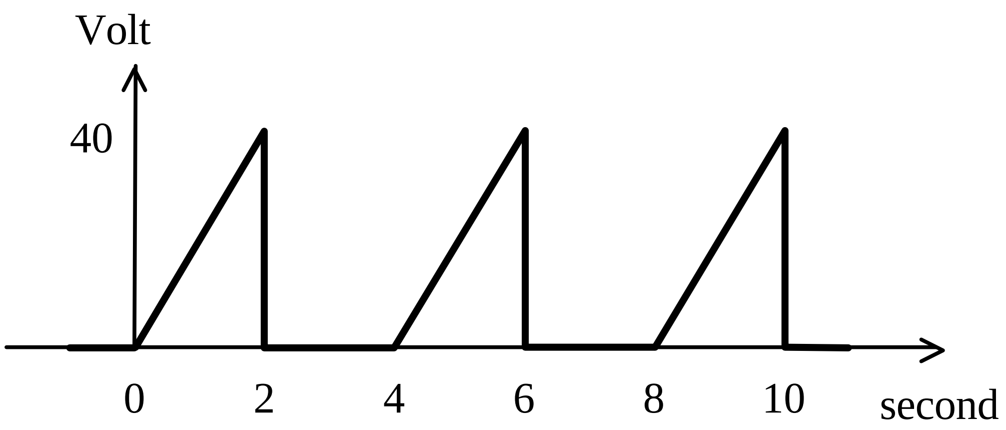


Fig. 4(a)

One period of the waveform is

$$v(t)=\begin{cases}20t, & 0\le t<2\\ 0, & 2\le t<4\end{cases}$$

$$v_m=\frac{\int_0^2 20t\,dt}{4}=\frac{20\left(\frac{2^2}{2}\right)}{4}=10$$

$$v_{ms}=\frac{\int_0^2 (20t)^2\,dt}{4}=\frac{400\left(\frac{2^3}{3}\right)}{4}=\frac{800}{3}$$

$$v_{rms}=\sqrt{\frac{800}{3}}$$

$$p=\frac{v_{rms}^{\ 2}}{20}=\frac{800}{3(20)}=\frac{40}{3}$$

(b) In the circuit of Fig. 4(b), a transformer is used to couple a loudspeaker to a amplifier. The loudspeaker is represented by an impedance of value $Z_L = 6 + j\,2$, while the amplifier is represented by a Thevenin equivalent circuit consisting of a voltage source in series with an impedance of $Z_S = 3 + j\,a$. Determine the voltage across the loudspeaker. Hence, find the value of a such that this voltage is maximized. Will maximum power be delivered to the loudspeaker under this condition?

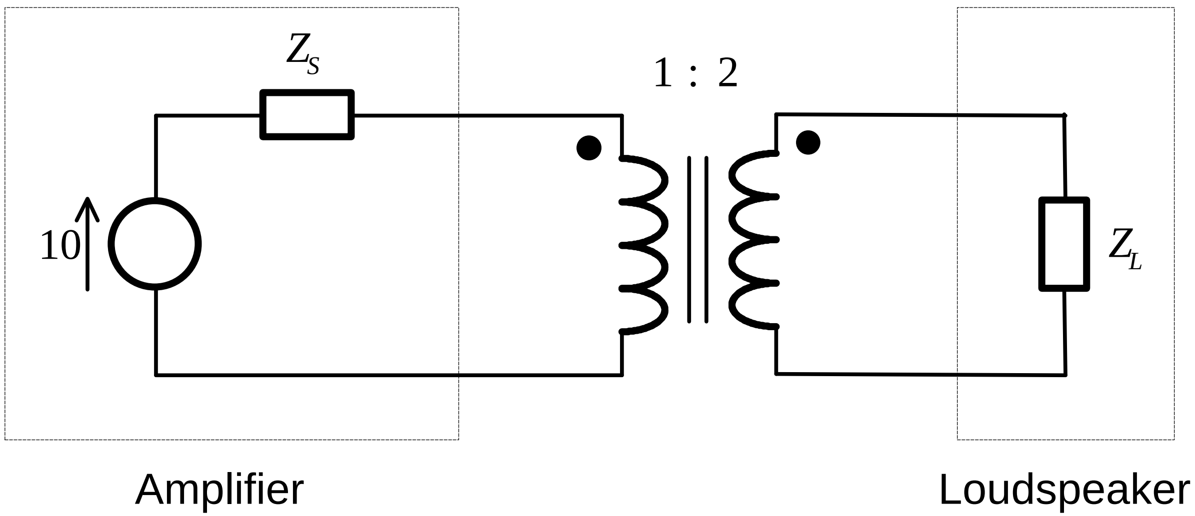


Fig. 4(b)

If V is the voltage across the loudspeaker, the currents in the primary & secondary windings are

$$I_2 = \frac{V}{Z_L} \qquad I_1 = 2I_2 = \frac{2V}{Z_L}$$

The primary voltage is $V_1 = \dfrac{V}{2}$

Applying KVL to the primary circuit: $10 = I_1 Z_S + V_1 = \dfrac{2VZ_S}{Z_L} + \dfrac{V}{2}$

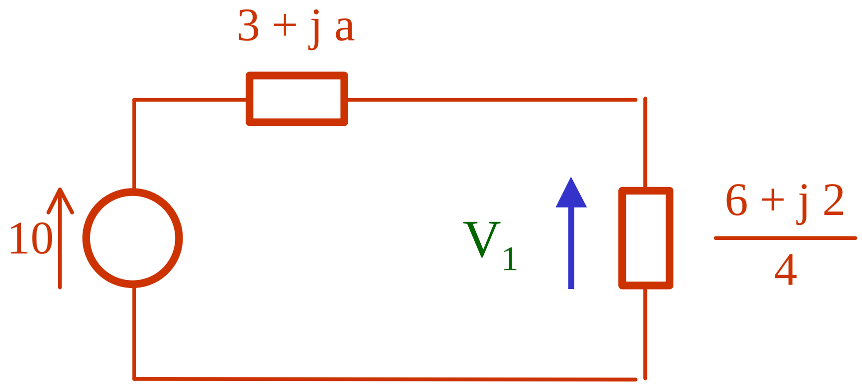
$$\Longrightarrow \quad V = \frac{10}{\dfrac{2Z_S}{Z_L} + \dfrac{1}{2}} = \frac{20}{1 + 4\left(\dfrac{3+aj}{6+2j}\right)} = \frac{20(6+2j)}{18 + j(4a+2)}$$

For the magnitude of this to be maximized, the denominator has to be minimized:

$$\max\left[\left|\frac{20(6+2j)}{18+j(4a+2)}\right|\right] = \min\left[\left|18 + j(4a+2)\right|\right] \quad \Longrightarrow \quad a = -\frac{2}{4} = -\frac{1}{2}$$

Maximum power will be delivered since power is proportional to $|V|^2$.

Method 2: The given circuit is equivalent to the following one,



Then, we have

$$V_1 = \left(\frac{10}{(3+ja) + \left(\frac{3}{2} + j\frac{1}{2} \right)} \right) \left(\frac{3}{2} + j\frac{1}{2} \right) = \frac{10(3+j)}{9 + j(2a+1)}$$

$$\Rightarrow \quad V_{load} = V_2 = nV_1 = \frac{20(3+j)}{9 + j(2a+1)}$$

The rest follows ......