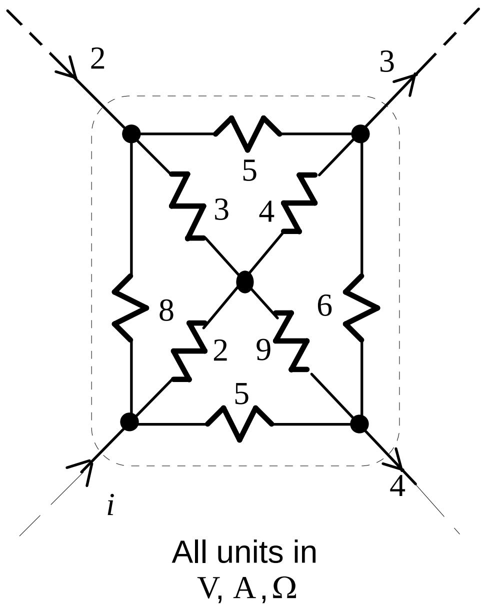
F

Tutorial Solutions

F.1 KCL, KVL, Power and Energy

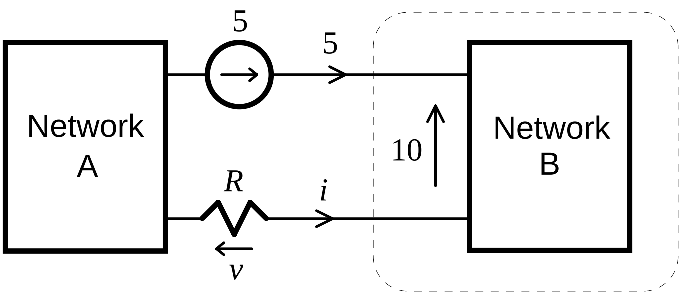
Q.1



Applying KCL to the dotted surface:

$$i + 2 = 3 + 4 \;\Rightarrow\; i = 5$$

Q.2



All units in V, A, Ω

Applying KCL to the dotted surface:

$$5 + i = 0 \;\Rightarrow\; i = -5\text{A}$$

regardless of the value of R. For $R = 2\Omega$,

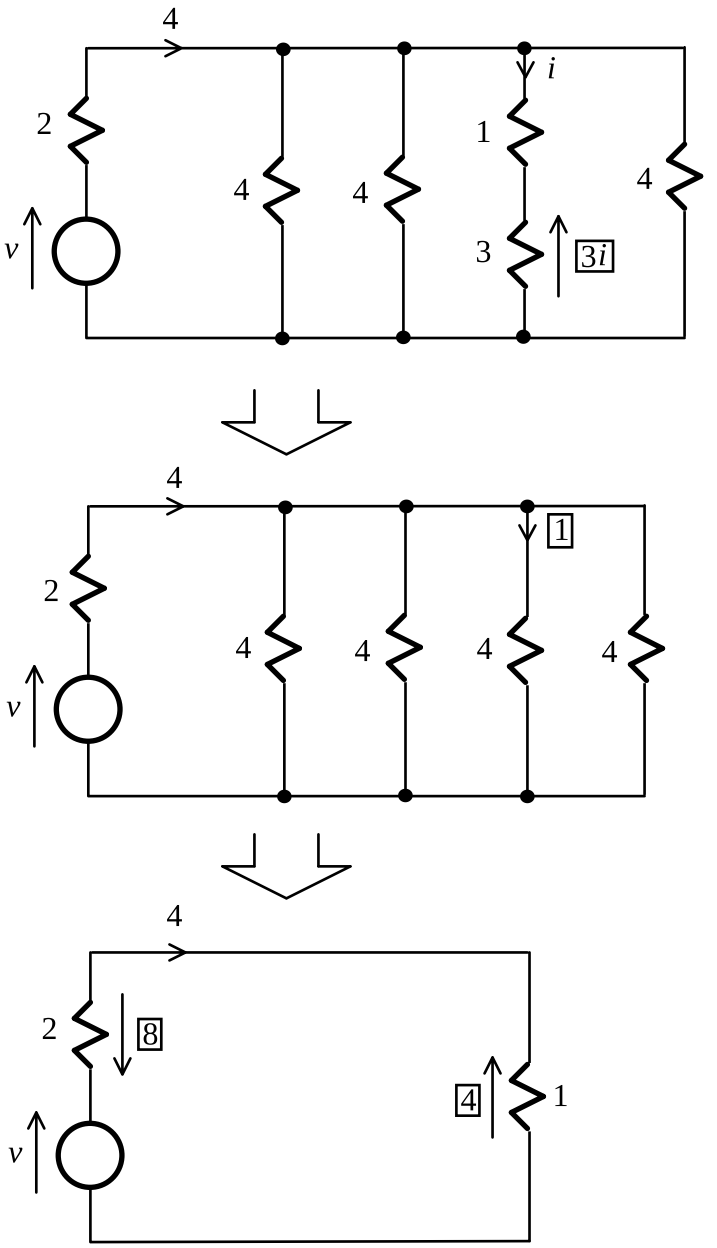
$$v = iR = -5 \times 2 = -10\,\text{V}$$

For $R = 50\,\text{K}\Omega$,

$$v = iR = -5 \times 50000 = -250\,\text{KV}$$

Q.3

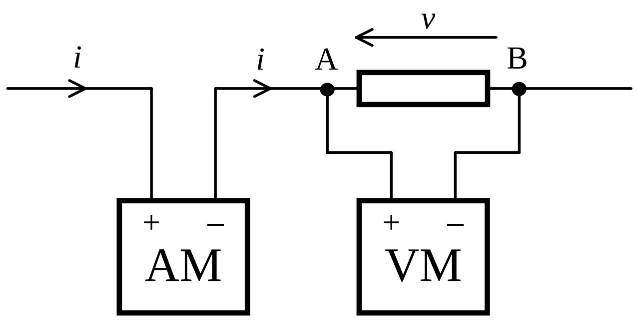
All units in
V, A, Ω



$v = 8 + 4 = 12\,\mathrm{V}$; $i = 1$; voltage across 3Ω resistor $= 3i = 3\,\mathrm{V}$

Q.4

(a) Both meters give positive readings



Since the arrows for v and i are in opposite directions

Power consumed $= vi$

Also, AM will give a positive reading if i is positive, while VM will give a positive readings if v is positive.

Since both i and v are positive in this case, vi is positive and power is consumed.

(b) Both meters give negative readings

Both i and v are negative, vi is positive and power is consumed.

(c) One meter gives a positive reading and the other gives a negative reading

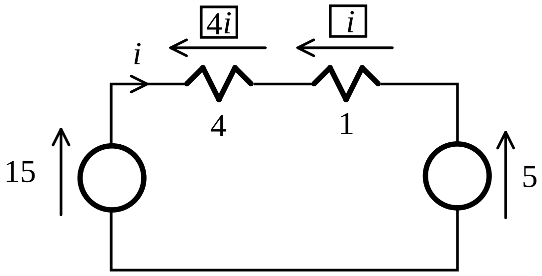
i and v have opposite signs, vi is negative and power is supplied by the device.

(d) One meter or both meter give zero readings

Power neither is consumed nor supplied by the device.

Q.5

Current in circuit



All units in
$\mathrm{V, A}, \Omega$

Applying KVL:

$15 = 4i + i + 5 \;\Rightarrow\; i = 2$

Power consumed/supplied

If the voltage and current arrows are in opposite directions,

Power consumed $=$ (voltage)(current)

Thus:

Power consumed by 4Ω resistor $= i(4i) = 16\,\mathrm{W}$

Power consumed by 1Ω resistor $= i(i) = 4\,\mathrm{W}$

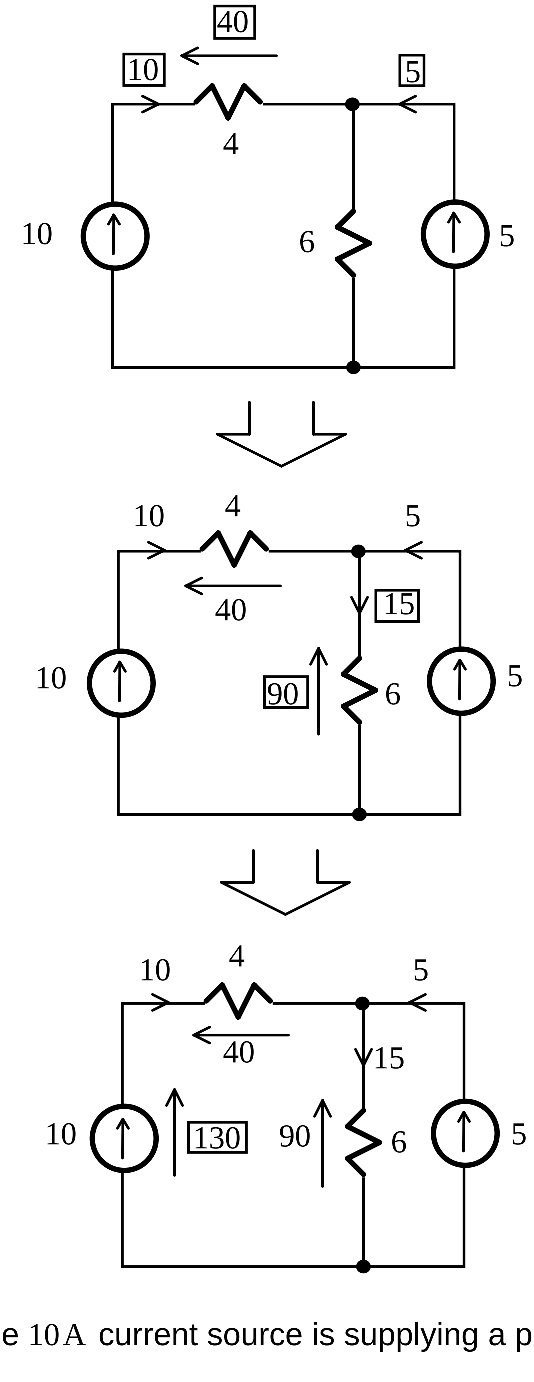
Power consumed by $5\,\mathrm{V}$ source $= i(5) = 10\,\mathrm{W}$

Power consumed by $15\,\mathrm{V}$ source $= -i(15) = -30\,\mathrm{W}$

or

Power supplied by $15\,\mathrm{V}$ source$= 30\,\mathrm{W}$

Q.6



All units in
V, A, Ω

The $10\,\mathrm{A}$ current source is supplying a power of $(130)(10) = 1300\mathrm{W}$.

Q.7

Efficiency

Electrical power supplied $= (100)(20) = 2000\,\mathrm{W}$

Mechanical power delivered $= (2.5\,\mathrm{h.p.})(746\,\mathrm{W/h.p.}) = 1865\,\mathrm{W}$

Efficiency $= \dfrac{1865}{2000} = 93.25\%$

Torque

Motor speed $= (100\,\mathrm{rev/min})\left(\dfrac{2\pi}{60}\dfrac{\mathrm{rad/rev}}{\mathrm{s/min}}\right) = \dfrac{20\pi}{6}\,\mathrm{rad/s}$

Torque $= \dfrac{\text{Mechanical power delivered}}{\text{motor speed}} = \dfrac{1865}{20\pi/6} = 178.1\,\mathrm{Nm}$

Energy lost

Power lost $= 2000 - 1865 = 135\,\mathrm{W}$

Energy lost per $\mathrm{min} = (135)(60) = 8100\,\mathrm{J}$

Q.8

Generator output power $= (100)(10) = 1000\,\mathrm{W}$

Generator input power $= \dfrac{1000}{0.9} = 1111.1\,\mathrm{W}$

Generator shaft speed $= \left(\dfrac{11000}{5}\,\mathrm{rev/min}\right)\left(\dfrac{2\pi}{60}\dfrac{\mathrm{rad/rev}}{\mathrm{s/min}}\right) = 230.4\,\mathrm{rad/s}$

Torque $= \dfrac{1111.1}{230.4} = 4.82\,\mathrm{Nm}$

Q.9

Voltage, current and power gains for system

Voltage gain $= g_v = \left|\dfrac{v_2}{v_1}\right| = \left|\dfrac{5v_1}{v_1}\right|$

$$= 5 = 20\log(5)\text{ dB} = 14\text{ dB}$$

Current gain $= g_i = \left|\dfrac{i_2}{i_1}\right| = \left|\dfrac{v_2/8}{v_1/10000}\right|$

$$= \left|\frac{5v_1/8}{v_1/10000}\right| = 6250 = 20\log(6250)\text{ dB} = 76\text{ dB}$$

Power gain $= g_p = \left|\dfrac{v_2 i_2}{v_1 i_1}\right| = g_v g_i$

$$= (5)(6250) = 10\log[(5)(6250)]\text{ dB}$$

$$= \frac{1}{2}\left[20\log(5) + 20\log(6250)\right]\text{ dB} = \frac{14+76}{2}\text{ dB} = 45\text{ dB}$$

Relationship between these gains

$$g_p = g_v g_i$$

$$\left(g_p\text{ dB}\right) = \frac{\left(g_v\text{ dB}\right) + \left(g_i\text{ dB}\right)}{2}$$

$g_p = g_v = g_i$ if load resistance equals amplifier's input resistance

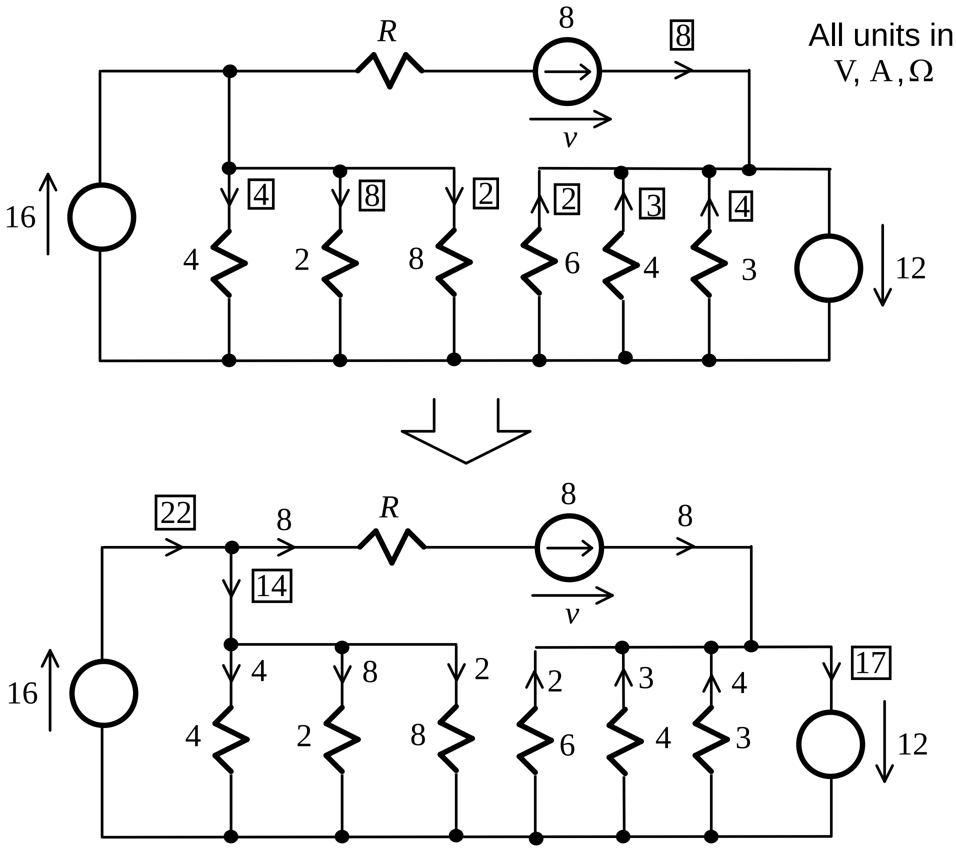
Audio amplifier

Most loudspeakers have resistances in the order of a few Ω. However, in order not to load the CD player or other audio input equipment, the input resistance of the amplifier will have to be large and is usually greater than many $\text{k}\Omega$.

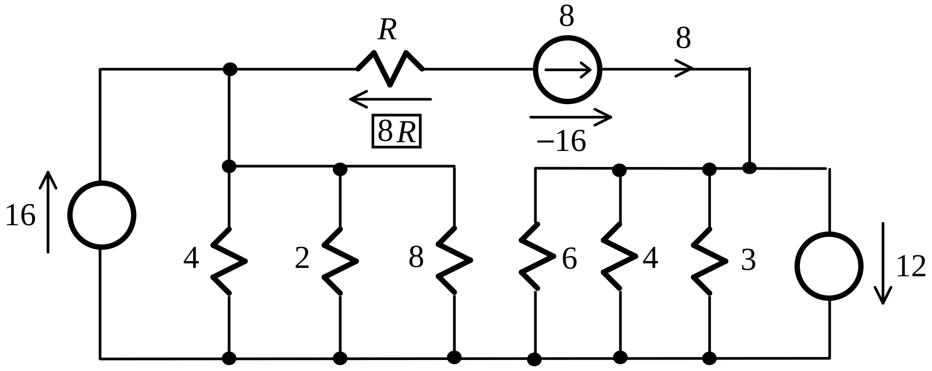
F.2 KCL, KVL and Grounding

Q.1

Currents



Value for $\underline{R}$

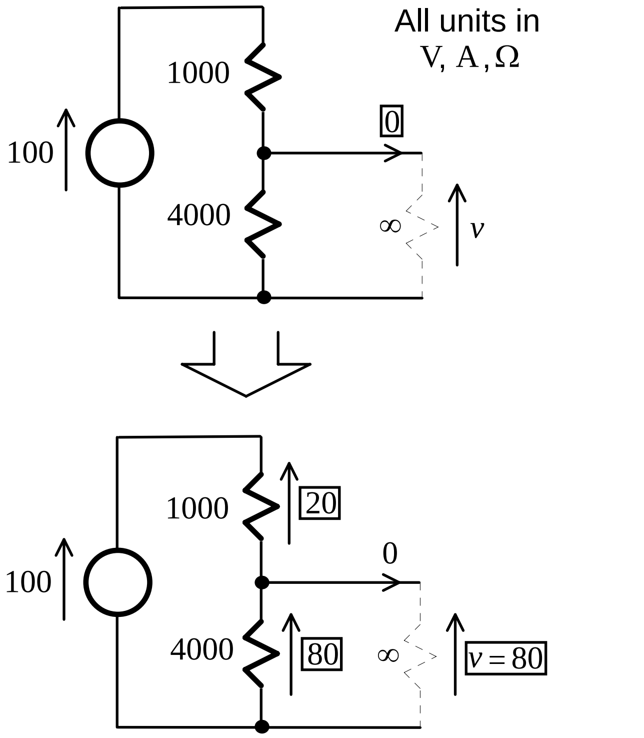


Applying KVL to the loop with the sources and R:

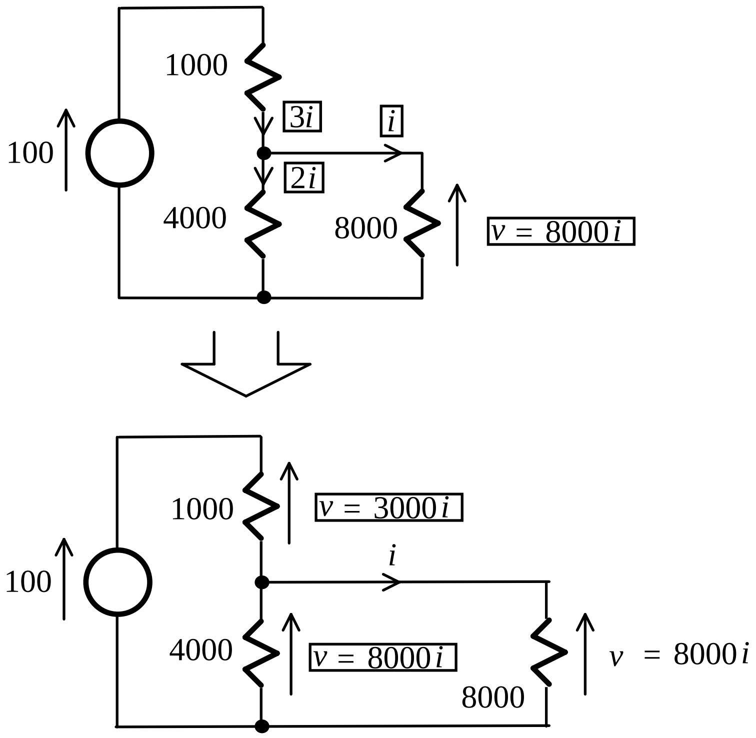
$$16 - 8R - 16 + 12 = 0 \;\Rightarrow\; R = 1.5$$

Q.2

(a) $R = \infty$ *(open circuit or no load situation)*



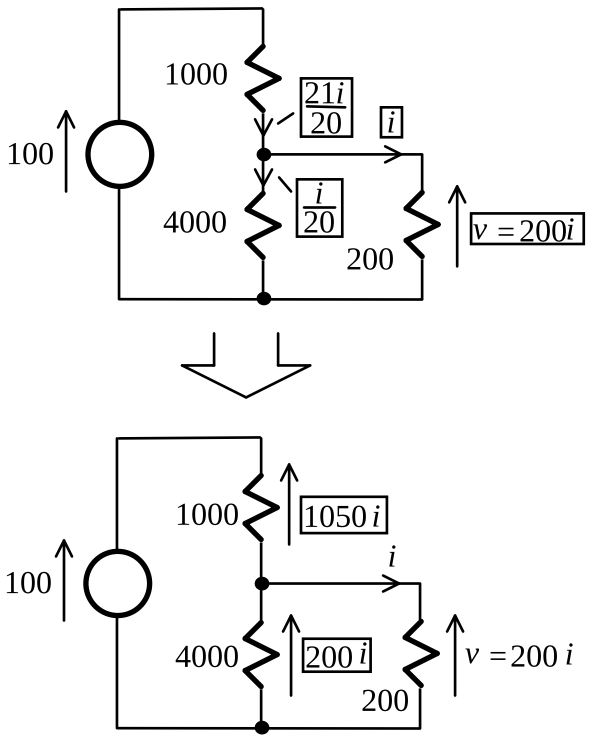
(b) $R = 8000\Omega$



$$100 = 3000i + 8000i \;\Rightarrow\; i = \frac{1}{110}\,\mathrm{A}$$

$v = 8000i = 72.73\text{V}$

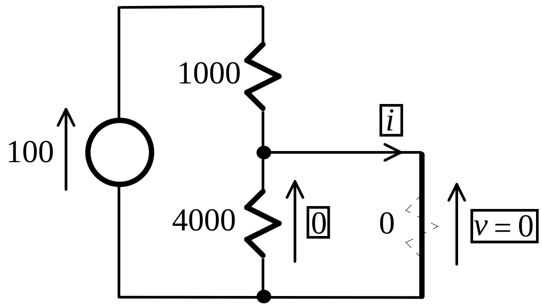
(c) $R = 200\Omega$

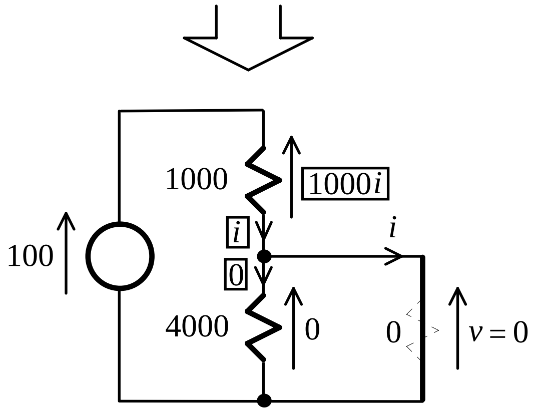


$$100 = 1050i + 200i \;\Rightarrow\; 100 = 1250i \;\Rightarrow\; i = \frac{2}{25}\text{A}$$

$v = 200i = 16\,\text{V}$

(d) $R = 0$ *(short circuit)*





$100 = 1000i \;\Rightarrow\; i = 0.1\,\mathrm{A}$

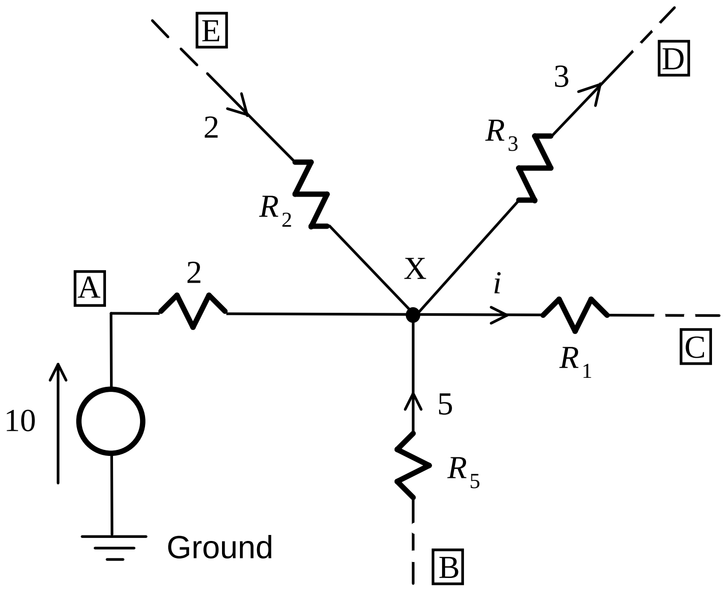
It may be slightly faster to derive two general formulas for v and i and then substitute the values for R.

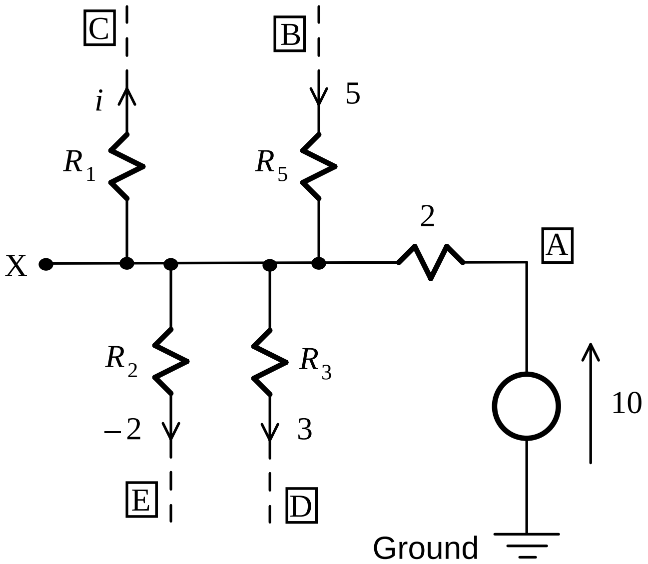
Q.3

Equivalence of circuits

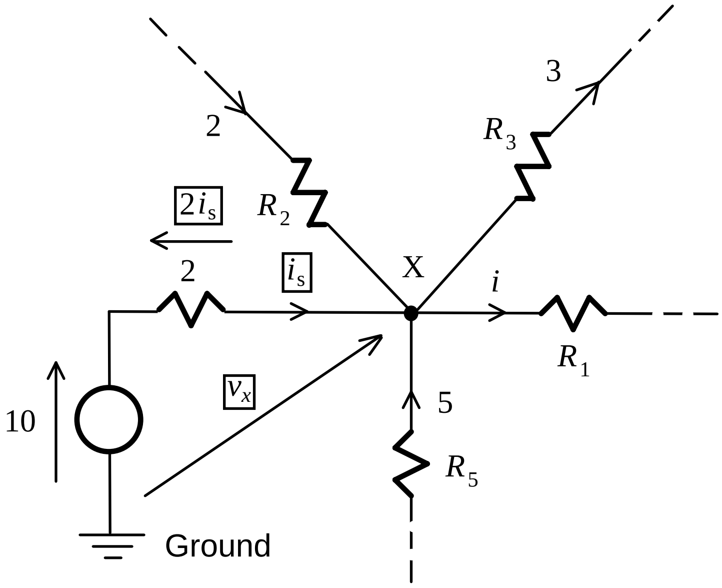
The two circuits are equivalent because the connections (topology), elements and currents between the various nodes are identical:

All units in
V, A, Ω





Current and voltage



Applying KCL to node X and then KVL:

$i_s = i + 3 - 2 - 5 = i - 4$

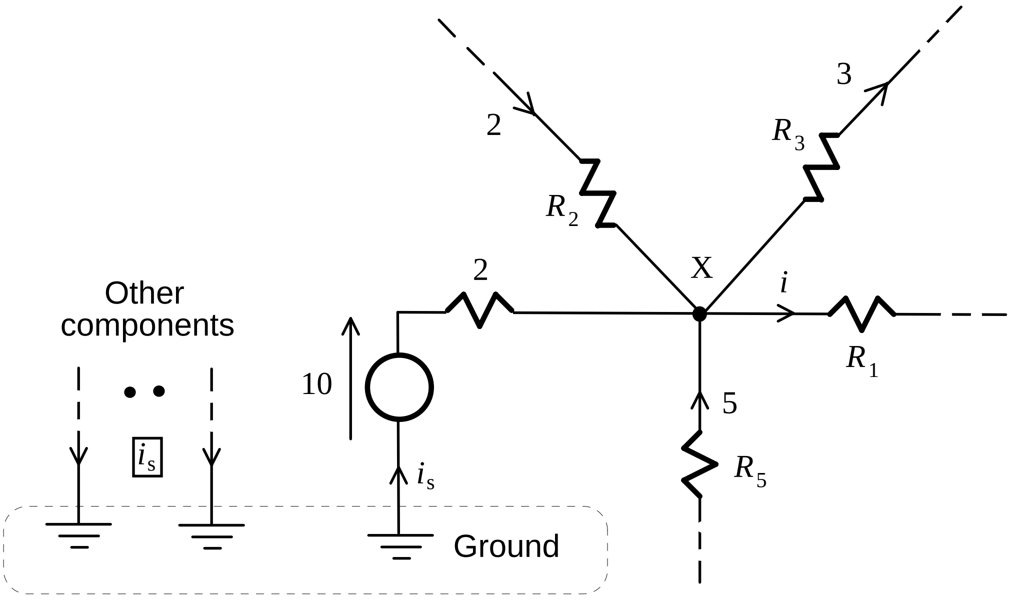
$v_x = 10 - 2i_s = 10 - 2(i-4) = 18 - 2i$

When $i = 2$, $i_s = -2\,\mathrm{A}$ and $v_x = 14\,\mathrm{V}$

When $i = -3$, $i_s = -7\,\mathrm{A}$ and $v_x = 24\,\mathrm{V}$

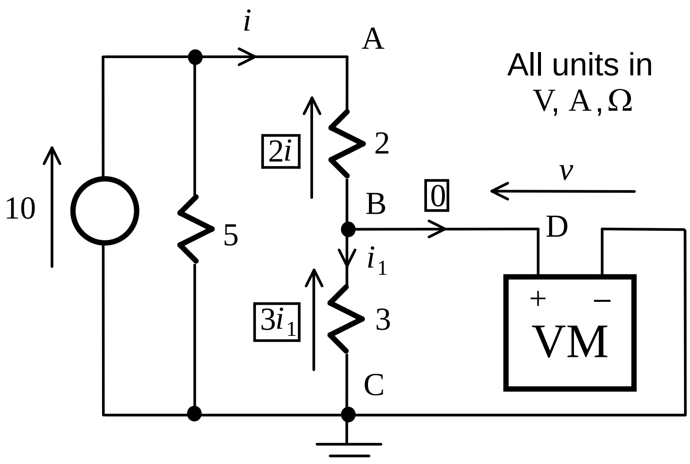
KCL for ground node

Since there may be other components connected to ground, the application of KCL must include all the other connections not shown in the original diagram. The implication is that all these other components must be delivering a combined current of i_s to ground:



Q.4

(a) Point C *grounded and Point* B *connected to Point* D



Applying KCL to node B:

$$i_1 = i - 0 = i$$

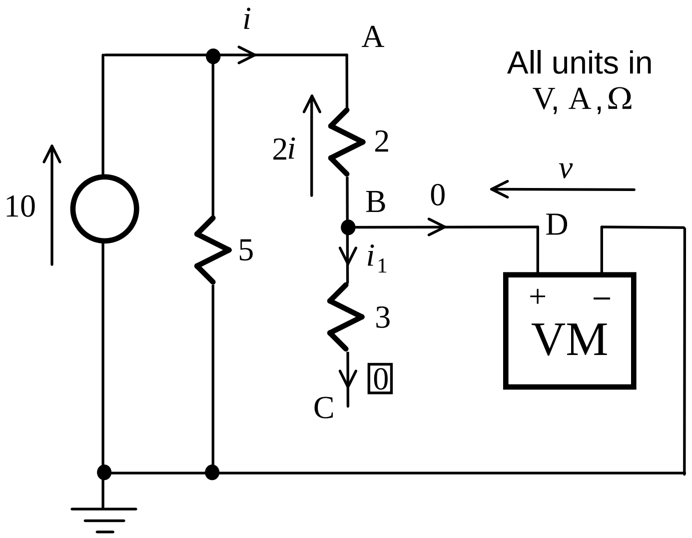
Applying KVL to loop with voltage source, A, B and C:

$$10 = 2i + 3i_1 = 5i \Rightarrow i = 2\,\mathrm{A}$$

Applying KVL to loop with B, C and VM:

$$v = 3i_1 = 3i = 6\,\mathrm{V}$$

(b) No connection for Point C *and Point* B *connected to Point* D



Applying KCL to node C and then to node B:

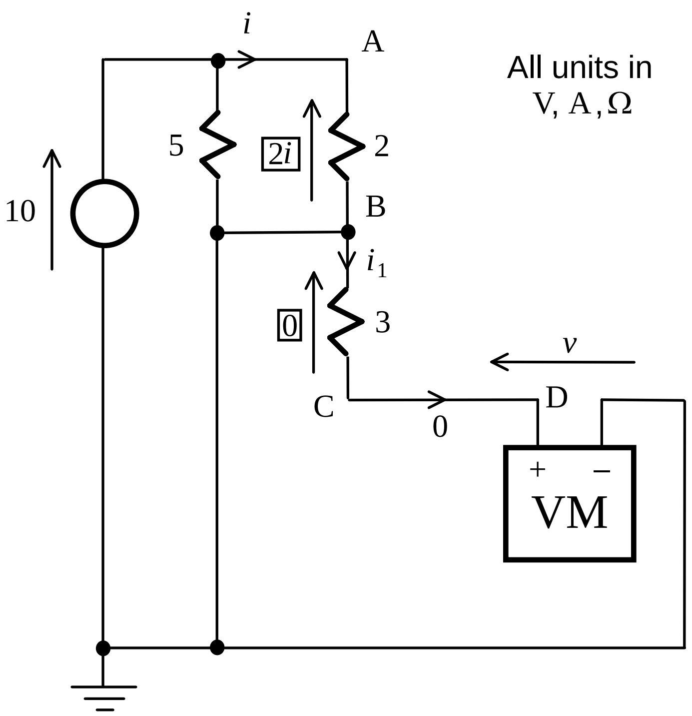
$$i_1 = 0\,\mathrm{A}$$

$$i = i_1 + 0 = 0\,\mathrm{A}$$

Applying KVL to loop with voltage source, A, B and VM:

$$10 = 2i + v \;\Rightarrow\; v = 10\,\mathrm{V}$$

(c) Point B *grounded and Point* C *connected to Point* D



Applying KVL to loop with voltage source, A and B:

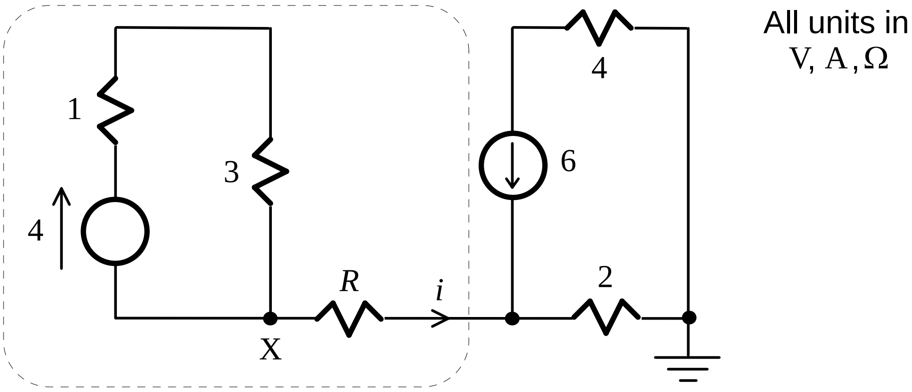
$$10 = 2i \Rightarrow i = 5\mathrm{A}$$

Applying KVL to loop with B, C, D and VM:

$$v + 3i_1 = 0 \Rightarrow v = 0\,\mathrm{V}$$

Q.5

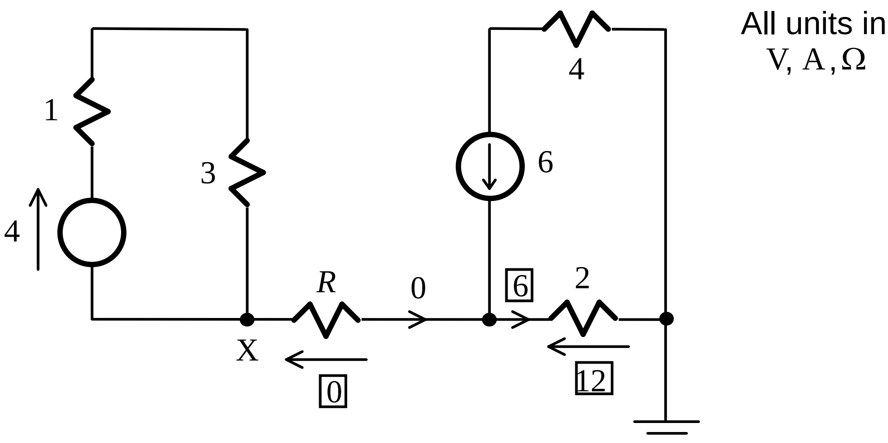
Original circuit



Applying KCL to the dotted surface:

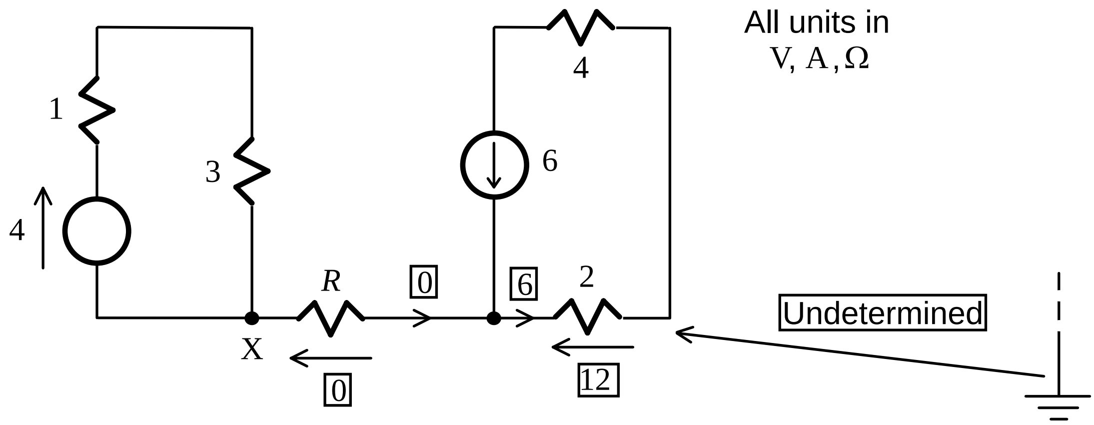
$$i = 0$$

Thus:



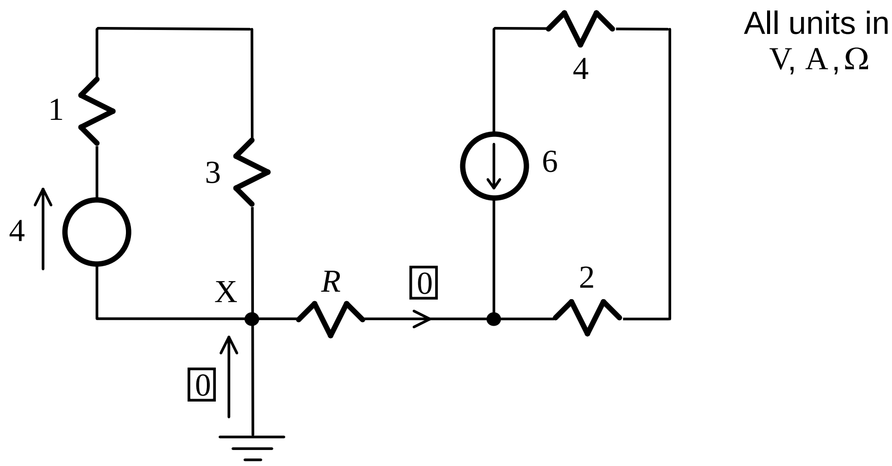
Potential of node X wrt ground $= 12\,\mathrm{V}$

When the circuit is not grounded



The potential of node X wrt ground cannot be determined. In practice, its value will depend on factors such as the existence of static charges and other electrical and magnetic effects.

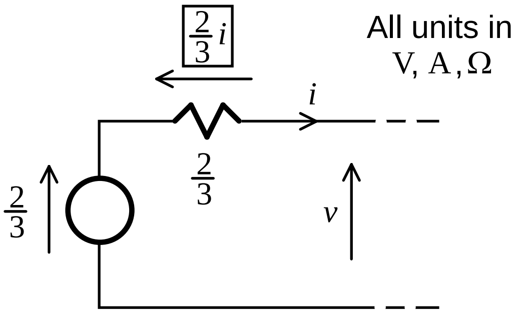
When Point X is grounded



The potential of node X wrt ground is now 0.

Q.6

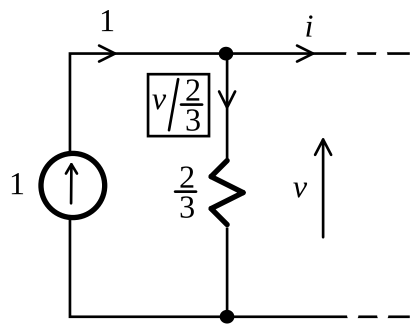
First circuit



Applying KVL:

$$\frac{2}{3} = v + \frac{2i}{3}$$

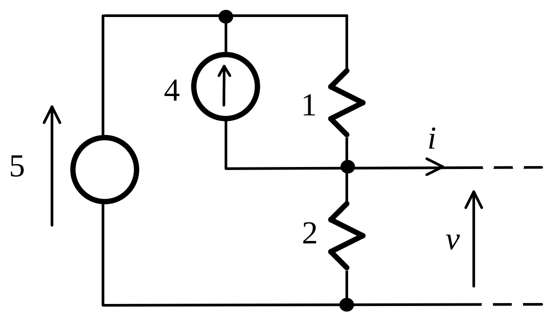
Second circuit

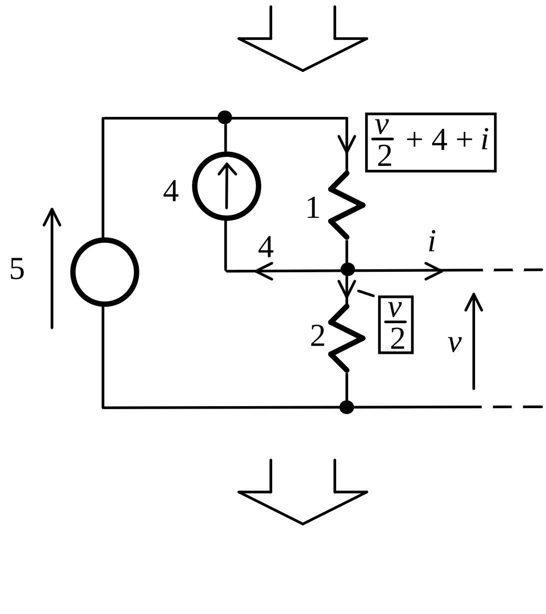


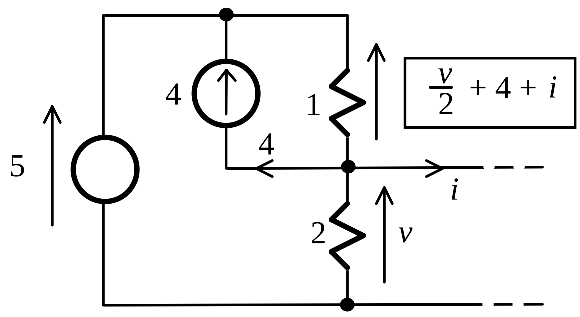
Applying KCL:

$$1 = \frac{v}{2/3} + i \Rightarrow \frac{2}{3} = v + \frac{2i}{3}$$

Third circuit







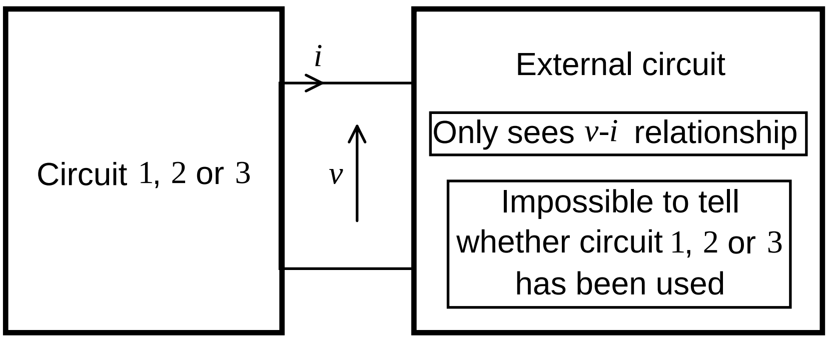
From KVL:

$$5 = \frac{v}{2} + 4 + i + v = \frac{3v}{2} + 4 + i$$

$$1 = \frac{3v}{2} + i \;\Rightarrow\; \frac{2}{3} = v + \frac{2i}{3}$$

Equivalence

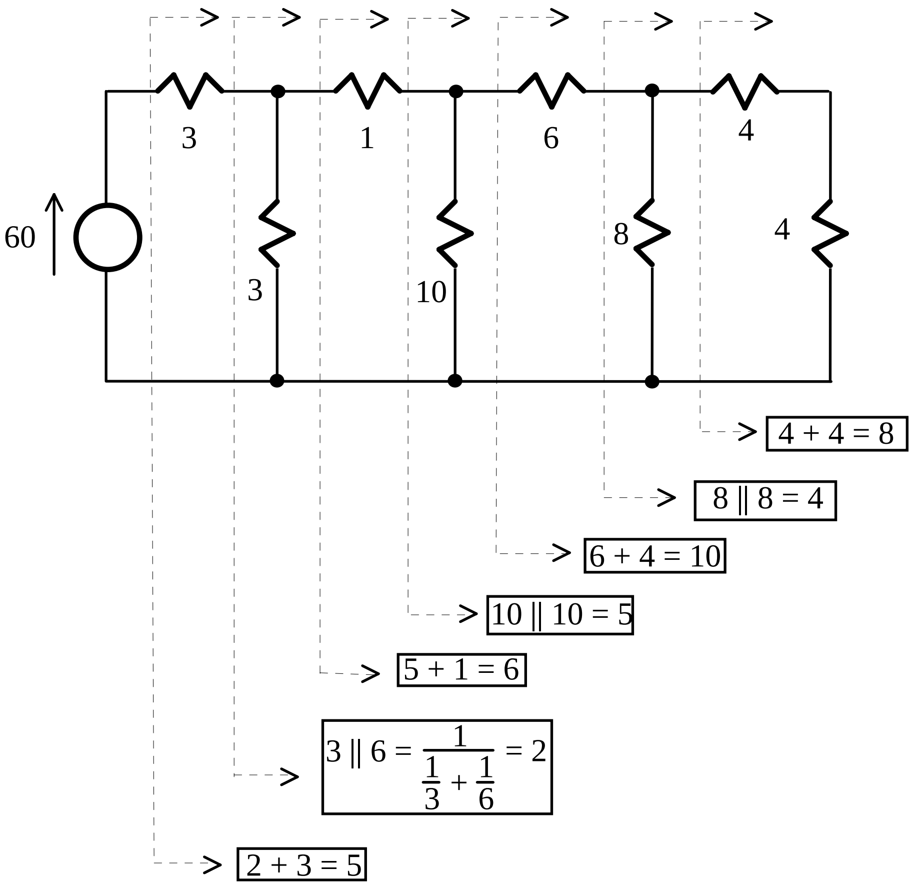
The three circuits are different in circuit topology and the components used. However, they have the same voltage-current relationship and are electrically equivalent from a voltage-current point of view. It is impossible for an external circuit connected to the outputs of these circuits to tell which circuit has actually been used:



F.3 DC Circuit Analysis I

Q.1

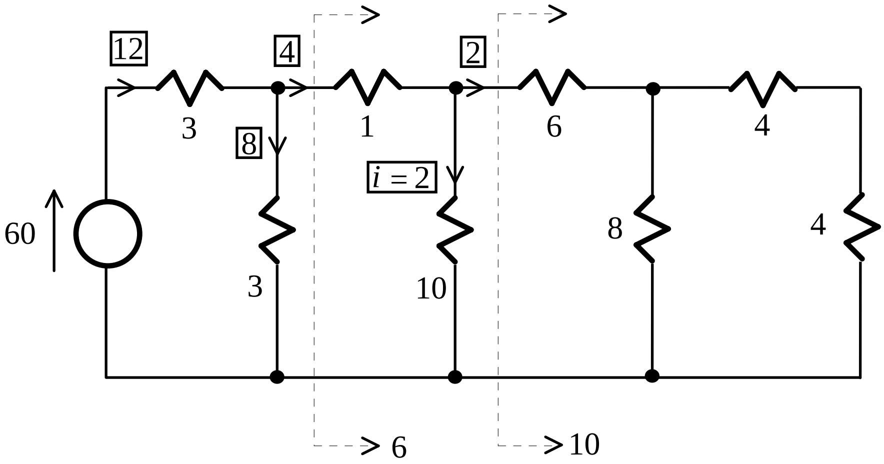
Source current



All units in
V, A, Ω

Source current $= \dfrac{60}{5} = 12\,\text{A}$

Value for i

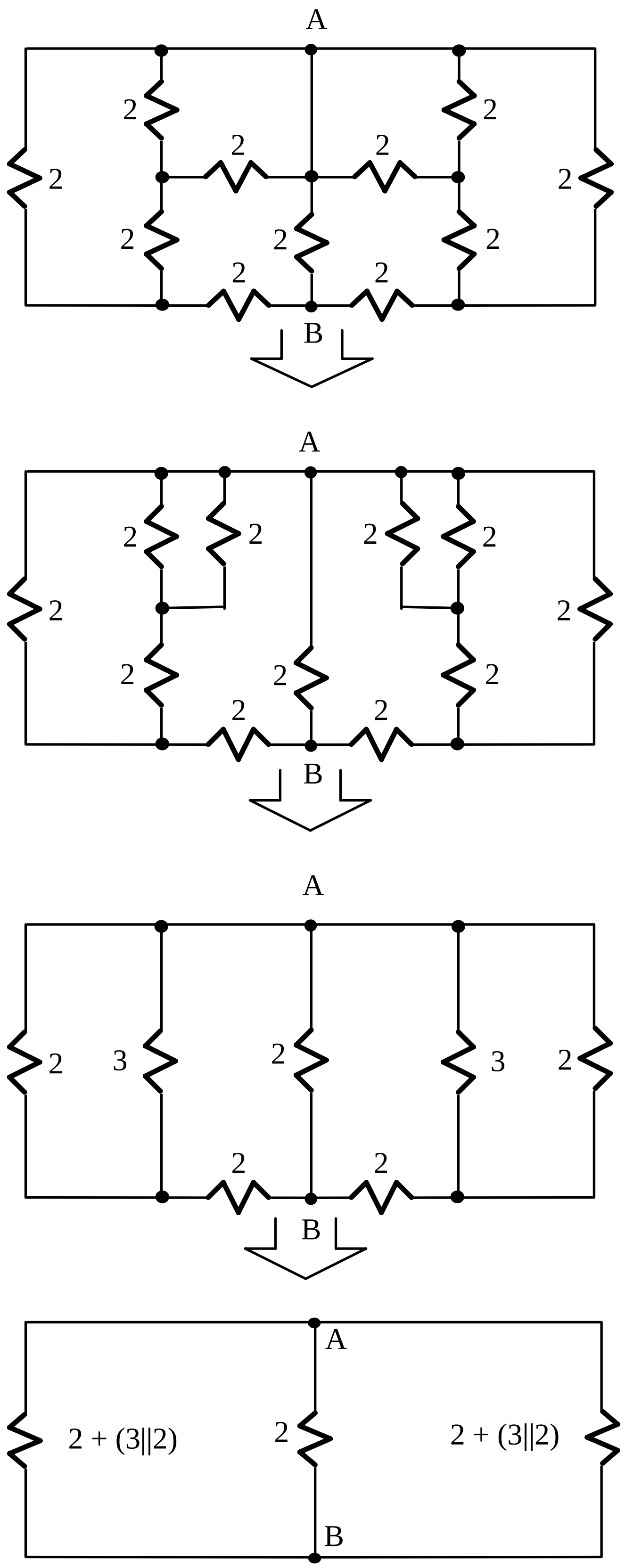


All units in
V, A, Ω

Q.2

Original circuit

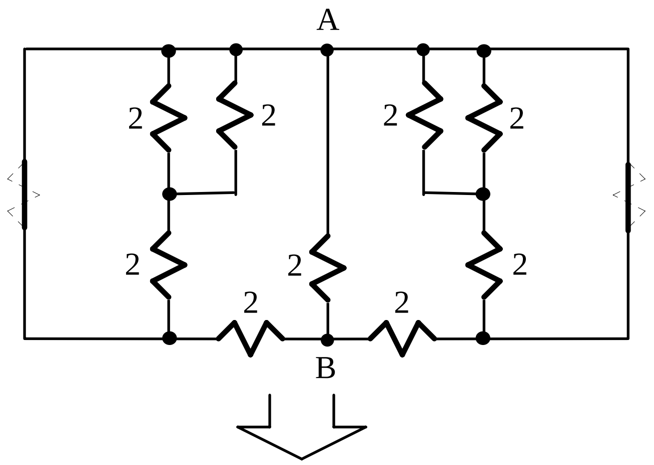
All units in
V, A, Ω

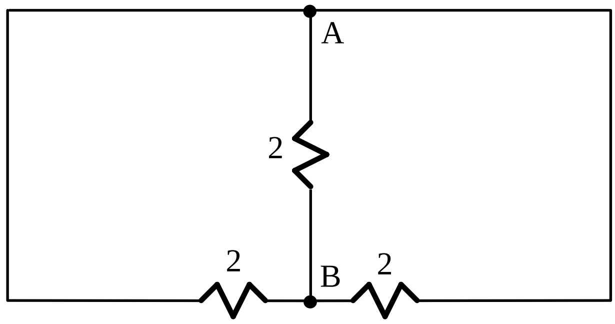


$$2+(3||2)=2+\frac{1}{\frac{1}{3}+\frac{1}{2}}=2+\frac{6}{5}=3.2$$

Equivalent resistance $=2\,||\,3.2\,||\,3.2=2\,||\,1.6=\dfrac{1}{0.5+0.625}=0.889\,\Omega$

When outer two resistors are short-circuited



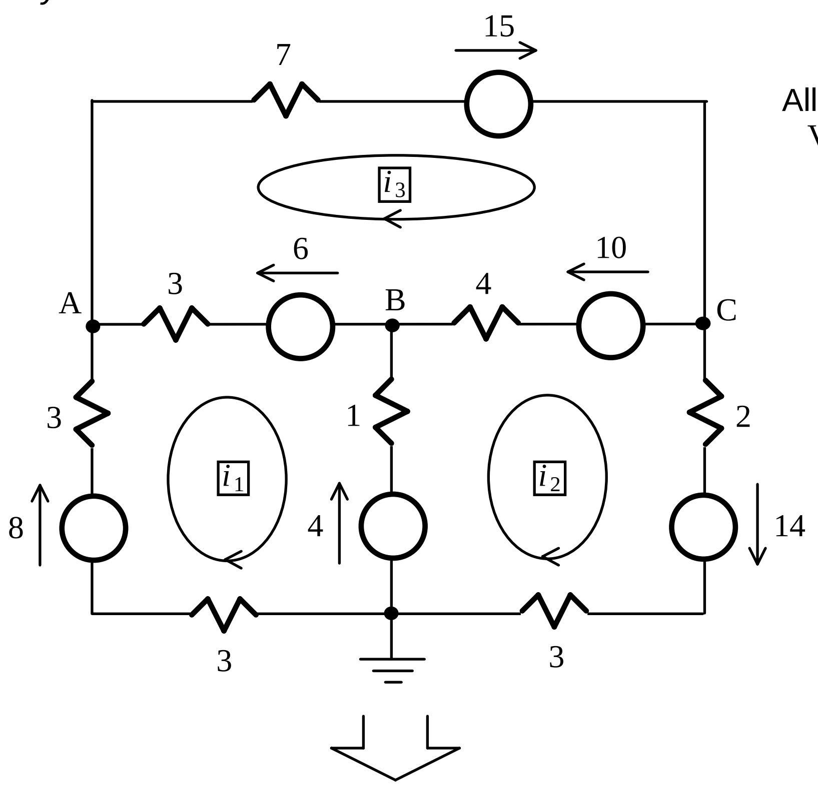


Equivalent resistance $=2||\,2\,||\,2=\dfrac{2}{3}=0.666\,\Omega$

Q.3

Mesh analysis

All units in V, A, Ω



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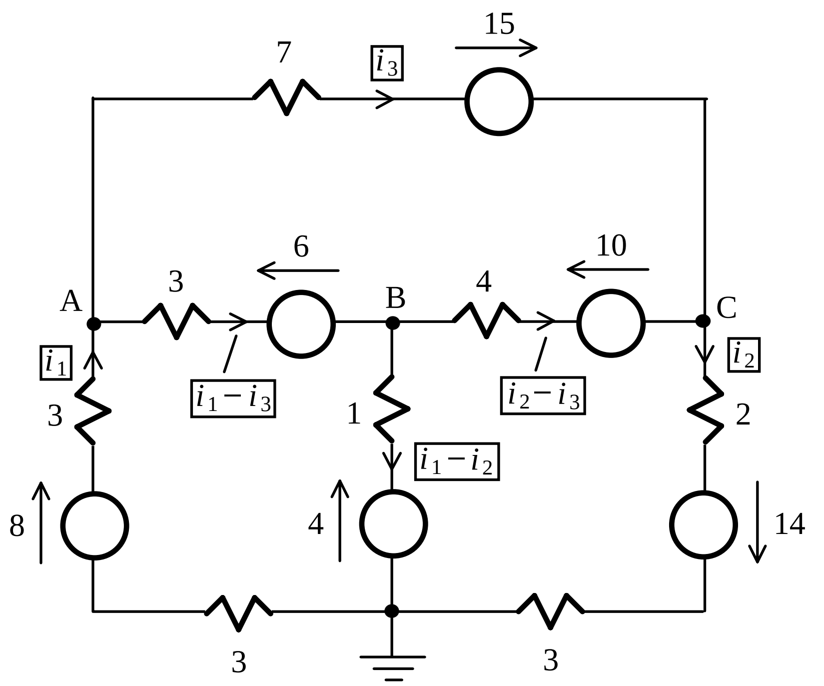


Applying KVL for the three loops shown:

$$8 - 3i_1 - 3(i_1 - i_3) - 6 + (i_1 - i_2) - 4 - 3i_1 = 0$$
$$14 - 3i_2 + 4 - (i_2 - i_1) - 4(i_2 - i_3) - 10 - 2i_2 = 0$$
$$15 + 10 - 4(i_3 - i_2) + 6 - 3(i_3 - i_1) - 7i_3 = 0$$

Simplifying:

$$-2 = 10i_1 - i_2 - 3i_3$$

$$8 = -i_1 + 10i_2 - 4i_3$$

$$31 = -3i_1 - 4i_2 + 14i_3$$

In matrix form:

$$\begin{bmatrix} 10 & -1 & -3 \\ -1 & 10 & -4 \\ -3 & -4 & 14 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 8 \\ 31 \end{bmatrix}$$

Solving (actually not required in this question):

$$-2 + 10(8) = 10i_1 - i_2 - 3i_3 + 10(-i_1 + 10i_2 - 4i_3) \quad \Rightarrow \quad 78 = 99i_2 - 43i_3$$

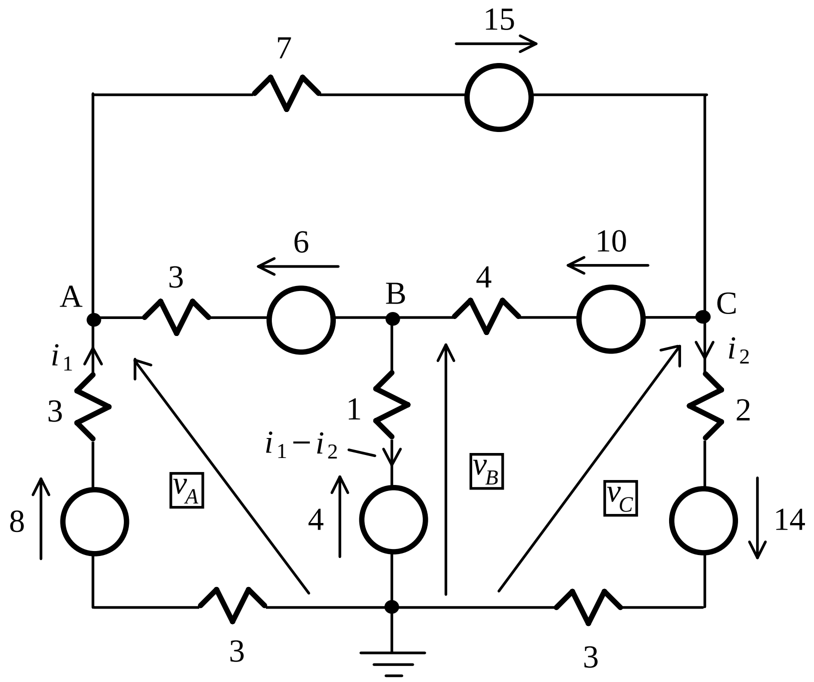
$$31 - 3(8) = -3i_1 - 4i_2 + 14i_3 - 3(-i_1 + 10i_2 - 4i_3) \;\Rightarrow\; 7 = -34i_2 + 26i_3$$

$$26(78) + 43(7) = 26(99i_2 - 43i_3) + 43(-34i_2 + 26i_3) \;\Rightarrow\; i_2 = \frac{26(78) + 43(7)}{26(99) - 43(34)} = \frac{2329}{1112}$$

$$34(78) + 99(7) = 34(99i_2 - 43i_3) + 99(-34i_2 + 26i_3) \;\Rightarrow\; i_3 = \frac{34(78) + 99(7)}{-34(43) + 99(26)} = \frac{3345}{1112}$$

$$i_1 = -8 + 10i_2 - 4i_3 = -8 + \frac{23290}{1112} - \frac{4(3345)}{1112} = \frac{1014}{1112}$$

Voltages of nodes A, B and C with respect to ground:

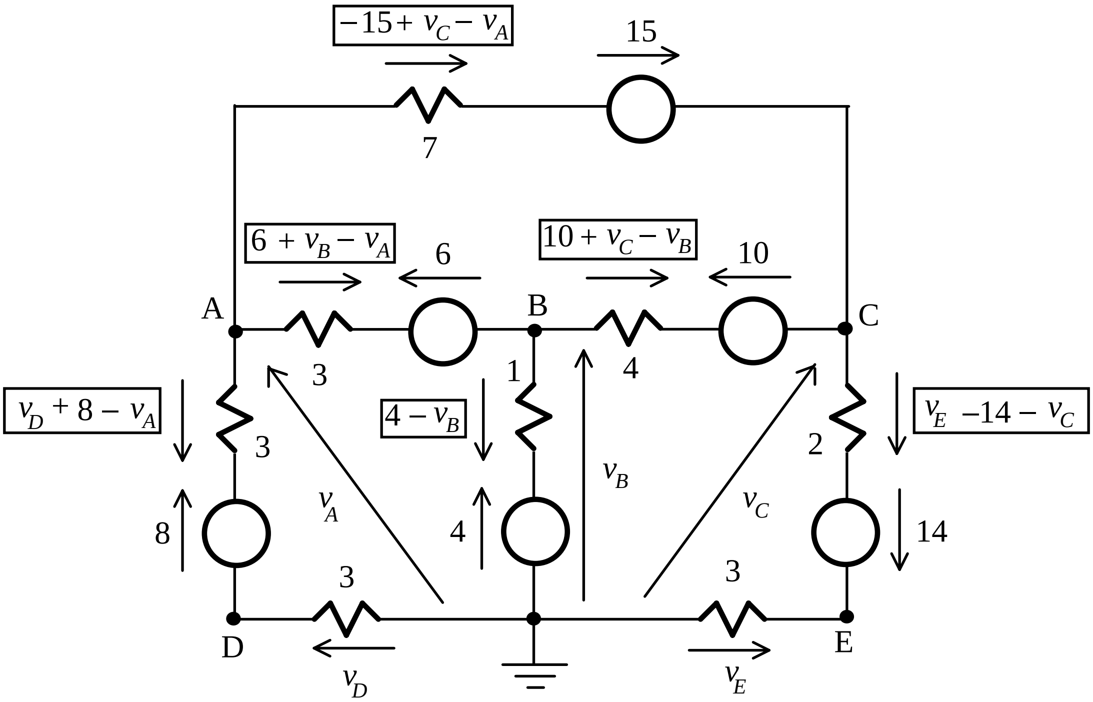


$$v_A = 8 - 6i_1$$

$$v_B = 4 + i_1 - i_2$$

$$v_C = -14 + 5i_2$$

Nodal analysis



Applying KCL to nodes A, B, C, D and E:

$$\frac{v_D + 8 - v_A}{3} + \frac{6 + v_B - v_A}{3} + \frac{v_C - v_A - 15}{7} = 0$$

$$\frac{4 - v_B}{1} + \frac{v_A - v_B - 6}{3} + \frac{v_C - v_B + 10}{4} = 0$$

$$\frac{v_E - 14 - v_C}{2} + \frac{v_A - v_C + 15}{7} + \frac{v_B - v_C - 10}{4} = 0$$

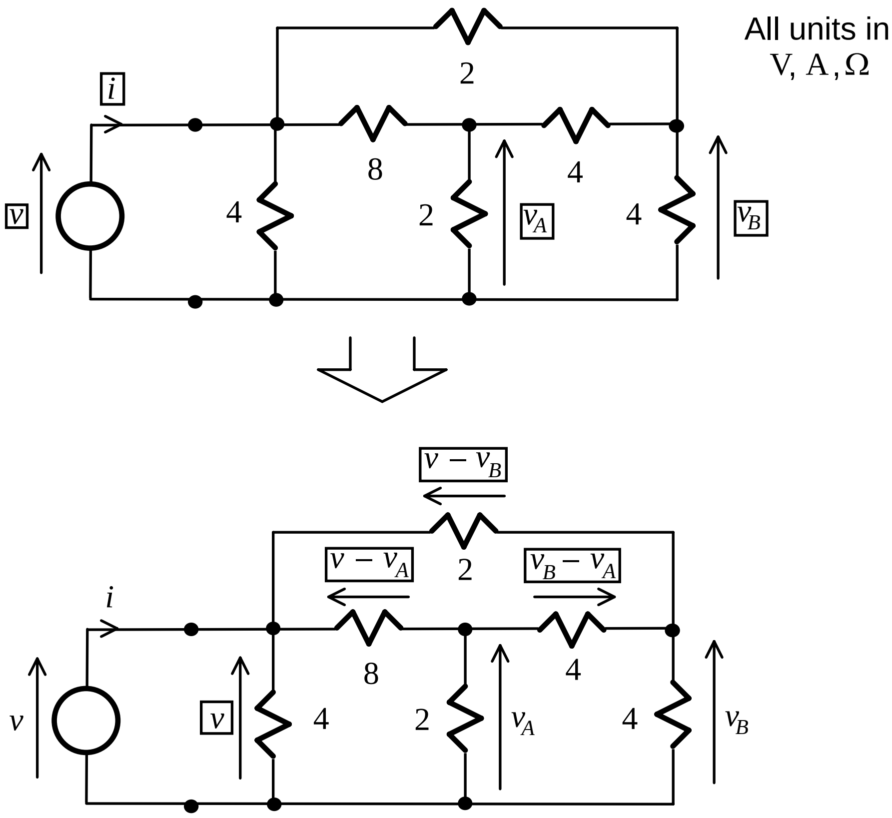
$$\frac{v_D + 8 - v_A}{3} + \frac{v_D}{3} = 0$$

$$\frac{v_E - 14 - v_C}{2} + \frac{v_E}{3} = 0$$

In matrix form:

$$\begin{bmatrix} \frac{1}{3}+\frac{1}{3}+\frac{1}{7} & -\frac{1}{3} & -\frac{1}{7} & -\frac{1}{7} & 0 \\ -\frac{1}{3} & 1+\frac{1}{3}+\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ -\frac{1}{7} & -\frac{1}{4} & \frac{1}{2}+\frac{1}{7}+\frac{1}{4} & 0 & -\frac{1}{2} \\ \frac{`}{3} & 0 & 0 & -\frac{1}{3}-\frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2}-\frac{1}{3} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \\ v_E \end{bmatrix} = \begin{bmatrix} \frac{8}{3}+\frac{6}{3}-\frac{15}{7} \\ \frac{1}{4}-2+\frac{10}{4} \\ -7+\frac{15}{7}-\frac{10}{4} \\ \frac{8}{3} \\ -\frac{14}{2} \end{bmatrix}$$

Q.4



Applying KCL:

$$i = \frac{v}{4} + \frac{v - v_A}{8} + \frac{v - v_B}{2}$$

$$\frac{v_A}{2} + \frac{v_A - v}{8} + \frac{v_A - v_B}{4} = 0 \;\Rightarrow\; 4v_A + v_A - v + 2v_A - 2v_B = 7v_A - 2v_B - v = 0$$

$$\frac{v_B}{4} + \frac{v_B - v_A}{4} + \frac{v_B - v}{2} = 0 \;\Rightarrow\; v_B + v_B - v_A + 2v_B - 2v = -v_A + 4v_B - 2v = 0$$

Eliminating v_A and v_B:

$$\left(7v_A - 2v_B - v\right) + 7\left(-v_A + 4v_B - 2v\right) = 0 \;\Rightarrow\; 26v_B - 15v = 0 \;\Rightarrow\; v_B = \frac{15}{26}v$$

$$2\left(7v_A - 2v_B - v\right) + \left(-v_A + 4v_B - 2v\right) = 0 \;\Rightarrow\; 13v_A - 4v = 0 \;\Rightarrow\; v_A = \frac{4}{13}v$$

$$i = \frac{v}{4} + \frac{v - v_A}{8} + \frac{v - v_B}{2} = \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right)v - \frac{1}{8}\left(\frac{4}{13}\right)v - \frac{1}{2}\left(\frac{15}{26}\right)v = \frac{57}{104}v$$

The equivalent resistance without the 13Ω resistor is therefore

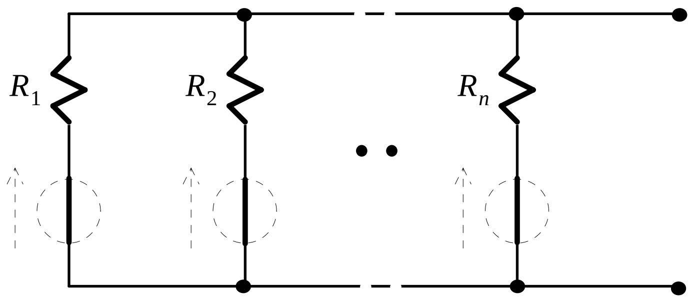
$$\frac{v}{i} = \frac{104}{57} = 1.82\,\Omega$$

and the equivalent resistance with the 13Ω series resistor is

Equivalent resistance $= 14.82\,\Omega$

Q.5

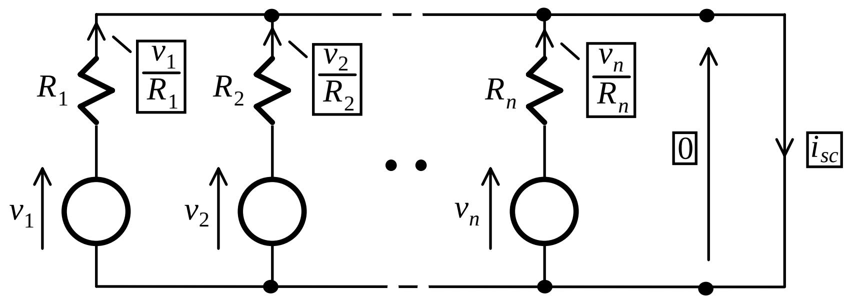
Equivalent resistance



Since the resistors are in parallel, the equivalent resistance R is

$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

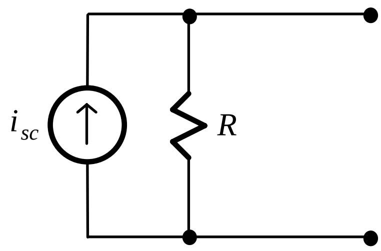
Short circuit current



Applying KCL:

$$i_{sc} = \frac{v_1}{R_1} + \cdots + \frac{v_n}{R_n}$$

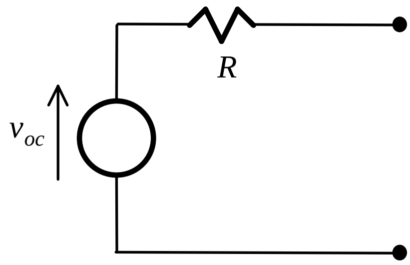
Norton's equivalent circuit



$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

$$i_{sc} = \frac{v_1}{R_1} + \cdots + \frac{v_n}{R_n}$$

Thevenin's equivalent circuit

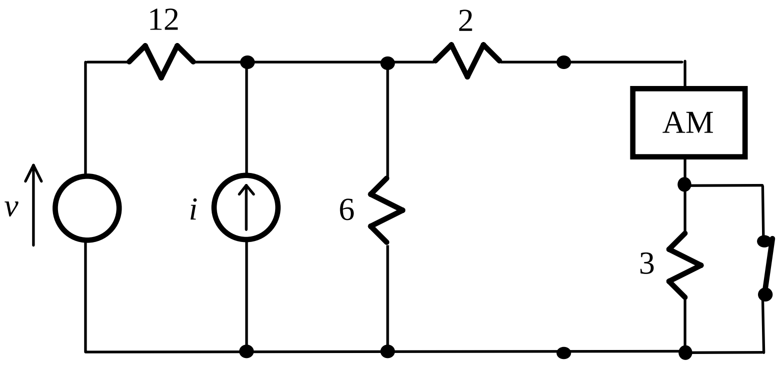


$$\frac{1}{R} = \frac{1}{R_1} + \cdots + \frac{1}{R_n}$$

$$v_{oc} = Ri_{sc} = \frac{\dfrac{v_1}{R_1} + \cdots + \dfrac{v_n}{R_n}}{\dfrac{1}{R_1} + \cdots + \dfrac{1}{R_n}}$$

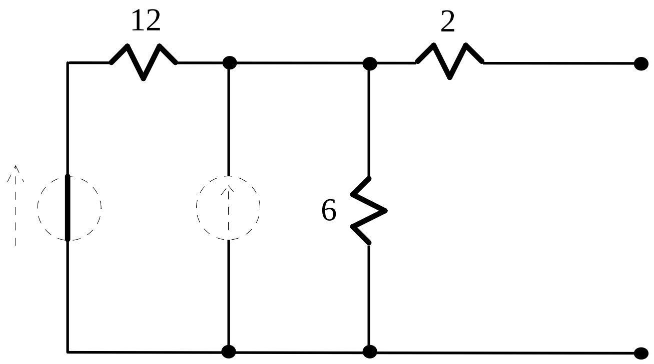
Q.6

Re-drawing original circuit



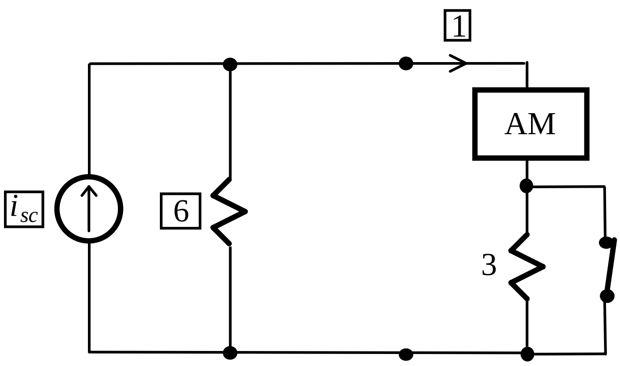
All units in V, A, Ω

Equivalent resistance without 3Ω *resistor*



Equivalent resistance $R = 2 + (6 \| 12) = 2 + \dfrac{1}{\dfrac{1}{6} + \dfrac{1}{12}} = 2 + 4 = 6\Omega$

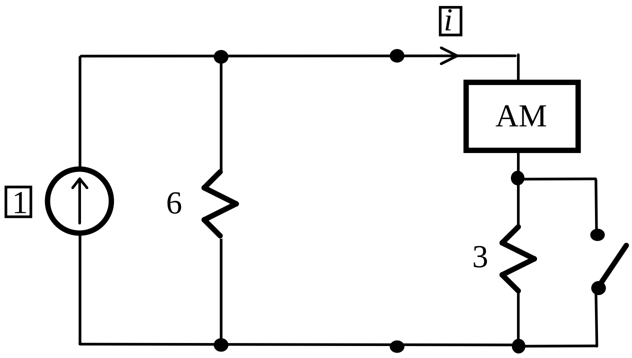
Using Norton's equivalent circuit



Since AM reads 1A

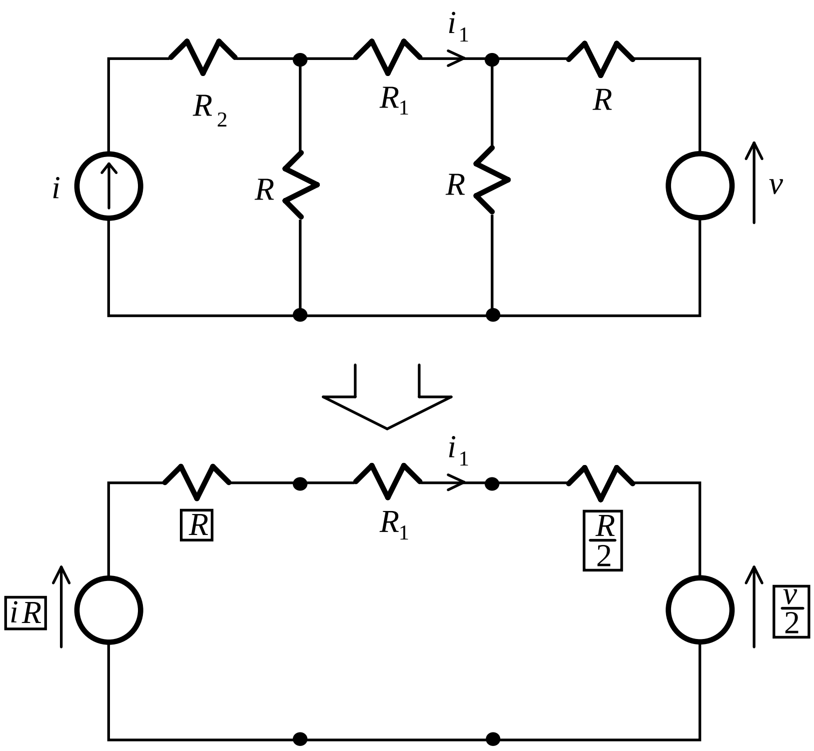
$i_{sc} = 1\text{A}$

Thus, when the switch is open



$$i = 1 \times \left(\frac{6}{6+3} \right) = \frac{2}{3} \mathrm{A}$$

Q.7

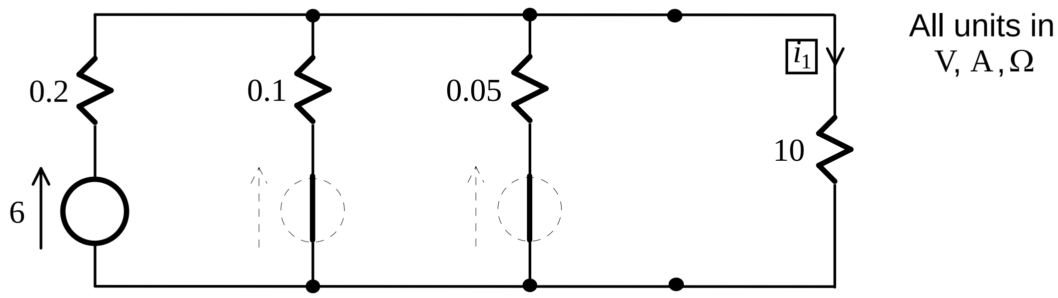


$$i_1 = \frac{iR - \frac{v}{2}}{\frac{3R}{2} + R_1} = \frac{2iR - v}{3R + 2R_1}$$

F.4 DC Circuit Analysis II

Q.1

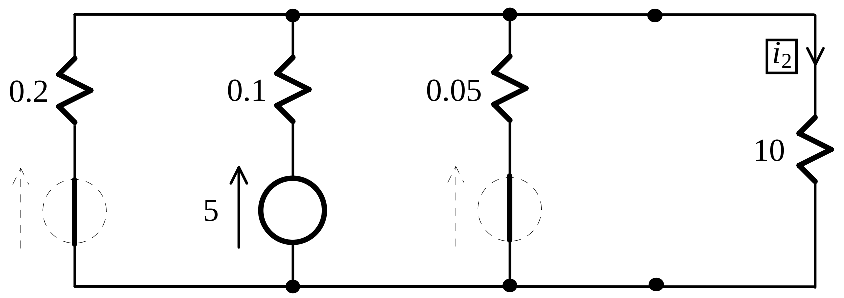
Load current due to Battery 1



$$\text{Current from source} = \frac{6}{0.2+0.1\|0.05\|10} = \frac{6}{0.2+\dfrac{1}{10+20+0.1}} = \frac{6}{0.2+0.033} = 25.75$$

$$i_1 = 25.75\times\left(\frac{0.1\|0.05}{10+0.1\|0.05}\right) = 25.75\times\left(\frac{\dfrac{1}{10+20}}{10+\dfrac{1}{10+20}}\right) = 25.75\times\left(\frac{0.033}{10.033}\right) = 0.0847\,\text{A}$$

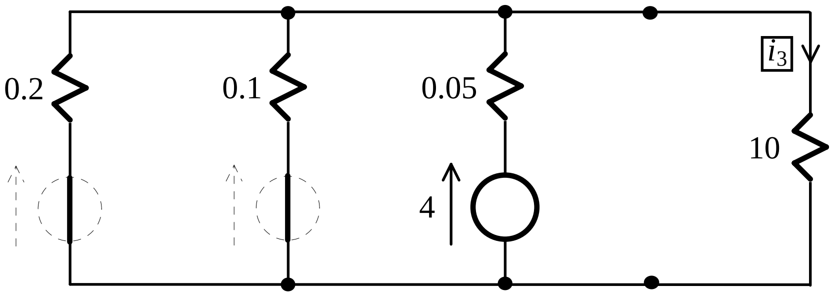
Load current due to Battery 2



$$\text{Current from source} = \frac{5}{0.1+0.2\|0.05\|10} = \frac{5}{0.1+\dfrac{1}{5+20+0.1}} = \frac{5}{0.1+0.04} = 35.71$$

$$i_2 = 35.71\times\left(\frac{0.2\|0.05}{10+0.2\|0.05}\right) = 35.71\times\left(\frac{\dfrac{1}{5+20}}{10+\dfrac{1}{5+20}}\right) = 35.71\times\left(\frac{0.04}{10.04}\right) = 0.1423\,\text{A}$$

Load current due to Battery 3



Current from source $= \dfrac{4}{0.05 + 0.2||0.1||10} = \dfrac{4}{0.05 + \dfrac{1}{5+10+0.1}} = \dfrac{4}{0.05+0.066} = 34.48$

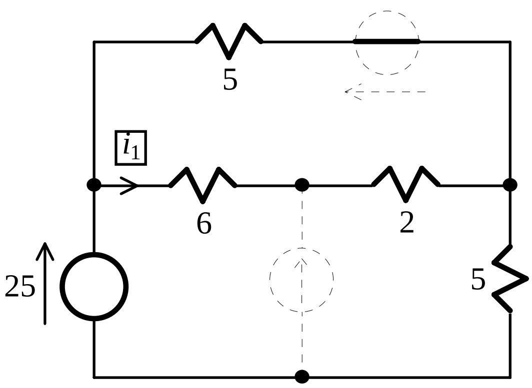
$$i_3 = 34.48 \times \left(\frac{0.2||0.1}{10 + 0.2||0.1} \right) = 34.48 \times \left(\frac{\dfrac{1}{5+10}}{10 + \dfrac{1}{5+10}} \right) = 34.48 \times \left(\frac{0.067}{10.07} \right) = 0.2294\,\mathrm{A}$$

Actual load current

$$i = i_1 + i_2 + i_3 = 0.0847 + 0.1423 + 0.2294 = 0.4564\,\mathrm{A}$$

Q.2

Current due to $25\,\mathrm{V}$ *voltage source*

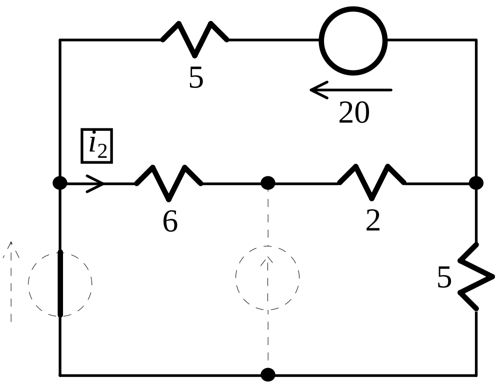


All units in
$\mathrm{V}, \mathrm{A}, \Omega$

Current from source $= \dfrac{25}{5 + 5||(6+2)} = \dfrac{25}{5 + \dfrac{1}{\dfrac{1}{5} + \dfrac{1}{8}}} = \dfrac{25}{5 + 3.08} = 3.09$

$$i_1 = 3.09 \left(\frac{5}{5+8} \right) = 1.19\,\mathrm{A}$$

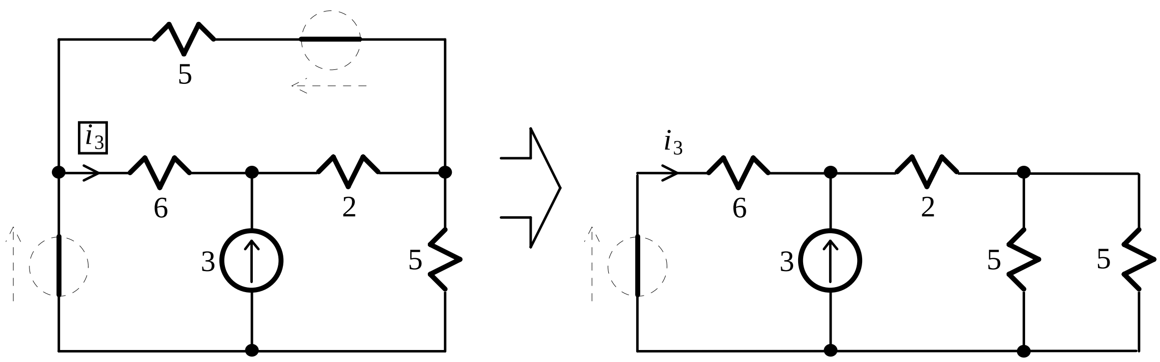
Current due to $20\,\text{V}$ *voltage source*



Current from source $= \dfrac{20}{5+5\|(6+2)} = \dfrac{20}{5+\dfrac{1}{\dfrac{1}{5}+\dfrac{1}{8}}} = \dfrac{20}{5+3.08} = 2.48$

$$i_2 = 2.48\left(\frac{5}{5+8}\right) = 0.954\,\text{A}$$

Current due to current source



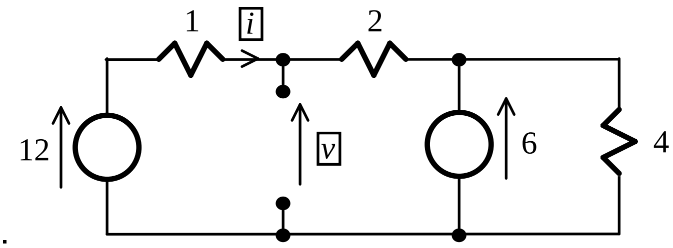
$$i_3 = -3\left[\frac{2+(5\|5)}{6+2+(5\|5)}\right] = -3\left[\frac{2+\dfrac{1}{0.2+0.2}}{8+\dfrac{1}{0.2+0.2}}\right] = -1.286\text{A}$$

Actual current

$$i = i_1 + i_2 + i_3 = 1.19 + 0.954 - 1.286 = 0.858\,\text{A}$$

Q.3

Open circuit voltage

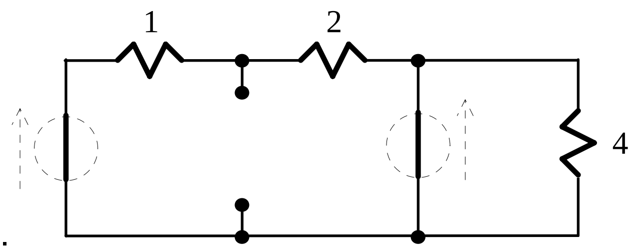


All units in
V, A, Ω

$$i = \frac{12-6}{1+2} = 2$$

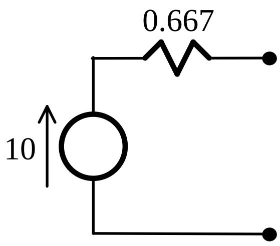
$$v = 12 - i = 10\,\mathrm{V}$$

Equivalent resistance



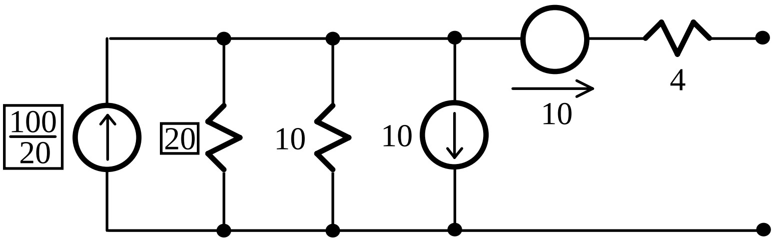
Resistance across terminals $= 1 \| 2 = \dfrac{1}{1+\dfrac{1}{2}} = 0.667\,\Omega$

Thevenin's equivalent circuit and maximum power



Maximum power transferable (with a $0.667\,\Omega$ load) $= \left(\dfrac{10}{2 \times 0.667}\right)^2 0.667 = 37.5\,\mathrm{W}$

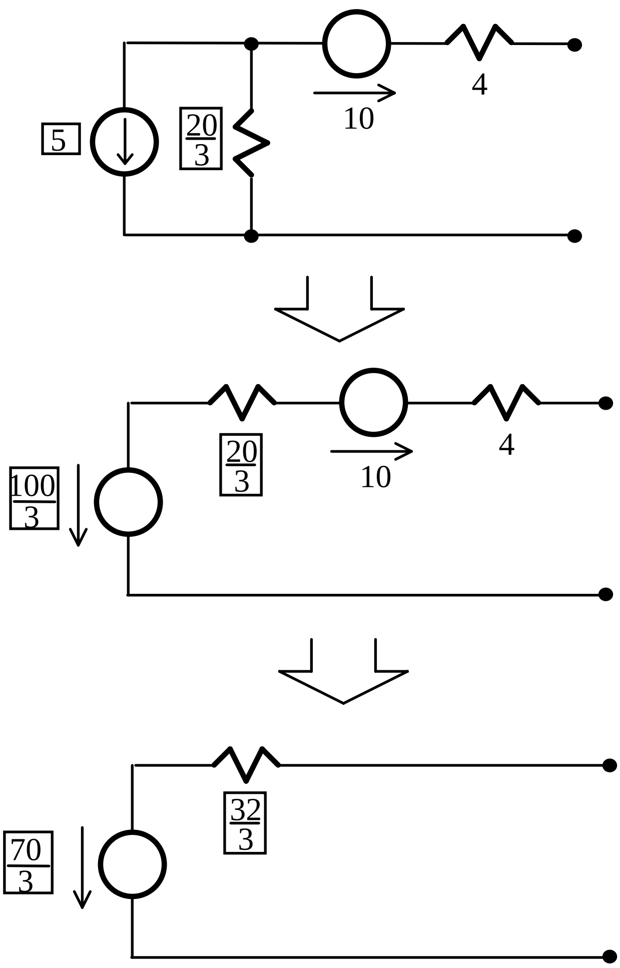
Q.4



All units in V, A, Ω

Combined current of current sources $= 10 - \dfrac{100}{20} = 5$

Equivalent parallel resistance $= 20 \,||\, 10 = \dfrac{1}{\dfrac{1}{20} + \dfrac{1}{10}} = \dfrac{20}{3}$



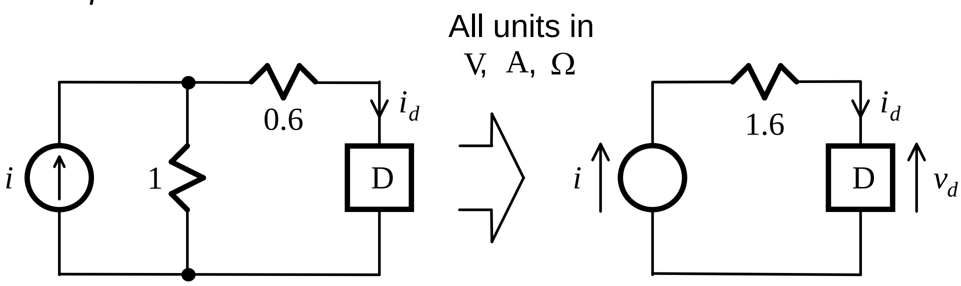
Resistor that draws $2\,\mathrm{A} = \dfrac{70}{3\times 2} - \dfrac{32}{3} = 1\Omega$

Resistor that absorbs the maximum power $= \dfrac{32}{3}\Omega$

Maximum power that can be transferred $= \left(\dfrac{70}{3} \Big/ \dfrac{32\times 2}{3}\right)^2 \dfrac{32}{3} = \dfrac{1225}{96}\mathrm{W}$

Q.5

Thevenin's equivalent circuit

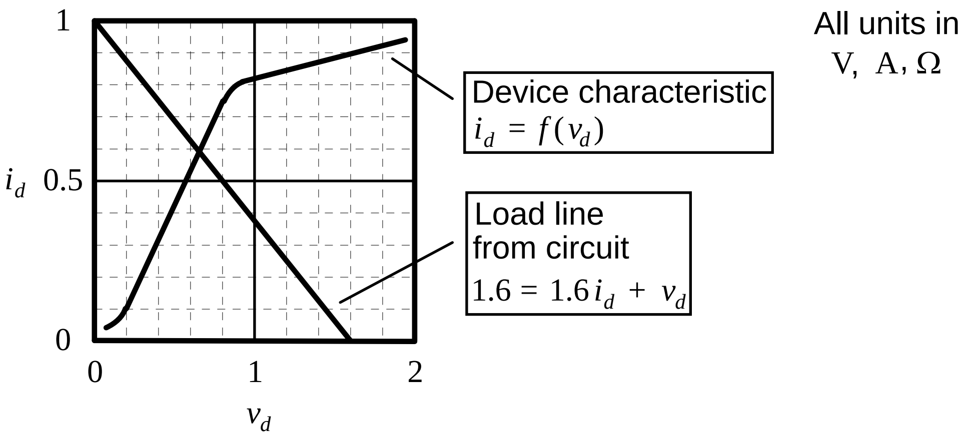


Device current given $i = 1.6\,\mathrm{A}$

Applying KVL:

$$i = 1.6 i_d + v_d = 1.6$$

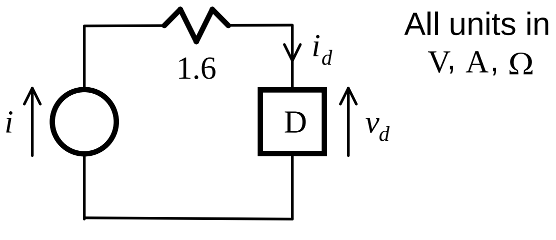
v_d and i_d can be found from solving this (which gives rise to the load line) and the relationship $i_d = f(v_d)$ given by the characteristic curve. Specifically, when $i_d = 0$, $v_d = 1.6$. Also, when $v_d = 0$, $i_d = 1$.



The point of intersection gives

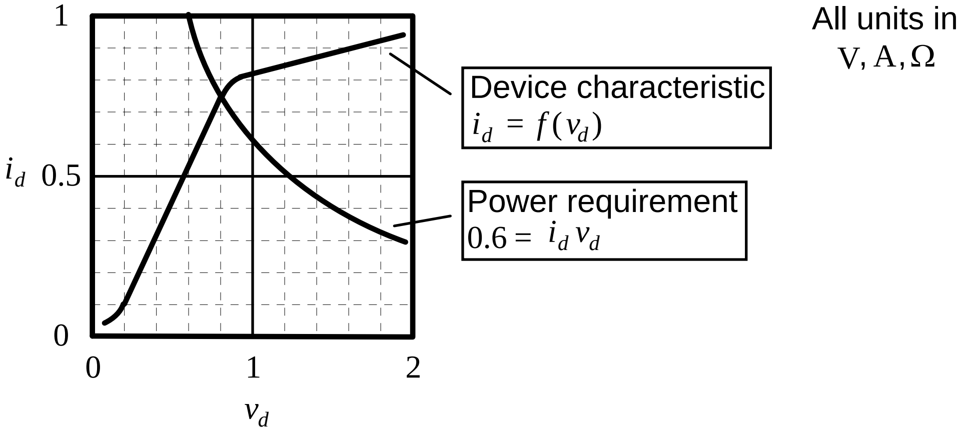
$$i_d = 0.6\,\mathrm{A}$$

Source current for power dissipated in D *to be* $0.6\,\mathrm{W}$



Power dissipated in $\mathrm{D} = i_d v_d = 0.6$

The device voltage and current can be found from solving this and the relationship $i_d = f(v_d)$ given by the characteristic curve:



The point of intersection gives

$$v_d \approx 0.8\,\mathrm{V}$$

$$i_d \approx 0.75\,\mathrm{A}$$

From KVL:

$$i = 1.6 i_d + v_d \approx 1.6(0.75) + 0.8 = 2\,\mathrm{A}$$

Q.6

Voltage gain

Applying KCL to the second half of the circuit:

$$v_2 = -20(50 i_1) = -1000 i_1$$

Applying KVL to the first half of the circuit:

$$v_1 = 2 i_1 + \frac{v_2}{5000}$$

Eliminating i_1:

$$v_1 = -2\left(\frac{v_2}{1000}\right) + \frac{v_2}{5000} = \frac{-1.8 v_2}{1000} \quad \Rightarrow \quad v_2 = -\frac{1000}{1.8} v_1$$

The voltage gain is

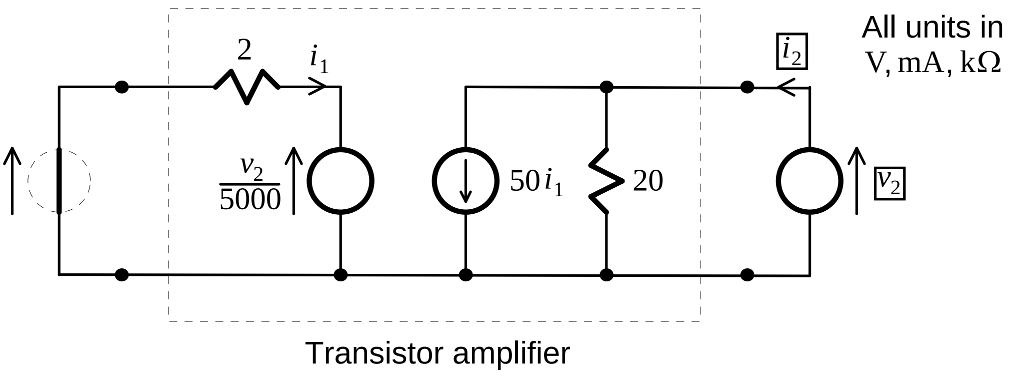
$$\frac{v_2}{v_1} = -\frac{1000}{1.8}$$

The gain in voltage magnitudes is

$$\left|\frac{v_2}{v_1}\right| = \frac{1000}{1.8} = 20\log\left(\frac{1000}{1.8}\right)\text{dB} = 55\text{dB}$$

Equivalent resistance

To determine the equivalent resistance as seen from the output terminals, all independent sources have to be replaced by their internal resistances and a voltage source has to be applied to these two terminals:



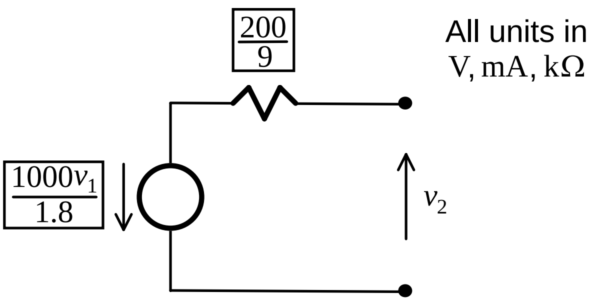
$$-2i_1 = \frac{v_2}{5000}$$

$$i_2 = 50i_1 + \frac{v_2}{20} = -50\left(\frac{v_2}{10000}\right) + \frac{v_2}{20} = \frac{9v_2}{200}$$

Equivalent resistance $= \frac{v_2}{i_2} = \frac{200}{9}\,\text{k}\Omega$

Note that in calculating this resistance or in using superposition, dependent sources must not be replaced by their internal resistances.

Thevenin's equivalent circuit



F.5 AC Circuit Analysis I

Q.1

	(a)	(b)
AC waveform	$5\sqrt{2}\sin(\omega t)$ $= 5\sqrt{2}\cos\left(\omega t - \frac{\pi}{2}\right)$ $= \mathrm{Re}\left[\left(5e^{-j\pi/2}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$	$5\sqrt{2}\cos(\omega t)$ $= \mathrm{Re}\left[\left(5e^{j0}\right)\left(\sqrt{2}e^{j\omega t}\right)\right]$
Peak value	$5\sqrt{2}$	$5\sqrt{2}$
Frequency	$\omega\,\mathrm{rad/s} = \frac{\omega}{2\pi}\,\mathrm{Hz}$	$\omega\,\mathrm{rad/s} = \frac{\omega}{2\pi}\,\mathrm{Hz}$
RMS value	5	5
Phase	$-\frac{\pi}{2} = -90^0$	0
Phasor	$5\ e^{-j\pi/2} = 5\ \underline{/-90^o}$	$5\ e^{j0} = 5\ \underline{/0^o} = 5$

	(c)	(d)
AC waveform	$10\sqrt{2}\sin\left(20t + 30^0\right)$ $= 10\sqrt{2}\cos\left(20t - 60^0\right)$ $= \mathrm{Re}\left[\left(10e^{-j\pi/3}\right)\left(\sqrt{2}e^{j20t}\right)\right]$	$120\sqrt{2}\cos\left(314t - 45^0\right)$ $= \mathrm{Re}\left[\left(120e^{-j\pi/4}\right)\left(\sqrt{2}e^{j314t}\right)\right]$
Peak value	$10\sqrt{2}$	$120\sqrt{2}$
Frequency	$20\,\mathrm{rad/s} = 3.18\,\mathrm{Hz}$	$314\,\mathrm{rad/s} = 50\,\mathrm{Hz}$
RMS value	10	120
Phase	$-\frac{\pi}{3} = -60^0$	$-\frac{\pi}{4} = -45^0$
Phasor	$10\,e^{-j\pi/3} = 10\underline{/-60^o}$	$120e^{-j\pi/4} = 120\underline{/-45^o}$

	(e)	(f)
AC waveform	$-50\sin\left(4t-\frac{\pi}{3}\right)$ $= 35.4\sqrt{2}\cos\left(4t-\frac{\pi}{3}-\frac{\pi}{2}+\pi\right)$ $= \mathrm{Re}\left[\left(35.4e^{j\pi/6}\right)\left(\sqrt{2}e^{j4t}\right)\right]$	$0.25\cos\left(2t+100^0\right)$ $= 0.177\sqrt{2}\cos\left(2t+1.75\right)$ $= \mathrm{Re}\left[\left(0.177e^{j1.75}\right)\left(\sqrt{2}e^{j2t}\right)\right]$
Peak value	50	0.25
Frequency	$4\ \mathrm{rad/s} = 0.637\ \mathrm{Hz}$	$2\ \mathrm{rad/s} = 0.318\ \mathrm{Hz}$
RMS value	35.4	0.177
Phase	$\frac{\pi}{6} = 30^0$	$1.75 = 100^0$
Phasor	$35.4e^{j\pi/6} = 35.4\angle 30^o$	$0.177e^{j1.75} = 0.177\angle 100^o$

Q.2

(a)	$\frac{100}{\sqrt{2}}e^{j30^0}\ \mathrm{V}$	$\mathrm{Re}\left[\left(\frac{100}{\sqrt{2}}e^{j\pi/6}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right] = 100\cos\left(314t+\frac{\pi}{6}\right)\mathrm{V}$
(b)	$115e^{j\pi/3}\ \mathrm{V}$	$\mathrm{Re}\left[\left(115e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right] = 115\sqrt{2}\cos\left(314t+\frac{\pi}{3}\right)\mathrm{V}$
(c)	$-0.12e^{-j\pi/4}\ \mathrm{A}$	$\mathrm{Re}\left[\left(-0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right] = \mathrm{Re}\left[\left(e^{j\pi}0.12e^{-j\pi/4}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $= 0.12\sqrt{2}\cos\left(314t+\frac{3\pi}{4}\right)\mathrm{A}$
(d)	$-0.69\angle 60^o\ \mathrm{A}$	$\mathrm{Re}\left[\left(-0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right] = \mathrm{Re}\left[\left(e^{j\pi}0.69e^{j\pi/3}\right)\left(\sqrt{2}e^{j100\pi t}\right)\right]$ $= 0.69\sqrt{2}\cos\left(314t+\frac{4\pi}{3}\right)$ $= 0.69\sqrt{2}\cos\left(314t-\frac{2\pi}{3}\right)\mathrm{A}$

Q.3

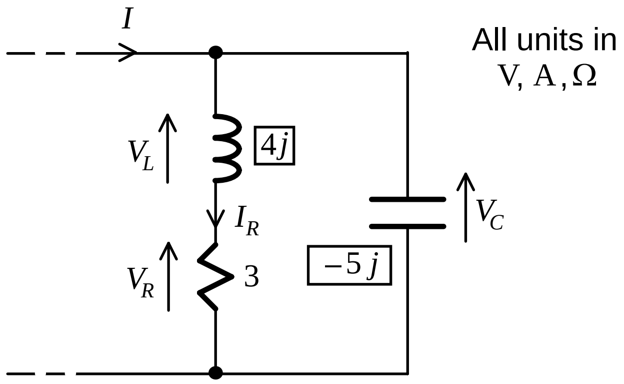
From $v_R(t) = 12\sqrt{2}\cos(2t)$ V

Frequency $= \omega = 2\ \mathrm{rad/s}$

Impedance of capacitor $= \dfrac{1}{j\omega 0.1} = \dfrac{1}{j0.2} = -5j\,\Omega$

Impedance of inductor $= j\omega 2 = 4j\,\Omega$

$V_R = 12e^{j0} = 12$

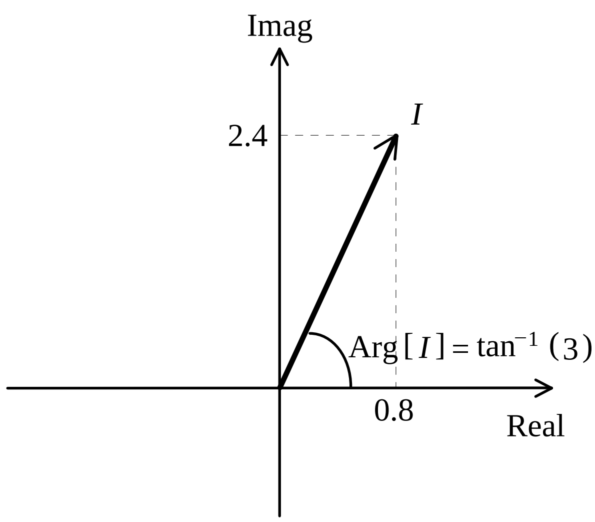


$$I_R = \frac{V_R}{3} = 4 \;\Rightarrow\; i_R(t) = 4\sqrt{2}\cos(2t)\ \mathrm{A}$$

$$V_L = (4j)I_R = 16j\ \mathrm{V} \;\Rightarrow\; v_L(t) = 16\sqrt{2}\cos\left(2t + \frac{\pi}{2}\right)\mathrm{V}$$

$$V_C = V_R + V_L = 12 + 16j$$

$$I = I_R + \frac{V_C}{-5j} = 4 - \frac{12+16j}{5j} = 4 + 2.4j - 3.2 = 0.8 + 2.4j$$



$$|I| = |0.8 + 2.4j| = \sqrt{0.8^2 + 2.4^2} = 2.53$$

$$\text{Arg}[I] = \text{Arg}[0.8 + 2.4j] = \tan^{-1}\left(\frac{2.4}{0.8}\right) = 1.25$$

$$I = 0.8 + 2.4j = 2.53e^{j1.25} \;\Rightarrow\; 2.53\sqrt{2}\cos(2t + 1.25)\text{ A}$$

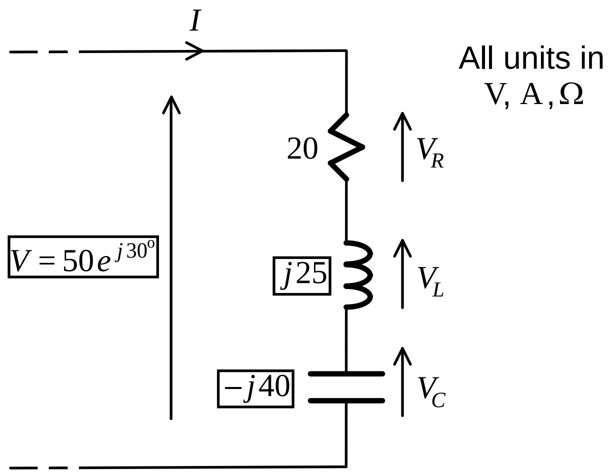
Q.4

Impedances and phasors

$$v(t) = 50\sqrt{2}\cos(1250t + 30^0)\text{ V} \;\Rightarrow\; V = 50e^{j30^0}\text{ V}.$$

Impedance of inductor $= j(1250)(0.02) = j25\Omega$

Impedance of capacitor $= \dfrac{1}{j(1250)(0.00002)} = -j40\Omega$



Total impedance $= 20 + j25 - j40 = 20 - j15 = \sqrt{20^2 + 15^2}\, e^{j\tan^{-1}\left(\frac{-15}{20}\right)} = 25e^{-36.9^0}\ \Omega$

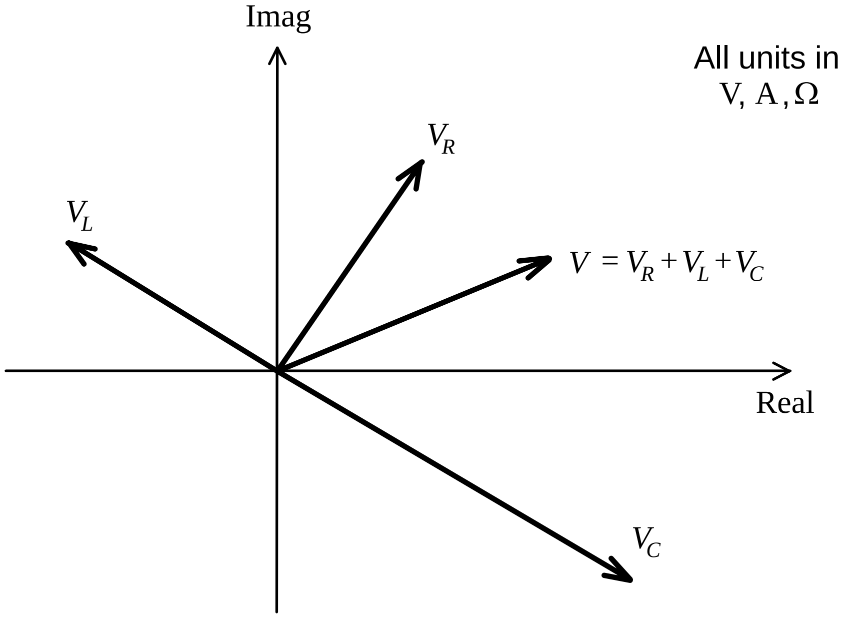
$$I = \frac{V}{20 + j25 - j40} = \frac{50e^{j30^0}}{25e^{j-36.9^0}} = 2e^{j66.9^0}\text{ A}$$

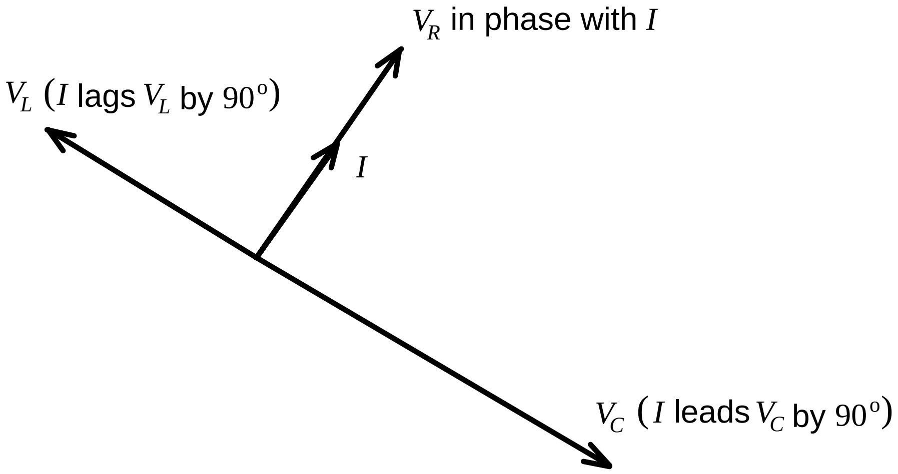
$$V_R = 20I = (20)\left(2e^{j66.9^0}\right) = 40e^{j66.9^0}\text{ V}$$

$$V_L = j25I = \left(25e^{j90^0}\right)\left(2e^{j66.9^0}\right) = 50e^{j156.9^0}\text{ V}$$

$$V_C = -j40I = \left(40e^{-j90^0}\right)\left(2e^{j66.9^0}\right) = 80e^{-j23.1^0}\text{ V}$$

Phasor diagram





Q.5

Components

The impedances of series RL, RC and LC circuits are

$$Z_{RL} = R + j\omega L = R + j100\pi L$$

$$Z_{RC} = R + \frac{1}{j\omega C} = R - \frac{j}{100\pi C}$$

$$Z_{LC} = j\omega L + \frac{1}{j\omega C} = j\left(100\pi L - \frac{1}{100\pi C}\right)$$

$20 + j30$ must correspond to a series RL circuit with components:

$$R_1 + j100\pi L_1 = 20 + j30 \;\Rightarrow\; R_1 = 20\Omega \text{ and } L_1 = \frac{30}{100\pi} = 0.0955\,\text{H}$$

$10 - j15$ must correspond to a series RC circuit with components:

$$R_2 - \frac{j}{100\pi C_2} = 10 - j15 \;\Rightarrow\; R_2 = 10\Omega \text{ and } C_2 = \frac{1}{100\pi(15)} = 212.3\,\mu\text{F}$$

Circuit admittance

$$\text{Circuit impedance} = Z = (20 + j30)\,\|\,(10 - j15) = \frac{1}{\dfrac{1}{20+j30} + \dfrac{1}{10-j15}}$$

$$\text{Circuit admittance} = \frac{1}{Z} = \frac{1}{20+j30} + \frac{1}{10-j15} = \frac{20-j30}{20^2+30^2} + \frac{10+j15}{10^2+15^2}$$

$$= 0.0154 - j0.0231 + 0.0308 + j0.0462 = (0.0462 + j0.0231)\,\Omega^{-1}$$

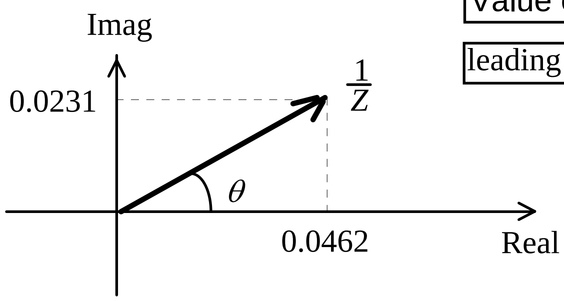
Power factor

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\text{Arg}(I) - \text{Arg}(V)] \\ \\ \text{leading / lagging} = \begin{cases} \text{leading, Arg}(I) - \text{Arg}(V) > 0 \\ \text{lagging, Arg}(I) - \text{Arg}(V) < 0 \end{cases} \end{array} \right\rangle$$

$$\text{Arg}(I) - \text{Arg}(V) = \text{Arg}\left(\frac{I}{V}\right) = \text{Arg}\left(\frac{1}{Z}\right) = -\text{Arg}(Z)$$

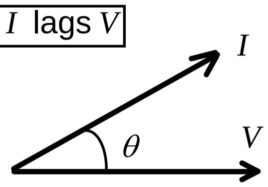
$$= \text{Arg}(0.0462 + j0.0231) = \tan^{-1}\left(\frac{0.0231}{0.0462}\right) = 0.464$$

$$\text{Power factor} \left\langle \begin{array}{c} \text{value} = \cos(0.464) = 0.894 \\ \\ \text{leading / lagging} = \begin{cases} \text{leading, } 0.464 > 0 \\ \text{lagging, } 0.464 < 0 \end{cases} \end{array} \right\rangle = \left\langle \begin{array}{c} 0.894 \\ \\ \text{leading} \end{array} \right\rangle$$



$\boxed{\text{Value of p.f.} = \cos\theta}$

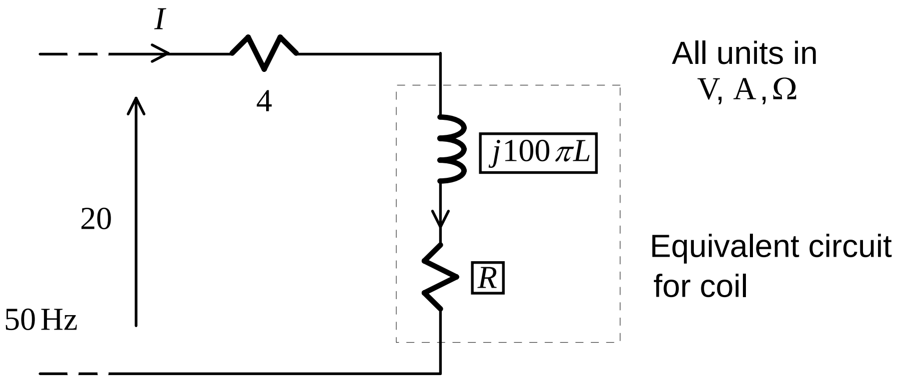
$\boxed{\text{leading p.f. as } I \text{ lags } V}$



F.6 AC Circuit Analysis II

Q.1

Circuit diagram



Main equations

$$\left|\text{Voltage across } 4\Omega\right| = |4I| = 9 \;\Rightarrow\; |I| = \frac{9}{4}$$

$$\left|\text{Voltage across coil}\right| = \left|I\left(R + j100\pi L\right)\right| = 14$$

$$\left|R + j314L\right| = \frac{14}{|I|} = 14\left(\frac{4}{9}\right) = 6.222 \;\Rightarrow\; R^2 + (314L)^2 = 38.72$$

$$\left|\text{Supply}\right| = \left|I\left(4 + R + j100\pi L\right)\right| = 20$$

$$\left|4 + R + j314L\right| = \frac{20}{|I|} = 20\left(\frac{4}{9}\right) = 8.889 \;\Rightarrow\; \left(R+4\right)^2 + (314L)^2 = 79.01$$

Component values

$$\left[\left(R+4\right)^2 + (314L)^2\right] - \left[R^2 + (314L)^2\right] = 79.01 - 38.72$$

$$8R + 16 = 40.29 \;\Rightarrow\; R = \frac{40.29 - 16}{8} = 3.04\,\Omega$$

$$R^2 + (314L)^2 = 38.72 \;\Rightarrow\; (314L)^2 = 38.72 - 3.04^2 = 29.48 \;\Rightarrow\; L = \frac{\sqrt{29.48}}{314} = 0.0173\,\text{H}$$

Power and power factor

$$\text{Power absorbed by coil} = |I|^2 R = \left(\frac{9}{4}\right)^2 (3.04) = 15.4\,\mathrm{W}$$

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\mathrm{Arg}(\text{current}) - \mathrm{Arg}(\text{voltage})] \\ \text{leading / lagging} = \begin{cases} \text{leading}, \mathrm{Arg}(\text{current}) - \mathrm{Arg}(\text{voltage}) > 0 \\ \text{lagging}, \mathrm{Arg}(\text{current}) - \mathrm{Arg}(\text{voltage}) < 0 \end{cases} \end{array} \right\rangle$$

$$\mathrm{Arg}(\text{current}) - \mathrm{Arg}(\text{voltage}) = \mathrm{Arg}\left(\frac{\text{current}}{\text{voltage}}\right)$$

$$= \mathrm{Arg}\left(\frac{1}{\text{Impedance}}\right) = -\mathrm{Arg}(\text{Impedance})$$

$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos[\mathrm{Arg}(\text{impedance})] \\ \text{leading / lagging} = \begin{cases} \text{leading}, \mathrm{Arg}(\text{impedance}) < 0 \\ \text{lagging}, \mathrm{Arg}(\text{impedance}) > 0 \end{cases} \end{array} \right\rangle$$

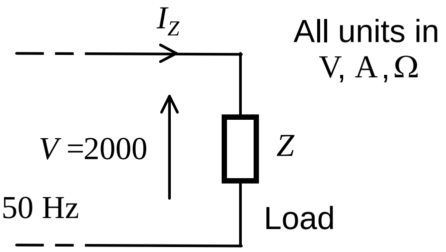
$$\mathrm{Arg}(\text{impedance}) = \mathrm{Arg}(R + 4 + j314L) = \mathrm{Arg}(3.04 + 4 + j314 \times 0.0173)$$

$$= \mathrm{Arg}(7.04 + j5.43) = \tan^{-1}\left(\frac{5.43}{7.04}\right) = 0.657$$

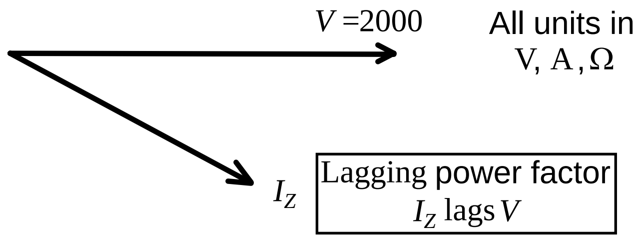
$$\text{Power factor} = \left\langle \begin{array}{l} \text{value} = \cos(0.657) = 0.792 \\ \text{leading / lagging} = \begin{cases} \text{leading}, 0.657 < 0 \\ \text{lagging}, 0.657 > 0 \end{cases} \end{array} \right\rangle = \left\langle \begin{array}{l} 0.792 \\ \text{lagging} \end{array} \right\rangle$$

Q.2

Load current



$$\text{Load power factor} = \frac{\text{actual power}}{\text{apparent power}} \Rightarrow 0.5 = \frac{10000}{2000|I_Z|} \Rightarrow |I_Z| = 10\,\text{A}$$



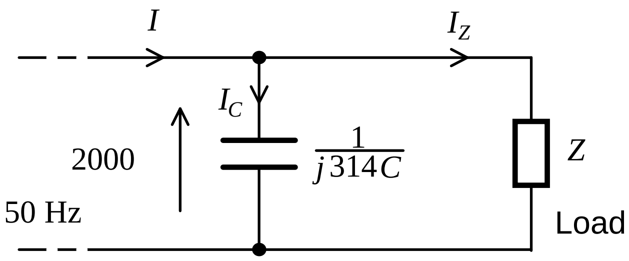
$$\begin{matrix}0.5\,\text{lagging}\\ \text{load p.f.}\end{matrix} \Rightarrow \left\langle \begin{matrix}\cos[\text{Arg}(I_Z) - \text{Arg}(V)] = 0.5\\ \text{Arg}(I_Z) - \text{Arg}(V) < 0\end{matrix} \right\rangle$$

$$\Rightarrow \left\langle \begin{matrix}\text{Arg}(I_Z) = \pm\cos^{-1}(0.5) = \pm 1.05\\ \text{Arg}(I_Z) < 0\end{matrix} \right\rangle$$

$$\Rightarrow \text{Arg}(I_Z) = -1.05$$

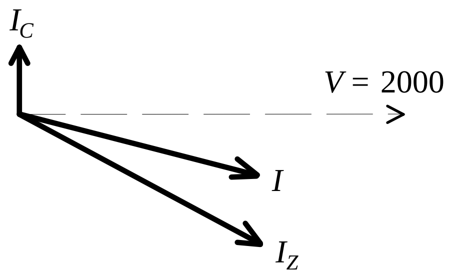
$$I_Z = |I_Z| e^{j\text{Arg}[I_Z]} = 10e^{-j1.05}\,\text{A}$$

Power factor improvement



$$I_C = (j314C)(2000) = j628000C$$

$$I = I_Z + I_C = 10e^{-j1.05} + j628000C = 5 + j(62800C - 8.66)$$



0.9 lagging overall p.f. $\Rightarrow \left\langle \begin{array}{l} \cos\left[\mathrm{Arg}(I) - \mathrm{Arg}(V)\right] = 0.9 \\ \mathrm{Arg}(I) - \mathrm{Arg}(V) < 0 \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{l} \mathrm{Arg}(I) = \pm 0.451 \\ \mathrm{Arg}(I) < 0 \end{array} \right\rangle$

$$\Rightarrow \mathrm{Arg}(I) = -0.451 \Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = -0.451$$

$$\Rightarrow C = \frac{8.66 - 5\tan(-0.451)}{628000} = 0.0000176\,\mathrm{F} = 17.6\,\mu\ \mathrm{F}$$

unity overall p.f. $\Rightarrow \left\langle \begin{array}{l} \cos\left[\mathrm{Arg}(I) - \mathrm{Arg}(V)\right] = 1 \\ \mathrm{Arg}(I) - \mathrm{Arg}(V) = 0 \end{array} \right\rangle \Rightarrow \mathrm{Arg}(I) = 0$

$$\Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = 0 \Rightarrow C = \frac{8.66}{628000} = 0.0000138\,\mathrm{F} = 13.8\,\mu\ \mathrm{F}$$

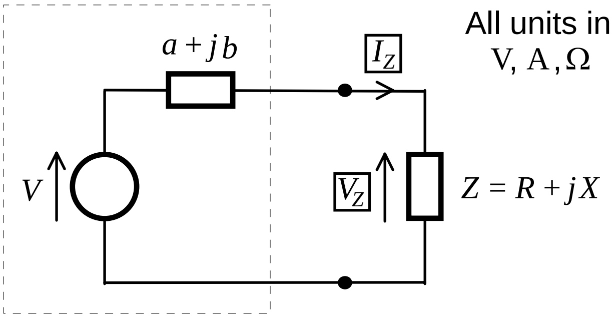
0.8 leading overall p.f. $\Rightarrow \left\langle \begin{array}{l} \cos\left[\mathrm{Arg}(I) - \mathrm{Arg}(V)\right] = 0.8 \\ \mathrm{Arg}(I) - \mathrm{Arg}(V) > 0 \end{array} \right\rangle \Rightarrow \left\langle \begin{array}{l} \mathrm{Arg}(I) = \pm 0.644 \\ \mathrm{Arg}(I) > 0 \end{array} \right\rangle$

$$\Rightarrow \mathrm{Arg}(I) = 0.644 \Rightarrow \tan^{-1}\left(\frac{628000C - 8.66}{5}\right) = 0.644$$

$$\Rightarrow C = \frac{8.66 + 5\tan(0.644)}{628000} = 0.0000078\,\mathrm{F} = 7.8\,\mu\ \mathrm{F}$$

Q.3

Power



Electrical system

$$I_Z = \frac{V}{(a + jb) + (R + jX)} = \frac{V}{(a+R) + j(b+X)}$$

$$V_Z = I_Z(R + jX)$$

Power absorbed $= p = \mathrm{Re}\left[I_Z^* V_Z\right] = \mathrm{Re}\left[\left|I_Z\right|^2 (R + jX)\right]$

$$= \left|I_Z\right|^2 \mathrm{Re}\left[R + jX\right] = R\left|I_Z\right|^2$$

$$= R\left|\frac{V}{(a+R) + j(b+X)}\right|^2 = \frac{R|V|^2}{(a+R)^2 + (b+X)^2}$$

Maximum power transfer

For maximum p, the denominator should be as small as possible. As the numerator does not depend on X and the smallest value for $(b + X)$ is 0, maximum power will be absorbed if

$$X = -b$$

so that

$$p = \frac{|V|^2 R}{(a + R)^2}$$

Differentiating:

$$\frac{dp}{dR} = |V|^2 \left[\frac{1}{(a+R)^2} - \frac{2R}{(a+R)^3}\right] = |V|^2 \left[\frac{a - R}{(a + R)^3}\right]$$

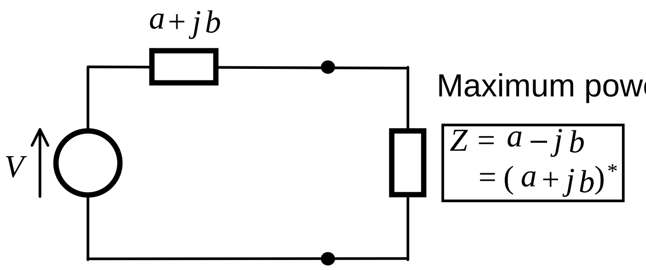
Thus, maximum p occurs when

$$R = a$$

and the maximum power transferable is

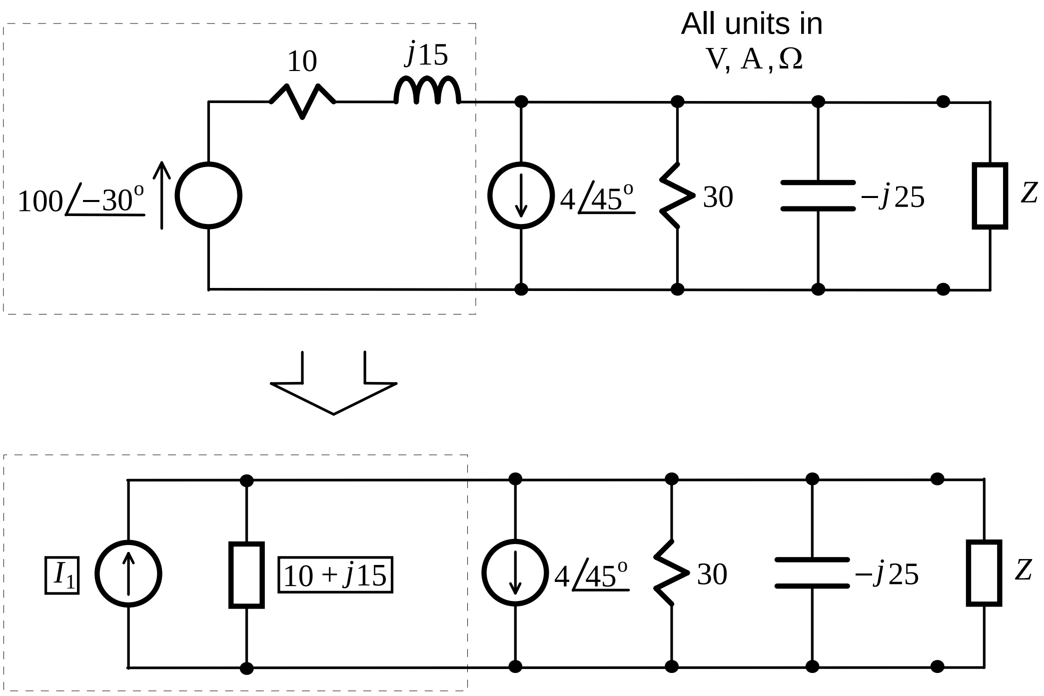
$$p = \frac{|V|^2 R}{\left(R + R\right)^2} = \frac{|V|^2}{4R} = \frac{|V|^2}{4a}\,\mathrm{W}$$

In general, maximum power transfer occurs when the load impedance is equal to the conjugate of the Thevenin's or Norton's impedance. When this occurs, the total impedance is purely resistive and the current and voltage in the circuit are in phase:

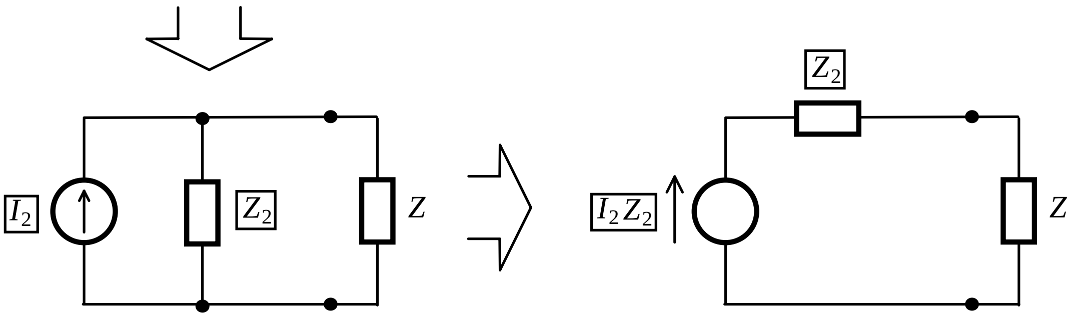


Q.4

Norton's and Thevenin's equivalent circuit



$$I_1 = \frac{100e^{-j30^0}}{10 + j15} = \frac{100e^{-j30^0}}{18e^{j56.3^0}} = 5.55e^{-j86.3^0} = 0.358 - j5.54$$



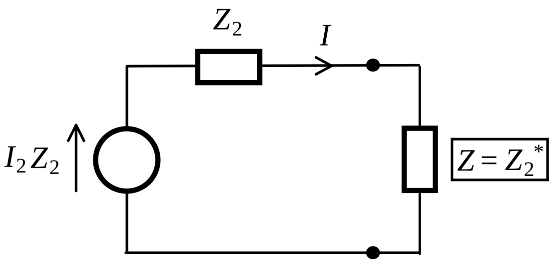
$$Z_2 = -j25 \,||\, 30 \,||\, (10 + j15) = \frac{1}{\frac{1}{-j25} + \frac{1}{30} + \frac{1}{10 + j15}} = \frac{1}{\frac{1}{-j25} + \frac{1}{30} + \frac{10 - j15}{10^2 + 15^2}}$$

$$= \frac{1}{0.0641 - j0.00615} = \frac{1}{0.0644e^{-j5.48^0}} = 15.5e^{j5.48^0}$$

$$I_2 = 0.359 - j5.55 - 4e^{j45^0} = -2.47 - j8.36 = 8.72e^{-j106^0}$$

Maximum power transfer

From the previous problem, this occurs when



$$Z = Z_2^* = \left(15.5e^{j5.48^0}\right)^* = 15.5e^{-j5.48^0}$$

Total impedance $= Z + Z_2 = Z_2^* + Z_2 = 15.5e^{j5.48^0} + 15.5e^{-j5.48^0} = 2\left[15.5\cos\left(5.48^0\right)\right]$

$$I = \frac{I_2 Z_2}{(Z + Z_2)} = \frac{8.72e^{-j106^0}\,15.5e^{j5.48^0}}{2\left[15.5\cos\left(5.48^0\right)\right]} = \frac{8.72e^{-j101^0}}{2\cos\left(5.48^0\right)}$$

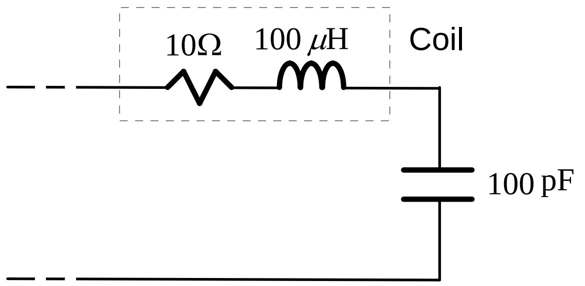
Thus, the maximum power transferable is

$$\mathrm{Re}\left[I^*(IZ)\right] = |I|^2\,\mathrm{Re}\left[Z_2^*\right]$$

$$= \left|\frac{8.72e^{-j101^0}}{2\cos\left(5.48^0\right)}\right|^2 \mathrm{Re}\left[15.5e^{-j5.48^0}\right] = \left|\frac{8.72}{2\cos\left(5.48^0\right)}\right|^2 15.5\cos\left(5.48^0\right)$$

$$= \frac{8.72^2 \times 15.5}{4\cos\left(5.48^0\right)} = 297\,\mathrm{W}$$

Q.5



$$\text{Resonant frequency} = \frac{1}{2\pi\sqrt{\left(100\times10^{-6}\right)\left(100\times10^{-12}\right)}} = 1.59\,\text{MHz}$$

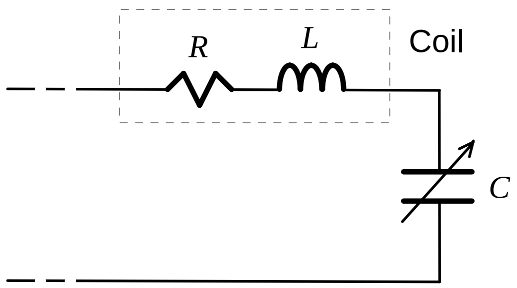
$$Q\ \text{factor} = \frac{2\pi\left(1.59\times10^{6}\right)\left(100\times10^{-6}\right)}{10} = 100$$

Since the Q factor is large, the circuit is bandpass in nature with

$$3\text{dB cutoff frequencies} \approx 1.59\left(1 \pm \frac{1}{200}\right)\text{MHz}$$

$$\text{Bandwidth} \approx \frac{1.59\times10^{6}}{100} = 15.9\,\text{kHz}$$

Q.6



$$C = 20 \cdots 500\,\text{pF} \Rightarrow \text{resonant frequency} = \frac{1}{2\pi\sqrt{L\left(500\times10^{-12}\right)}} \cdots \frac{1}{2\pi\sqrt{L\left(20\times10^{-12}\right)}}$$

$$= \frac{7.12}{\sqrt{L}} \cdots \frac{35.6}{\sqrt{L}}\,\text{kHz}$$

For the lowest tunable frequency to be $666\,\text{kHz}$:

$$666 = \frac{7.12}{\sqrt{L}} \Rightarrow L = \left(\frac{7.12}{666}\right)^2 = 0.114\,\text{mH}$$

The highest tunable frequency is then $\dfrac{35.6}{\sqrt{L}} = \dfrac{35.6}{\sqrt{0.114\times10^{-3}}} = 3.42\,\text{MHz}$

F.7 Periodic Signals

Q.1

(a) Period of voltage and current waveforms

Since the shortest time needed for the waveforms to repeat themselves is 6s, the period is 6s.

(b) Average or mean value of voltage and current waveforms

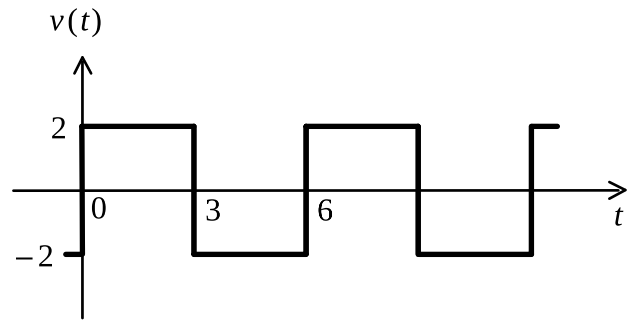
The average values of both $v(t)$ and $i(t)$ are obviously 0.

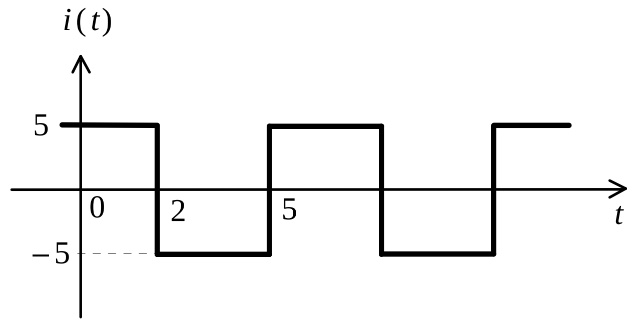
(c) Time when the device is consuming power and when it is supplying power

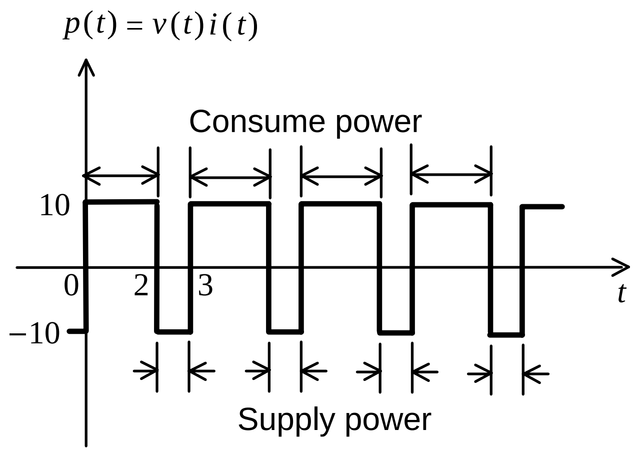
Since the voltage and current arrows are in opposite directions, the instantaneous power consumed by the device is

$$p(t) = v(t)i(t)$$

Graphically:







(d) Period of instantaneous power curve

From the above curve, the period of $p(t)$ is 3s.

(e) Average power consumed by device

In 1 period of 3s, the device consumes 10W of power for 2s and supplies 10W of power for 1s. Thus:

Net energy consumed in 3s $= (10)(2) - (10)(1) = 10\text{J}$

Average power consumed $= \dfrac{10}{3}\text{W}$

Q.2

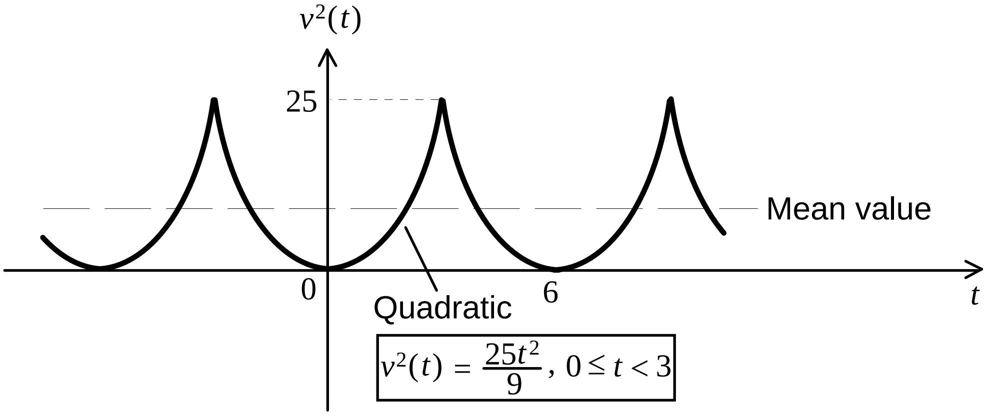
(a) Period and mean value of $v(t)$

From the waveform itself:

Period of $v(t) = T = 6\,\mathrm{s}$

Mean value of $v(t) = \dfrac{5}{2}\,\mathrm{V}$

(b) RMS value



Period of $v^2(t) = T = 6\,\mathrm{s}$

Mean value of $v^2(t) =$ mean value of $v^2(t)$ from $t=0$ to $t=3$

$$= \frac{\text{area under } v^2(t) \text{ from } t=0 \text{ to } t=3}{3}$$

$$= \frac{1}{3}\int_0^3 \frac{25t^2}{9}\,dt = \frac{1}{3}\left[\frac{25t^3}{27}\right]_0^3 = \frac{25}{3}\,\mathrm{V}^2$$

RMS value of $v(t) = v_{rms} = \sqrt{\text{mean value of } v^2(t)} = \dfrac{5}{\sqrt{3}}\,\mathrm{V}$

(c) Average power

Instantaneous power consumed $= p(t) = \dfrac{v^2(t)}{R}$

Average power consumed $=$ mean value of $\dfrac{v^2(t)}{R} = \dfrac{\text{mean value of } v^2(t)}{R}$

$$= \frac{\left[\sqrt{\text{mean value of } v^2(t)}\right]^2}{R} = \frac{{v_{rms}}^2}{R}$$

Q.3

(a) $v_1(t) = 10\sqrt{2}\cos\left(2\pi t + 2^0\right)$ *and* $v_2(t) = 20\sqrt{2}\cos\left(2\pi t + 23^0\right)$

To an observer with a time origin of $t = 0$:

$$v_1(t) = 10\sqrt{2}\cos\left(2\pi t + 2^0\right) \Rightarrow V_1 = 10e^{j2^0} \text{ and frequency} = 1$$

$$v_2(t) = 20\sqrt{2}\cos\left(2\pi t + 23^0\right) \Rightarrow V_2 = 20e^{j23^0} \text{ and frequency} = 1$$

Phase difference $= 23^0 - 2^0 = 21^0, V_2$ leading

To an observer with a time origin of $t = 0.5$:

$$v_1\left(t + 0.5\right) = 10\sqrt{2}\cos\left(2\pi t + \pi + 2^0\right) \Rightarrow V_1' = 10e^{j182^0} \text{ and frequency} = 1$$

$$v_2\left(t + 0.5\right) = 20\sqrt{2}\cos\left(2\pi t + \pi + 23^0\right) \Rightarrow V_2' = 20e^{j203^0} \text{ and frequency} = 1$$

Phase difference $= 203^0 - 182^0 = 21^0, V_2'$ leading

Since the time origin has changed, the voltages observed are time shifted and the phasors change in phase. However, since the frequencies are the same, the relative phase difference remains constant.

(b) $v_1(t) = 10\sqrt{2}\cos\left(2\pi t + 2^0\right)$ *and* $v_2(t) = 20\sqrt{2}\cos\left(\sqrt{2}\pi t + 23^0\right)$

To an observer with a time origin of $t = 0$:

$$v_1(t) = 10\sqrt{2}\cos\left(2\pi t + 2^0\right) \Rightarrow V_1 = 10e^{j2^0} \text{ and frequency} = 1$$

$$v_2(t) = 20\sqrt{2}\cos\left(\sqrt{2}\pi t + 23^0\right) \Rightarrow V_2 = 20e^{j23^0} \text{ and frequency} = \frac{1}{\sqrt{2}}$$

To an observer with a time origin of $t = 0.5$:

$$v_1\left(t + 0.5\right) = 10\sqrt{2}\cos\left(2\pi t + \pi + 2^0\right) \Rightarrow V_1' = 10e^{j182^0} \text{ and frequency} = 1$$

$$v_2\left(t + 0.5\right) = 20\sqrt{2}\cos\left(\sqrt{2}\pi t + 0.5\sqrt{2}\pi + 23^0\right) \Rightarrow V_2' = 20e^{j150^0} \text{ and frequency} = \frac{1}{\sqrt{2}}$$

Since the frequencies are not the same, the phase difference between $v_1(t)$ and $v_2(t)$ is not constant and depends on the time origin. In fact, it is neither meaningful nor useful to compare the phases of $v_1(t)$ and $v_2(t)$, as these signals have different frequencies.

Phasor analysis can only be used when all the sources are sinusoidal and have the same frequencies. In situations where this is not the case but the sources are still sinusoidal, superposition has to be used together with phasor analysis.

Q.4

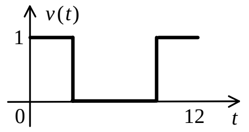
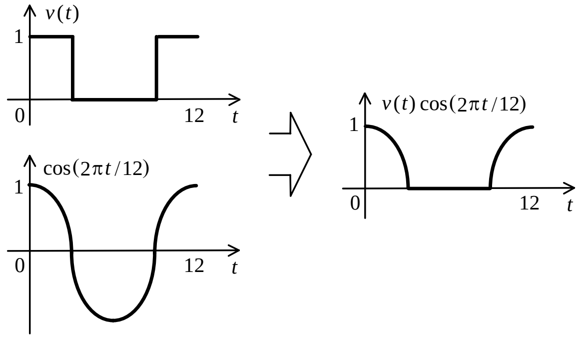
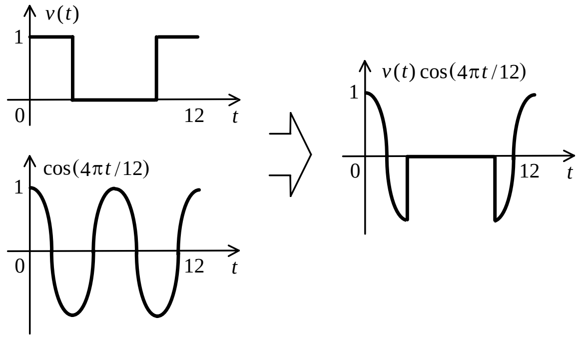
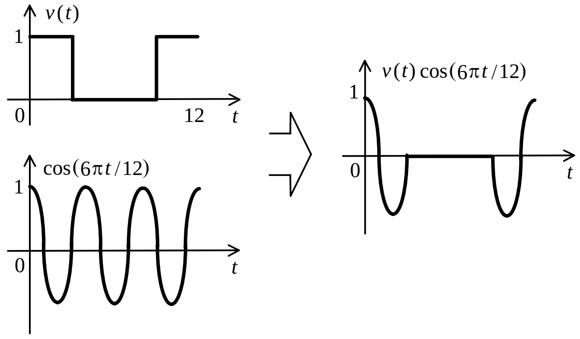
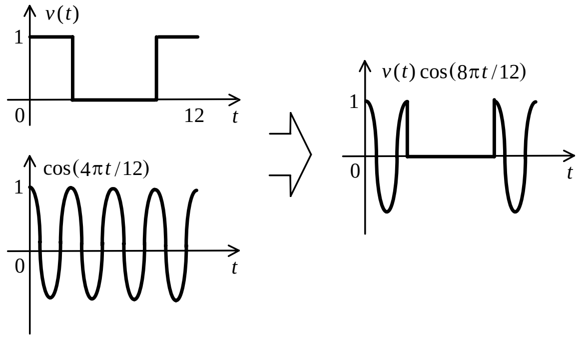
In general, for a signal $v(t)$ with period T:

$$v(t) = a_0 + \left[a_1 \cos\left(\frac{2\pi t}{T}\right) + b_1 \sin\left(\frac{2\pi t}{T}\right)\right] + \left[a_2 \cos\left(\frac{4\pi t}{T}\right) + b_2 \sin\left(\frac{4\pi t}{T}\right)\right] + \cdots$$

a_0	$\frac{1}{T}\int_0^T v(t)dt$	Average value of $v(t)$
$a_n, n \geq 1$	$\frac{2}{T}\int_0^T v(t)\cos\left(\frac{2n\pi t}{T}\right)dt$	Twice the average value of $v(t)\cos\left(\frac{2\pi nt}{T}\right)$
$b_n, n \geq 1$	$\frac{2}{T}\int_0^T v(t)\sin\left(\frac{2n\pi t}{T}\right)dt$	Twice the average value of $v(t)\sin\left(\frac{2\pi nt}{T}\right)$

For the periodic waveform above:

$T = 12\,\text{s}$

$a_0 = \text{mean of } v(t)$ $= \dfrac{1}{2}$	
$a_1 = 2\left[\text{mean of } v(t)\cos\left(\dfrac{2\pi t}{12}\right)\right]$ $= \dfrac{2}{6}\int_0^3 \cos\left(\dfrac{2\pi t}{12}\right)dt = \dfrac{1}{3}\left[\dfrac{12}{2\pi}\sin\left(\dfrac{2\pi t}{12}\right)\right]_0^3$ $= \dfrac{2}{\pi}$	
$a_2 = 2\left[\text{mean of } v(t)\cos\left(\dfrac{4\pi t}{12}\right)\right]$ $= 0$	
$a_3 = 2\left[\text{mean of } v(t)\cos\left(\dfrac{6\pi t}{12}\right)\right]$ $= \dfrac{2}{6}\int_0^3 \cos\left(\dfrac{6\pi t}{12}\right)dt = \dfrac{1}{3}\left[\dfrac{12}{6\pi}\sin\left(\dfrac{2\pi t}{12}\right)\right]_0^3$ $= -\dfrac{2}{3\pi}$	
$a_4 = 2\left[\text{mean of } v(t)\cos\left(\dfrac{8\pi t}{12}\right)\right]$ $= 0$	

Mathematically, for $n > 0$:

$$a_n = 2\left[\text{mean of } v(t)\cos\left(\frac{2n\pi t}{12}\right)\right] = \frac{2}{12}\left[\int_0^3 \cos\left(\frac{2n\pi t}{12}\right)dt + \int_9^{12}\cos\left(\frac{2n\pi t}{12}\right)dt\right]$$

$$= \frac{1}{6}\left\{\left[\frac{12}{2n\pi}\sin\left(\frac{2n\pi t}{12}\right)\right]_0^3 + \left[\frac{12}{2n\pi}\sin\left(\frac{2n\pi t}{12}\right)\right]_9^{12}\right\} = \frac{1}{n\pi}\left\{\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{3n\pi}{2}\right)\right\}$$

$$a_1, a_2, a_3, a_4, a_5, a_6, \cdots = \frac{2}{\pi}, 0, -\frac{2}{3\pi}, 0, \frac{2}{5\pi}, 0, \cdots$$

$$b_n = 2\left[\text{mean of } v(t)\sin\left(\frac{2n\pi t}{12}\right)\right] = \frac{2}{12}\left[\int_0^3 \sin\left(\frac{2n\pi t}{12}\right)dt + \int_9^{12}\sin\left(\frac{2n\pi t}{12}\right)dt\right]$$

$$= -\frac{1}{6}\left\{\left[\frac{12}{2n\pi}\cos\left(\frac{2n\pi t}{12}\right)\right]_0^3 + \left[\frac{12}{2n\pi}\cos\left(\frac{2n\pi t}{12}\right)\right]_9^{12}\right\} = -\frac{1}{n\pi}\left\{\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{3n\pi}{2}\right)\right\}$$

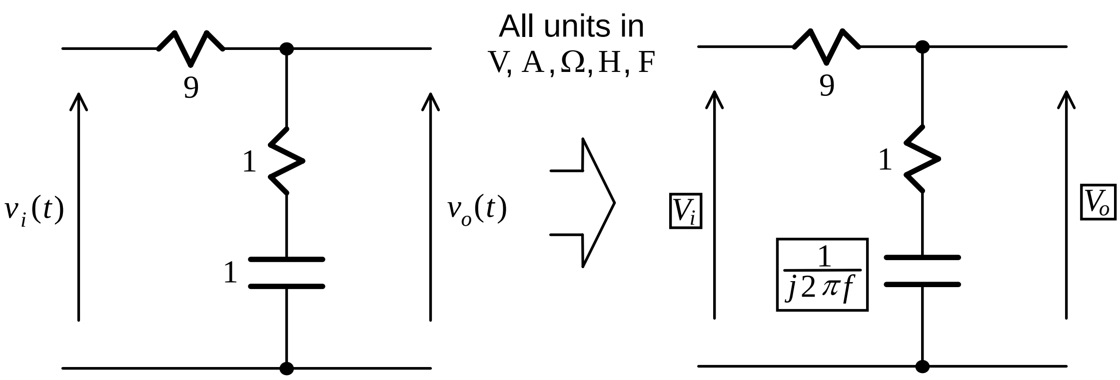
$$b_1, b_2, b_3, b_4, b_5, b_6, \cdots = 0, 0, 0, 0, 0, 0, \cdots$$

The Fourier series representation for $v(t)$ is thus:

$$v(t) = \frac{1}{2} + \frac{2}{\pi}\cos\left(\frac{2\pi t}{12}\right) - \frac{2}{3\pi}\cos\left(\frac{6\pi t}{12}\right) + \frac{2}{5\pi}\cos\left(\frac{10\pi t}{12}\right) - \cdots$$

Q.5

Frequency response



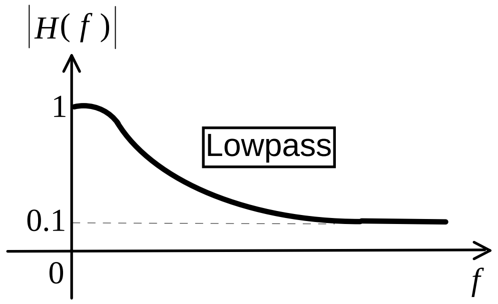
$$\text{Frequency response} = H(f) = \frac{V_o}{V_i} = \frac{1 + \dfrac{1}{j2\pi f}}{10 + \dfrac{1}{j2\pi f}} = \frac{1 + j2\pi f}{1 + j20\pi f}$$

Magnitude response

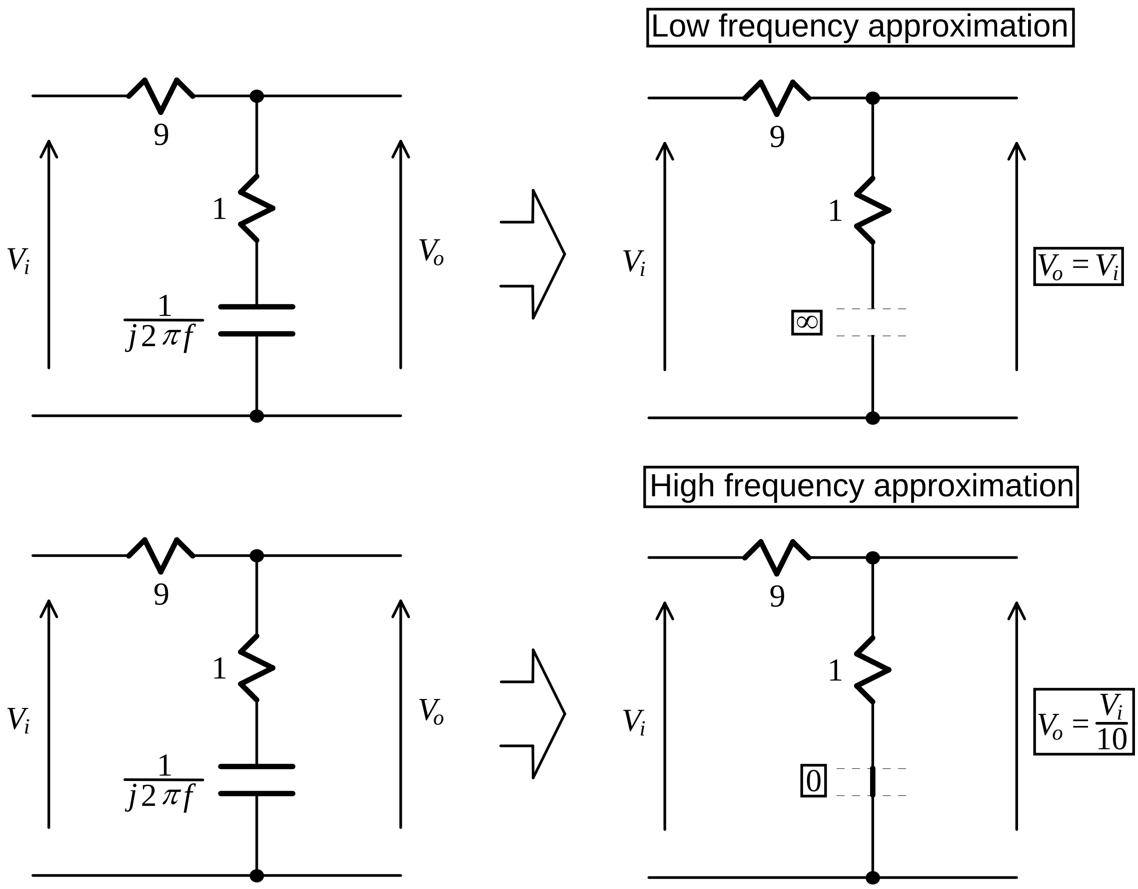
$$\text{Magnitude response} = |H(f)| = \left|\frac{1 + j2\pi f}{1 + j20\pi f}\right| = \sqrt{\frac{1 + 4\pi^2 f^2}{1 + 400\pi^2 f^2}}$$

$$|H(0)| = \sqrt{\frac{1+0}{1+0}} = 1$$

$$|H(f \to \infty)| \to \sqrt{\frac{4\pi^2 f^2}{400\pi^2 f^2}} = \sqrt{\frac{4}{400}} = \frac{1}{10}$$



The response is obviously lowpass in nature. This can also be deduced from the low and high frequency characteristics of the capacitors:



Non-periodic excitation with 2 sinusoidal components

The output due to a general sinusoidal excitation can be found as follows:

$$v_i(t) = r\cos(2\pi ft + \theta) \;\Rightarrow\; V_i = \frac{re^{j\theta}}{\sqrt{2}}$$

$$H(f) = \frac{V_o}{V_i} = \;\Rightarrow\; V_o = H(f)V_i = |H(f)|e^{j\mathrm{Arg}[H(f)]}\frac{re^{j\theta}}{\sqrt{2}} = \frac{|H(f)|r}{\sqrt{2}}e^{j\{\theta + \mathrm{Arg}[H(f)]\}}$$

$$\begin{aligned} v_o(t) &= r|H(f)|\cos\{2\pi ft + \theta + \mathrm{Arg}[H(f)]\} \\ &= r\left|\frac{1+j2\pi f}{1+j20\pi f}\right|\cos\left\{2\pi ft + \theta + \mathrm{Arg}\left[\frac{1+j2\pi f}{1+j20\pi f}\right]\right\} \\ &= r\sqrt{\frac{1+4^2\pi^2 f}{1+j400\pi^2 f}}\cos\left[2\pi ft + \theta + \tan^{-1}(2\pi f) - \tan^{-1}(20\pi f)\right] \end{aligned}$$

Thus, from superposition, the output due to $v_i(t) = \cos(2\pi t) + \sin(\sqrt{2}\pi t)$ is given by

$$\begin{aligned} v_i(t) = \cos(2\pi t) \;\Rightarrow\; v_o(t) &= \left|\frac{1+j2\pi}{1+j20\pi}\right|\cos\left[2\pi t + \mathrm{Arg}\left(\frac{1+j2\pi}{1+j20\pi}\right)\right] \\ &= \sqrt{\frac{1+4\pi^2}{1+400\pi^2}}\cos\left[2\pi t + \tan^{-1}(2\pi) - \tan^{-1}(20\pi)\right] \\ &= 0.101\cos(2\pi t - 0.142) \end{aligned}$$

$$\begin{aligned} v_i(t) = \sin(\sqrt{2}\pi t) \;\Rightarrow\; v_o(t) &= \left|\frac{1+j\sqrt{2}\pi}{1+j10\sqrt{2}\pi}\right|\sin\left[\sqrt{2}\pi t + \mathrm{Arg}\left(\frac{1+j\sqrt{2}\pi}{1+j10\sqrt{2}\pi}\right)\right] \\ &= \sqrt{\frac{1+2\pi^2}{1+200\pi^2}}\sin\left[\sqrt{2}\pi t + \tan^{-1}(\sqrt{2}\pi) - \tan^{-1}(10\sqrt{2}\pi)\right] \\ &= 0.102\sin(\sqrt{2}\pi t - 0.199) \end{aligned}$$

$$v_i(t) = \cos(2\pi t) + \sin(\sqrt{2}\pi t) \;\Rightarrow\; v_o(t) = 0.101\cos(2\pi t - 0.142) + 0.102\sin(2\pi t - 0.199)$$

Periodic square excitation

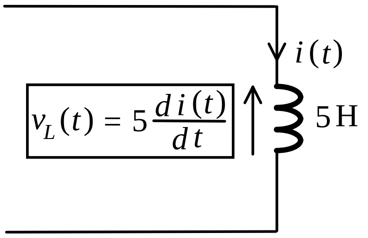
$$v_i(t) = \sin(2\pi t) + \frac{\sin(6\pi t)}{3} + \frac{\sin(10\pi t)}{5} + \cdots = \sum_{n=1,3,5,\cdots}^{\infty}\frac{\sin(2\pi nt)}{n}$$

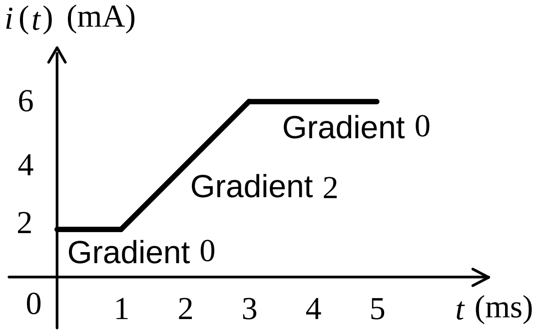
$$\begin{aligned} v_o(t) &= \sum_{n=1,3,5,\cdots}^{\infty}\frac{1}{n}|H(f=n)|\sin\{2\pi nt + \mathrm{Arg}[H(f=n)]\} \\ &= \sum_{n=1,3,5,\cdots}^{\infty}\frac{1}{n}\sqrt{\frac{1+4\pi^2 n^2}{1+400\pi^2 n^2}}\sin\{2\pi nt + \tan^{-1}(2\pi n) - \tan^{-1}(20\pi n)\} \end{aligned}$$

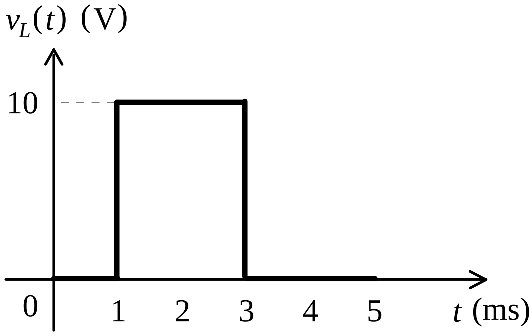
F.8 Transients I

Q.1

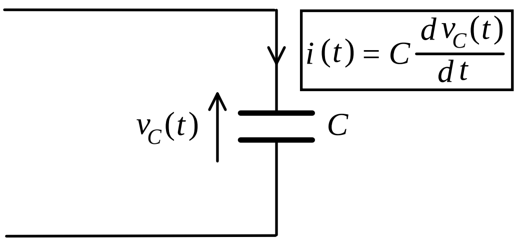
Voltages across inductor







Voltages across capacitor



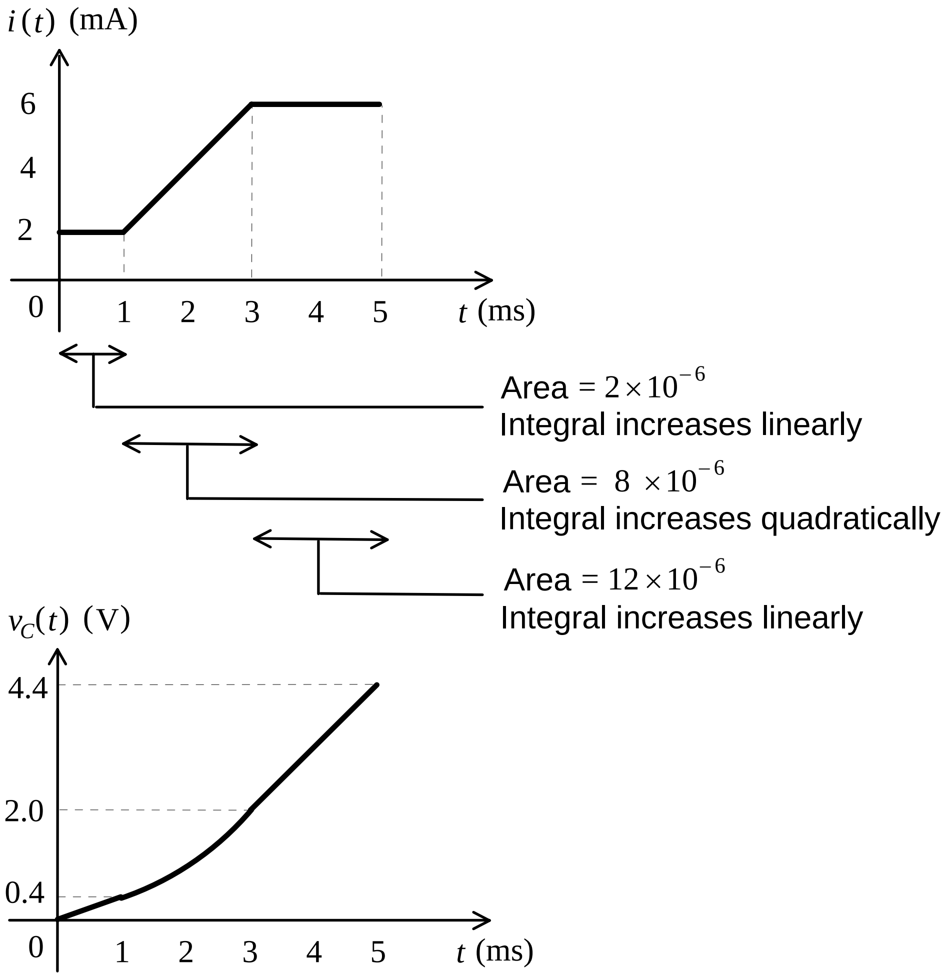
$$\int_0^t i(t)dt = \int_0^t C\frac{dv_C(t)}{dt}dt = \int_0^t Cdv_C(t) = \left[Cv_C(t)\right]_0^t = C\left[v_C(t) - v_C(0)\right]$$

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i(t)dt$$

$$= [\text{initial voltage at } t=0] + \frac{1}{C}[\text{area under } i(t) \text{ from } 0 \text{ to } t]$$

For this problem:

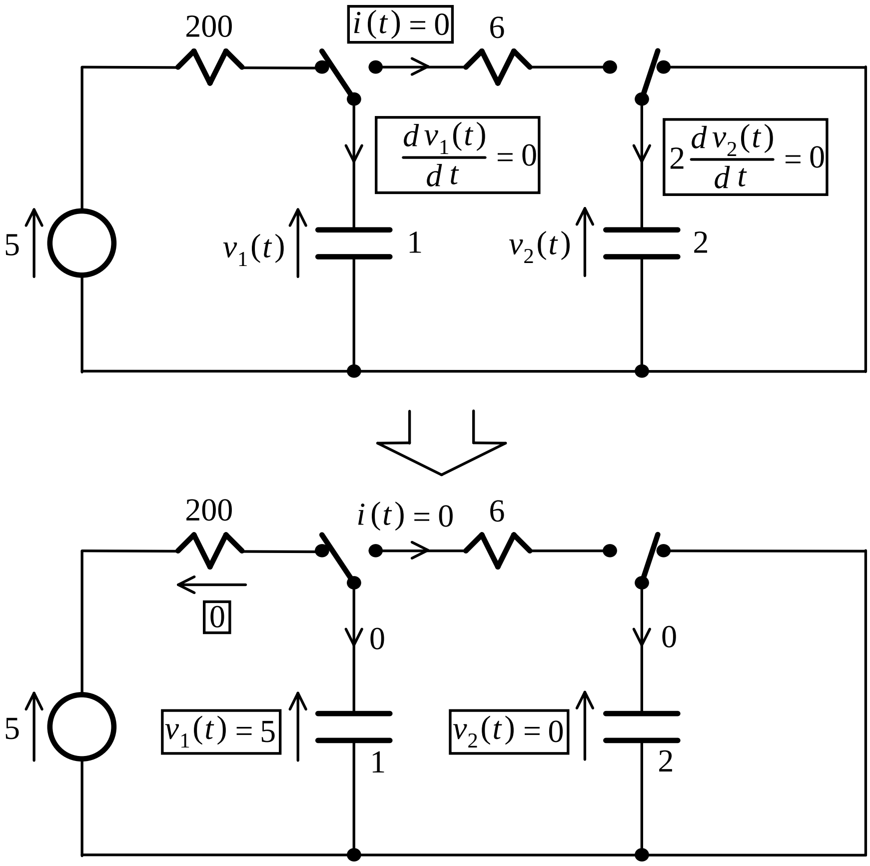
$$v_C(t) = \frac{1}{5\times10^{-6}}[\text{area under } i(t) \text{ from } 0 \text{ to } t]$$



Q.2

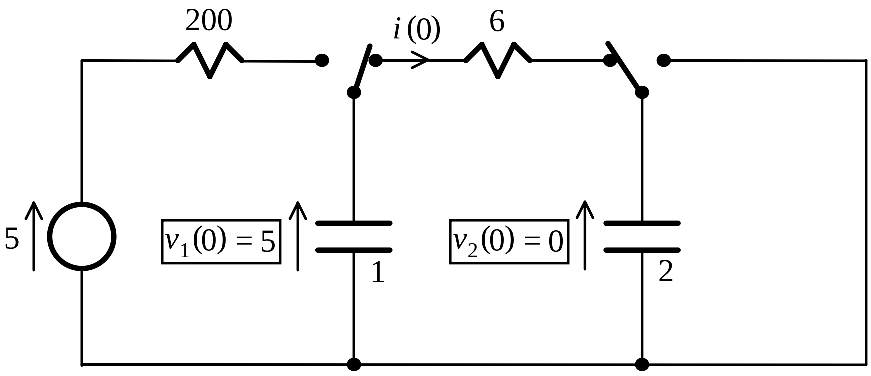
(a) Voltages and currents for $t < 0$

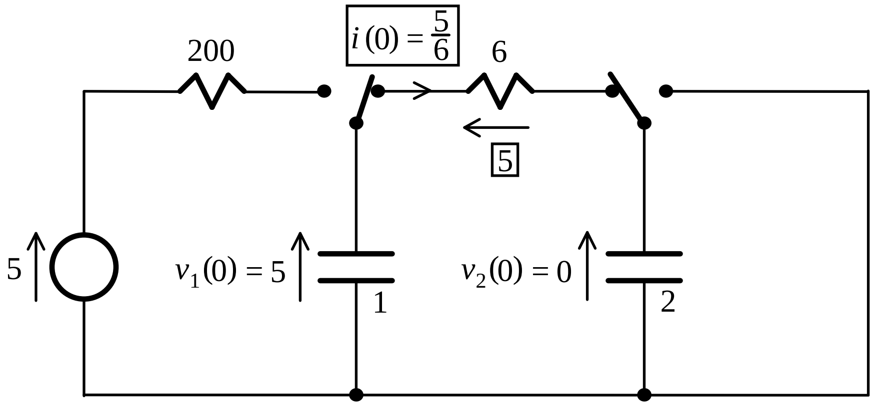
With the source being a dc one and taking the switches to be in the positions shown starting from $t = -\infty$, all the voltages and currents will have settled down to constant values for practically all $t < 0$:



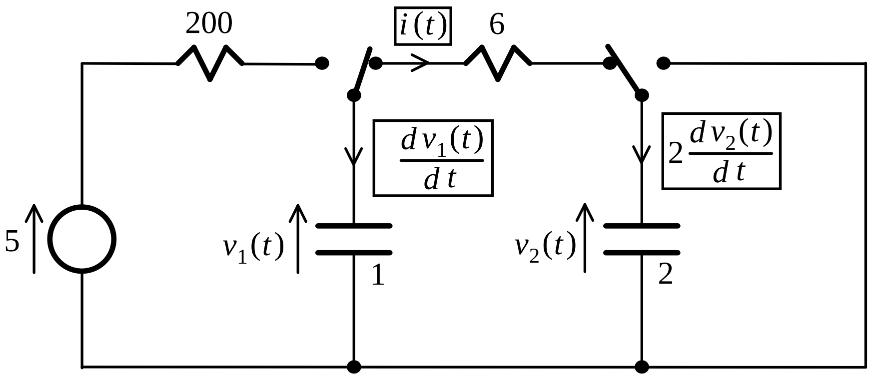
(b) Voltages and currents at $t = 0$ *(just after the switches are thrown)*

Since the voltages across capacitors must be continuous, $v_1(0)$ and $v_2(0)$ must have the same values before the switches are thrown:





(c) Voltages and current for $t \geq 0$



From KCL and KVL:

$$i(t) = 2\frac{dv_2(t)}{dt} = -\frac{dv_1(t)}{dt}$$

$$v_1(t) - 6i(t) - v_2(t) = 0$$

Eliminating $v_1(t)$ and $v_2(t)$:

$$\frac{dv_1(t)}{dt} - 6\frac{di(t)}{dt} - \frac{dv_2(t)}{dt} = \left(-1 - \frac{1}{2}\right)i(t) - 6\frac{di(t)}{dt} = 0 \Rightarrow \frac{di(t)}{dt} + \frac{i(t)}{4} = 0$$

Solving this homogeneous equation:

$$i(t) = ke^{-\frac{t}{4}}, t \ge 0$$

Applying initial condition:

$$i(0) = k = \frac{5}{6} \Rightarrow i(t) = \frac{5}{6}e^{-\frac{t}{4}}, t \ge 0$$

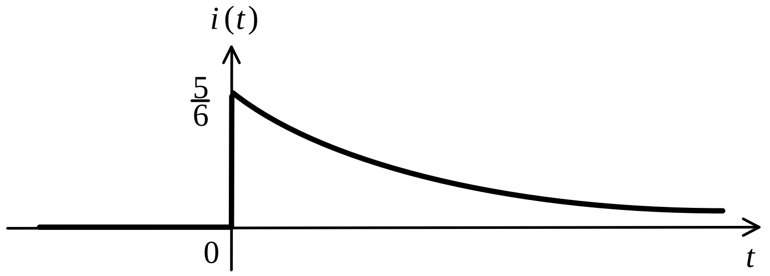
Solving for $v_1(t)$ and $v_2(t)$:

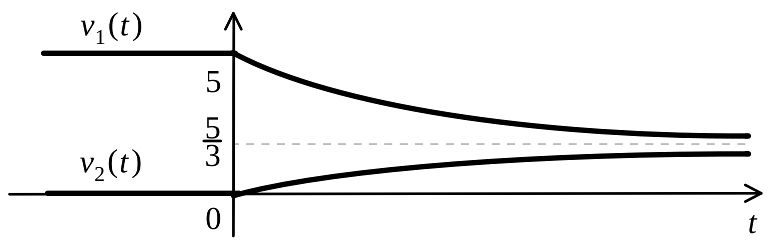
$$\frac{dv_1(t)}{dt} = -i(t) = -\frac{5}{6}e^{-\frac{t}{4}} \Rightarrow v_1(t) = k_1 + \frac{10}{3}e^{-\frac{t}{4}}$$

$$v_1(0) = k_1 + \frac{10}{3} = 5 \Rightarrow k_1 = \frac{5}{3} \Rightarrow v_1(t) = \frac{5}{3} + \frac{10}{3}e^{-\frac{t}{4}}, t \ge 0$$

$$v_2(t) = v_1(t) - 6i(t) = \frac{5}{3} + \frac{10}{3}e^{-\frac{t}{4}} - 5e^{-\frac{t}{4}} = \frac{5}{3} - \frac{5}{3}e^{-\frac{t}{4}}, t \ge 0$$

(d) Voltages and current waveforms

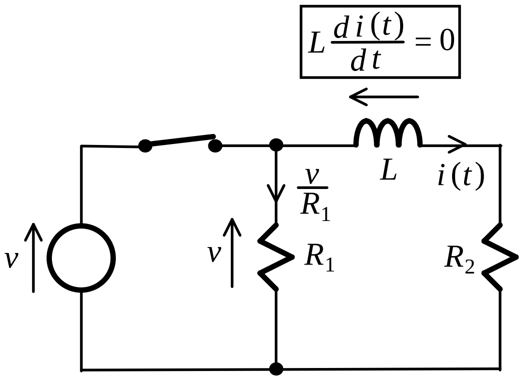


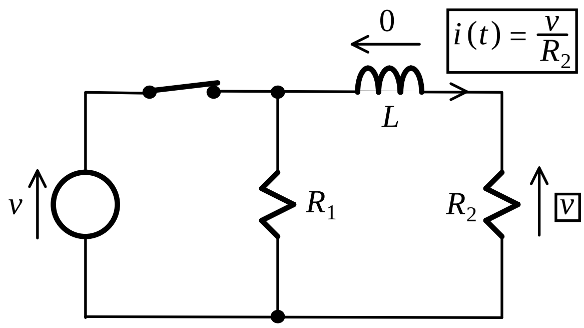


Q.3

(a) Voltages and current and energy stored for $t < 0$

With the source being a dc one and taking the switches to be in the positions shown starting from $t = -\infty$, all the voltages and currents will have settled down to constant values for practically all $t < 0$:

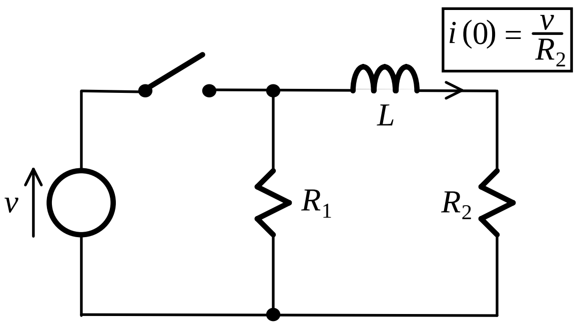




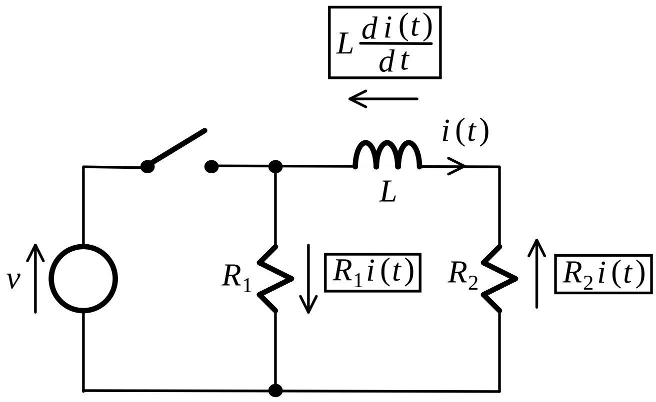
Energy stored in inductor $= \frac{Li^2(t)}{2} = \frac{Lv^2}{2R_2{}^2}$

(b) Current at $t = 0$ (just after the switches is opened)

Since the current in the inductor must be continuous, $i(0)$ must have the same value before the switch is opened:



(c) Current for $t \geq 0$



$$L\frac{di(t)}{dt} + R_1 i(t) + R_2 i(t) = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \left(\frac{R_1+R_2}{L}\right) i(t) = 0$$

$$i(t) = ke^{-\left(\frac{R_1+R_2}{L}\right)t},\ t \geq 0$$

$$i(0) = k = \frac{v}{R_2} \;\Rightarrow\; i(t) = \frac{v}{R_2} e^{-\left(\frac{R_1+R_2}{L}\right)t},\ t \geq 0$$

(d) Energy lost for $t \geq 0$

Instantaneous power lost in resistors $= (R_1 + R_2) i^2(t) = \dfrac{v^2(R_1+R_2)}{R_2^{\,2}} e^{-2\left(\frac{R_1+R_2}{L}\right)t},\ t \geq 0$

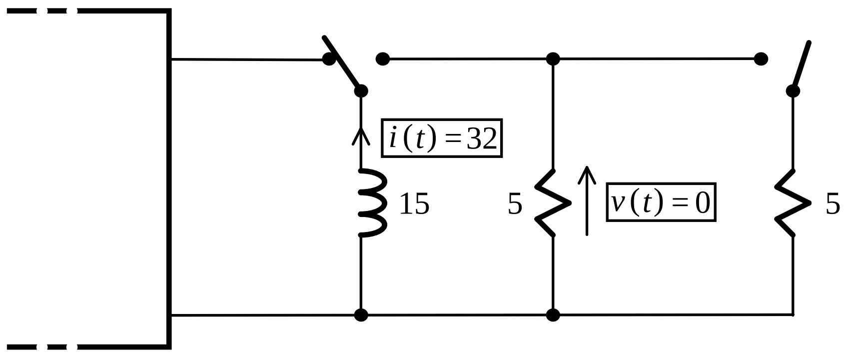
$$\text{Total energy lost} = \int_0^\infty \frac{v^2(R_1+R_2)}{R_2^{\,2}} e^{-2\left(\frac{R_1+R_2}{L}\right)t} dt = \frac{v^2(R_1+R_2)}{R_2^{\,2}} \left[\frac{e^{-2\left(\frac{R_1+R_2}{L}\right)t}}{-2\left(\frac{R_1+R_2}{L}\right)} \right]_0^\infty = \frac{Lv^2}{2R_2^{\,2}}$$

From the conservation of energy, this must also be equal to the decrease in energy stored by the inductor:

$$\text{Decrease in energy stored by inductor} = \frac{Li^2(0)}{2} - \frac{Li^2(\infty)}{2} = \frac{Lv^2}{2R_2^{\,2}}$$

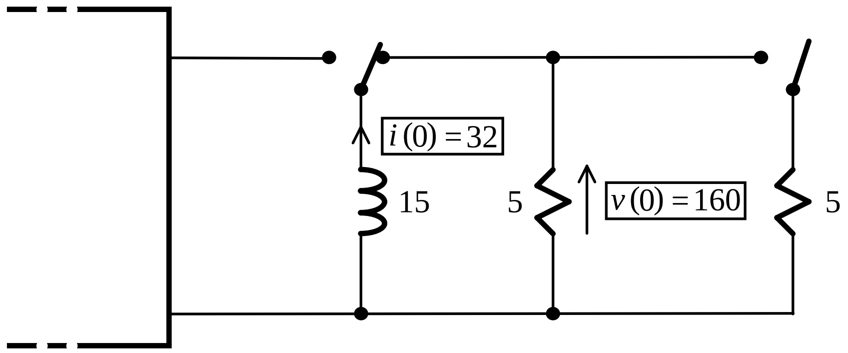
Q.4

Time $t < 0$

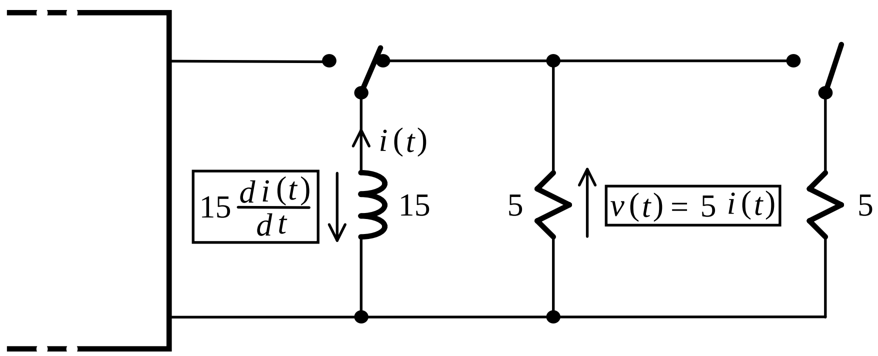


Time $t = 0$ *just after the first switch is activated*

Since the current in the inductor must be continuous, the inductor must be carrying the same 16A of current just after the first switch is activated:



Time $t \geq 0$ *and* $t < 3$ *after activating 1st switch but before closing 2nd switch*



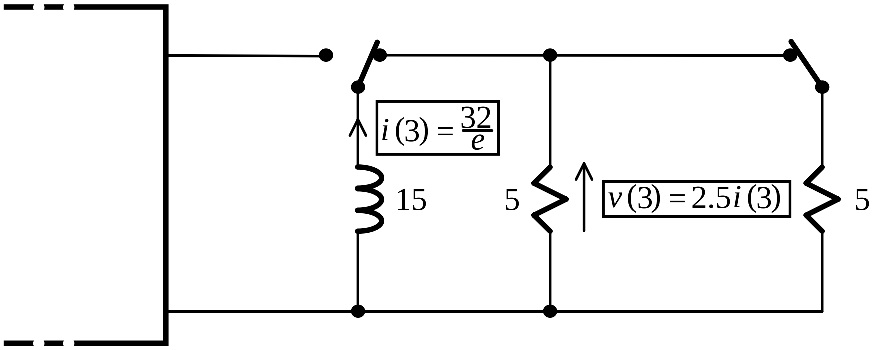
$$15\frac{di(t)}{dt} + 5i(t) = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \frac{i(t)}{3} = 0 \;\Rightarrow\; i(t) = ke^{-\frac{t}{3}},\, 0 \leq t < 3$$

$$i(0) = k = 32 \;\Rightarrow\; i(t) = 32e^{-\frac{t}{3}},\, 0 \leq t < 3$$

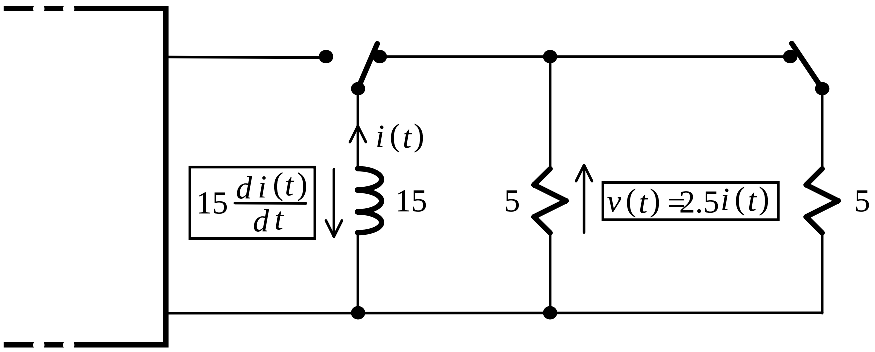
$$v(t) = 5i(t) = 160e^{-\frac{t}{3}},\, 0 \leq t < 3$$

Time $t = 3$ *just after closing second switch*

Since the current in the inductor must be continuous, $i(t)$ must be the same as $32e^{-3/3} = 32e^{-1}\,\mathrm{A}$ just before closing this switch:



Time $t \geq 3$ *after closing second switch*

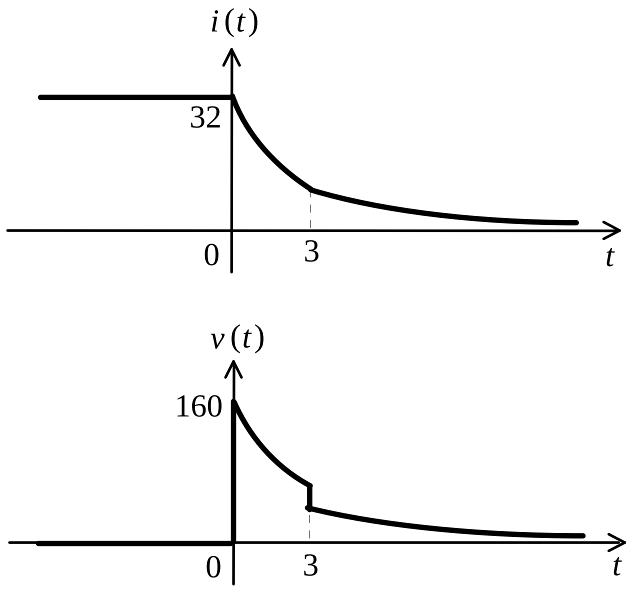


$$15\frac{di(t)}{dt} + 2.5i(t) = 0 \;\Rightarrow\; \frac{di(t)}{dt} + \frac{i(t)}{6} = 0 \;\Rightarrow\; i(t) = he^{-\frac{t}{6}}, 3 \leq t$$

$$i(3) = he^{-\frac{3}{6}} = 32e^{-1} \;\Rightarrow\; h = 32e^{-\frac{1}{2}} \;\Rightarrow\; i(t) = 32e^{-1}e^{-\frac{(t-3)}{6}}, 3 \leq t$$

$$v(t) = 2.5i(t) = 80e^{-1}e^{-\frac{(t-3)}{6}}, 3 \leq t$$

Voltage and current waveforms

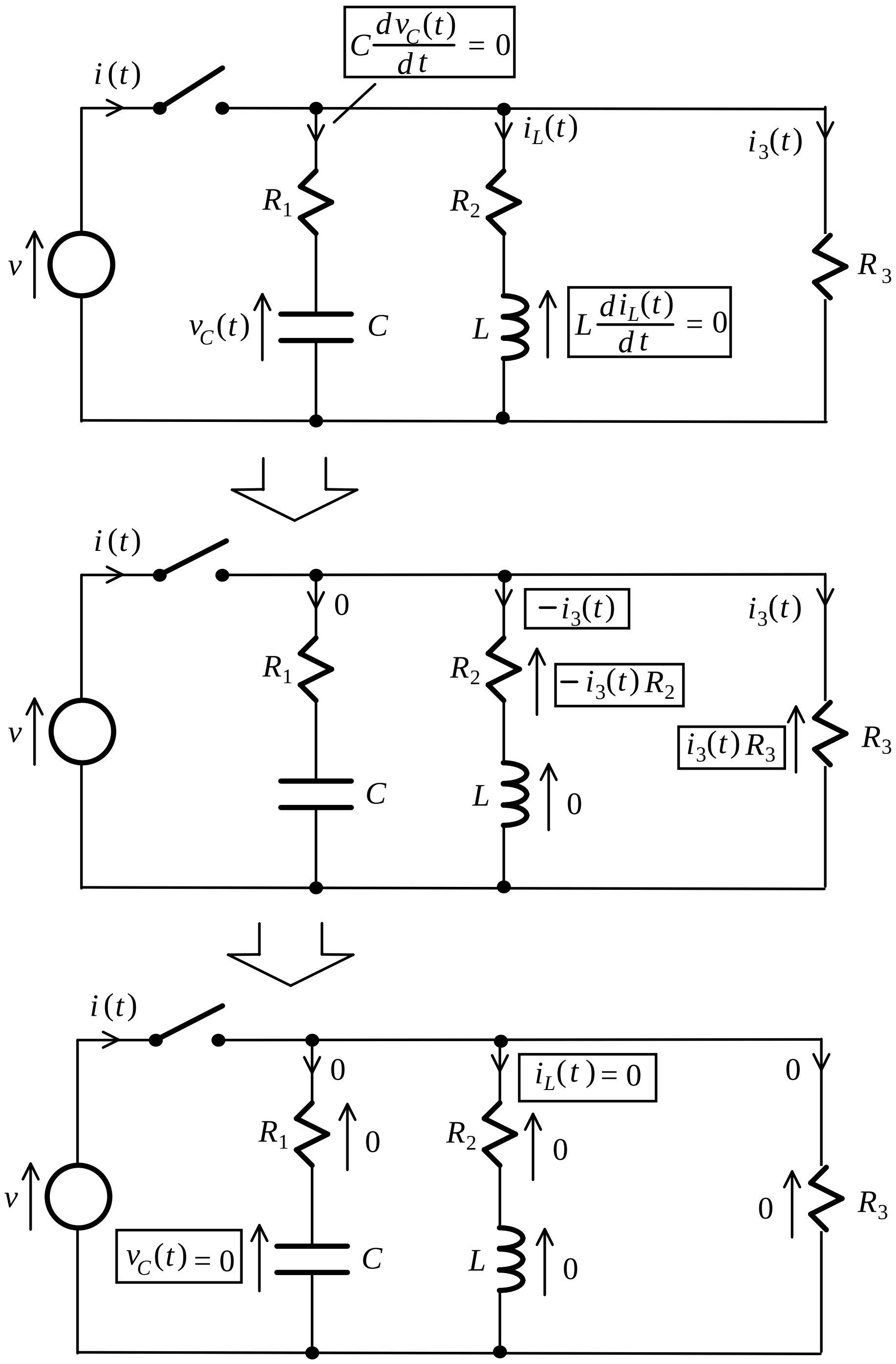

i(t)
32
0
3
t
v(t)
160
0
3
t

F.9 Transients II

Q.1

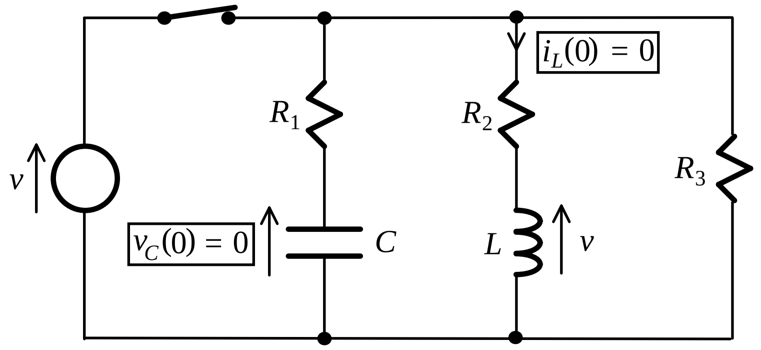
Voltages and currents for $t < 0$

Taking the switches to be in the positions shown starting from $t = -\infty$, all the voltages and currents will have settled down to constant values for practically all $t < 0$.

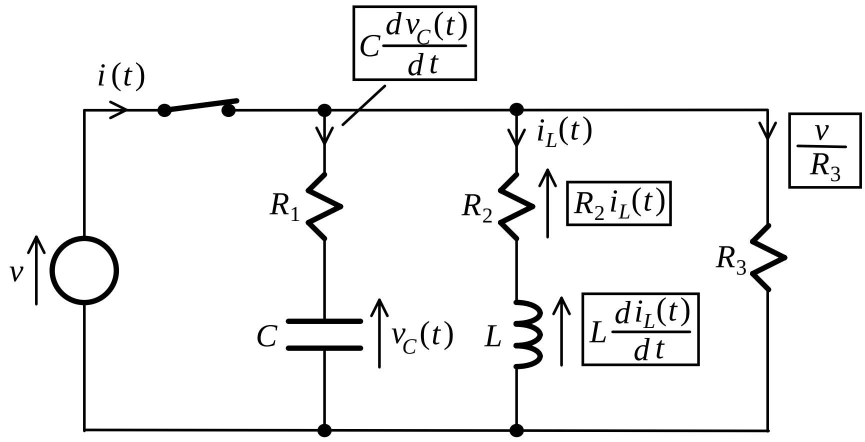


Voltages and currents just after closing switch at $t = 0$

Since the voltage across a capacitor and the current through an inductor must be continuous, the initial conditions are



Voltages and currents for $t \geq 0$



$$v = v_C(t) + CR_1 \frac{dv_C(t)}{dt} \;\Rightarrow\; \frac{dv_C(t)}{dt} + \frac{v_C(t)}{CR_1} = \frac{v}{CR_1}$$

$$v = R_2 i_L(t) + L\frac{di_L(t)}{dt} \;\Rightarrow\; \frac{di_L(t)}{dt} + \frac{R_2}{L} i_L(t) = \frac{v}{L}$$

The general solutions are

$$v_C(t) = v_{ss}(t) + v_{tr}(t)$$

$$i_L(t) = i_{ss}(t) + i_{tr}(t)$$

The steady state responses are

$$v_{ss}(t) = k_v,\, t \geq 0$$

$$\frac{dv_{ss}(t)}{dt} + \frac{v_{ss}(t)}{CR_1} = \frac{k_v}{CR_1} = \frac{v}{CR_1} \;\Rightarrow\; k_v = v \;\Rightarrow\; v_{ss}(t) = v,\, t \geq 0$$

$$i_{ss}(t) = k_i,\, t \geq 0$$

$$\frac{di_{ss}(t)}{dt}+\frac{R_2}{L}i_{ss}(t)=\frac{R_2k_i}{L}=\frac{v}{L} \Rightarrow k_i=\frac{v}{R_2} \Rightarrow i_{ss}(t)=\frac{v}{R_2}, t\geq 0$$

The transient responses are given by

$$\frac{dv_{tr}(t)}{dt}+\frac{v_{tr}(t)}{CR_1}=0 \Rightarrow v_{tr}(t)=h_v e^{-\frac{t}{CR_1}}, t\geq 0$$

$$\frac{di_{tr}(t)}{dt}+\frac{R_2}{L}i_{tr}(t)=0 \Rightarrow i_{tr}(t)=h_i e^{-\frac{R_2 t}{L}}, t\geq 0$$

Combining and applying initial conditions:

$$v_C(t)=v_{ss}(t)+v_{tr}(t)=v+h_v e^{-\frac{t}{CR_1}}, t\geq 0$$

$$v_C(0)=v+h_v=0 \Rightarrow h_v=-v \Rightarrow v_C(t)=v-ve^{-\frac{t}{CR_1}}, t\geq 0$$

$$i_L(t)=i_{ss}(t)+i_{tr}(t)=\frac{v}{R_2}+h_i e^{-\frac{R_2 t}{L}}, t\geq 0$$

$$i_L(0)=\frac{v}{R_2}+h_i=0 \Rightarrow h_i=-\frac{v}{R_2} \Rightarrow i_L(t)=\frac{v}{R_2}-\frac{v}{R_2}e^{-\frac{R_2 t}{L}}, t\geq 0$$

Source current: for $t\geq 0$

$$i(t)=C\frac{dv_C(t)}{dt}+i_L(t)+\frac{v}{R_3}=\frac{Cv}{CR_1}e^{-\frac{t}{CR_1}}+\frac{v}{R_2}-\frac{v}{R_2}e^{-\frac{R_2 t}{L}}+\frac{v}{R_3}=\frac{v}{R_2}+\frac{v}{R_3}+v\left(\frac{e^{-\frac{t}{CR_1}}}{R_1}-\frac{e^{-\frac{R_2 t}{L}}}{R_2}\right)$$

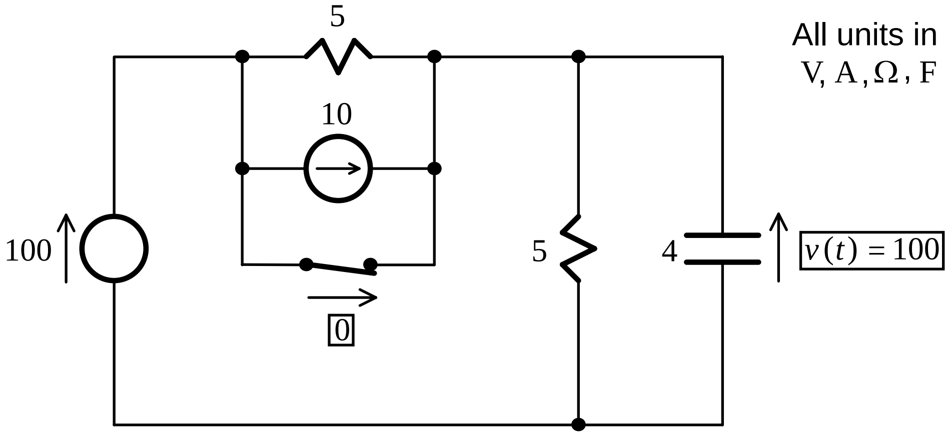
For this to be time independent:

$$R_1=R_2$$

$$CR_1=\frac{L}{R_2} \Rightarrow R_1^{\,2}=R_2^{\,2}=\frac{L}{C}$$

Q.2

Voltages and currents for $t < 0$ before the switch is opened

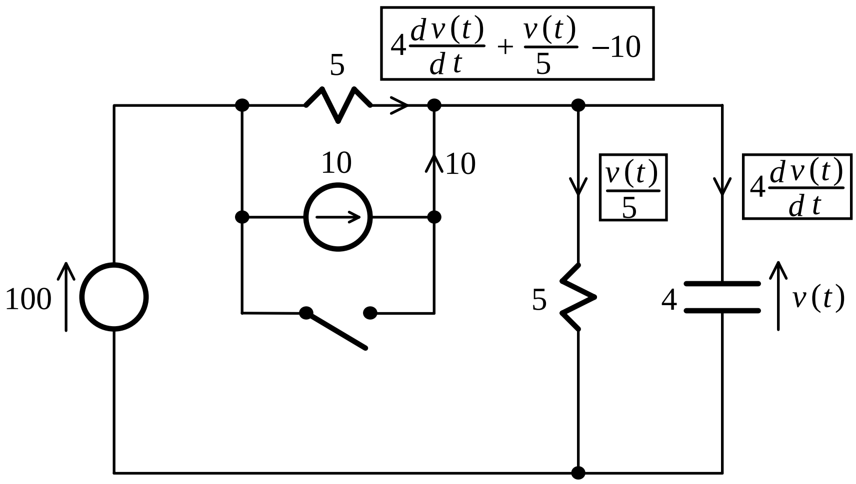


$v(t)$ *at* $t = 0$ *just after the switch is opened*

Since the voltage across the capacitor must be continuous, $v(0)$ must be

$$v(0) = 100$$

$v(t)$ *for* $t \geq 0$ *after the switch is opened*



$$100 = 5\left[4\frac{dv(t)}{dt} + \frac{v(t)}{5} - 10\right] + v(t) \;\Rightarrow\; 20\frac{dv(t)}{dt} + 2v(t) = 150$$

The general solution of this is given by

$$v(t) = v_{ss}(t) + v_{tr}(t)$$

$v_{ss}(t)$ is the steady state response and can be found by trying

$$v_{ss}(t) = k,\ t \geq 0$$

$$20\frac{dv_{ss}(t)}{dt}+2v_{ss}(t)=0+2k=150 \;\Rightarrow\; k=75 \;\Rightarrow\; v_{ss}(t)=75, t\geq 0$$

$v_{tr}(t)$ is the transient response and is equal to the general solution of

$$20\frac{dv_{tr}(t)}{dt}+2v_{tr}(t)=0 \;\Rightarrow\; v_{tr}(t)=he^{-\frac{t}{10}}, t\geq 0$$

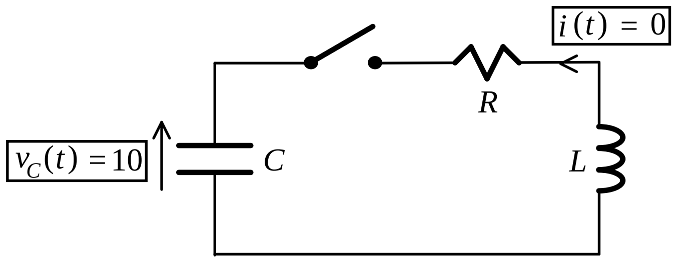
Combining:

$$v(t)=v_{ss}(t)+v_{tr}(t)=75+he^{-\frac{t}{10}}, t\geq 0$$

$$v(0)=75+h=100 \;\Rightarrow\; h=25 \;\Rightarrow\; v(t)=75+25e^{-\frac{t}{10}}, t\geq 0$$

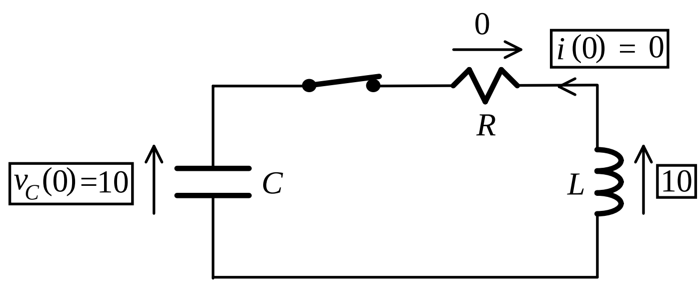
Q.3

(a) Voltage and current before closing switch

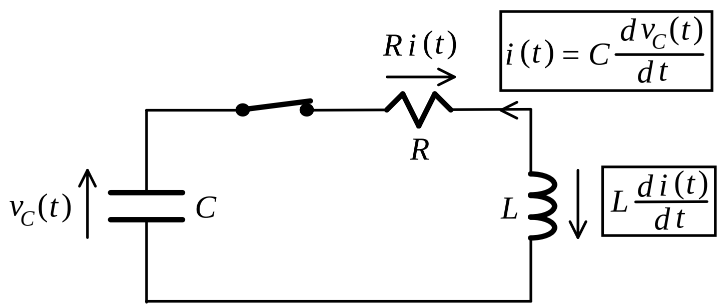


(b) Voltages and current just after closing switch at $t=0$

Since the voltage across a capacitor and the current through an inductor must be continuous, the initial conditions are



(c) Governing differential equation for current for $t\geq 0$



$$L\frac{di(t)}{dt}+Ri(t)+v_C(t)=0 \;\Rightarrow\; \frac{d^2i(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{L}\frac{dv_C(t)}{dt}=0$$

$$\frac{d^2i(t)}{dt^2}+\frac{R}{L}\frac{di(t)}{dt}+\frac{1}{LC}i(t)=0,\; t\geq 0$$

with initial conditions

$$i(0)=0$$

$$v_C(0)=10 \;\Rightarrow\; L\left.\frac{di(t)}{dt}\right|_{t=0}=-10 \;\Rightarrow\; \left.\frac{di(t)}{dt}\right|_{t=0}=-\frac{10}{L}$$

Substituting

Resonant frequency $=\omega=\dfrac{1}{\sqrt{LC}}$

$$Q=\frac{\omega L}{R}=\sqrt{\frac{L}{R^2C}}$$

the governing differential equation is

$$\frac{d^2i(t)}{dt^2}+\frac{\omega}{Q}\frac{di(t)}{dt}+\omega^2 i(t)=0,\; t\geq 0$$

(d) Overdamped situation when $Q<\frac{1}{2}$

The roots of the polynomial

$$\left.\frac{d^2i(t)}{dt^2}+\frac{\omega}{Q}\frac{di(t)}{dt}+\omega^2 i(t)\right|_{\frac{di(t)}{dt}\text{ replaced by } z}=z^2+\frac{\omega}{Q}z+\omega^2$$

are

$$z_1, z_2=\frac{-\frac{\omega}{Q}\pm\sqrt{\frac{\omega^2}{Q^2}-4\omega^2}}{2}=\frac{-\omega}{2Q}\left(1\pm\sqrt{1-4Q^2}\right)$$

When $Q<\frac{1}{2}$, both roots are real, negative and distinct. Thus:

$$i(t)=k_1e^{z_1t}+k_2e^{z_2t},\; t\geq 0$$

$$z_1=\frac{-\omega}{2Q}\left(1-\sqrt{1-4Q^2}\right)<0$$

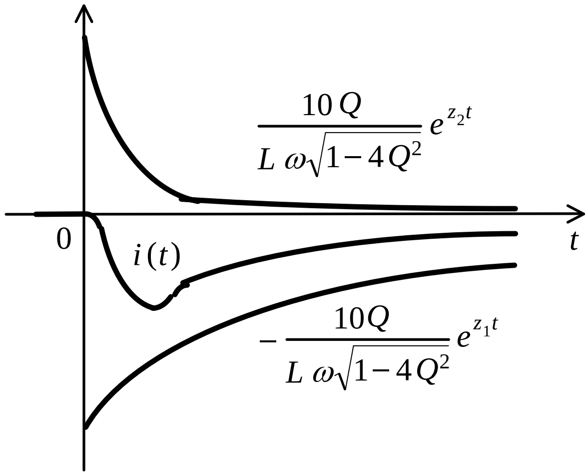
$$z_2=\frac{-\omega}{2Q}\left(1+\sqrt{1-4Q^2}\right)<z_1$$

Using initial conditions:

$$i(0)=0 \;\Rightarrow\; k_1+k_2=0 \;\Rightarrow\; k_1=-k_2$$

$$\left.\frac{di(t)}{dt}\right|_{t=0} = -\frac{10}{L} \Rightarrow k_1 z_1 + k_2 z_2 = -k_2(z_1 - z_2) = -\frac{k_2\omega}{Q}\sqrt{1-4Q^2} = -\frac{10}{L}$$

$$i(t) = k_2\left(e^{z_2 t} - e^{z_1 t}\right) = \frac{10Q}{L\omega\sqrt{1-4Q^2}}\left(e^{z_2 t} - e^{z_1 t}\right), t \ge 0$$



(e) Underdamped situation when $Q > \frac{1}{2}$

When $Q > \frac{1}{2}$, the two roots form a complex pair:

$$z_1 = \frac{-\omega}{2Q}\left(1 - j\sqrt{4Q^2-1}\right) = -\frac{\omega}{2Q} + j\omega\sqrt{1-\frac{1}{4Q^2}}$$

$$z_2 = \frac{-\omega}{2Q}\left(1 + j\sqrt{4Q^2-1}\right) = -\frac{\omega}{2Q} - j\omega\sqrt{1-\frac{1}{4Q^2}} = z_1^*$$

$$i(t) = h_1 e^{z_1 t} + h_2 e^{z_2 t}, t \ge 0$$

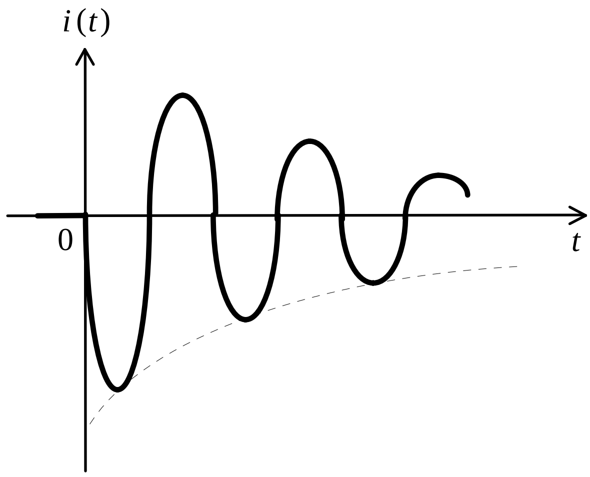
Using initial conditions:

$$i(0) = 0 \Rightarrow h_1 + h_2 = 0 \Rightarrow h_1 = -h_2$$

$$\left.\frac{di(t)}{dt}\right|_{t=0} = -\frac{10}{L} \Rightarrow h_1 z_1 + h_2 z_2 = -h_2(z_1 - z_2) = -jh_2 2\omega\sqrt{1-\frac{1}{4Q^2}} = -\frac{10}{L}$$

$$i(t) = h_2\left(e^{z_2 t} - e^{z_1 t}\right) = \frac{-j10}{2L\omega\sqrt{1-\frac{1}{4Q^2}}}\left[e^{\left(-\frac{\omega}{2Q} - j\omega\sqrt{1-\frac{1}{4Q^2}}\right)t} - e^{\left(-\frac{\omega}{2Q} + j\omega\sqrt{1-\frac{1}{4Q^2}}\right)t}\right]$$

$$= \frac{-j10e^{-\frac{\omega}{2Q}t}}{2L\omega\sqrt{1-\frac{1}{4Q^2}}}\left[e^{-j\omega\sqrt{1-\frac{1}{4Q^2}}t} - e^{j\omega\sqrt{1-\frac{1}{4Q^2}}t}\right] = \frac{-10e^{-\frac{\omega}{2Q}t}}{L\omega\sqrt{1-\frac{1}{4Q^2}}}\sin\left(\omega\sqrt{1-\frac{1}{4Q^2}}t\right), t \ge 0$$

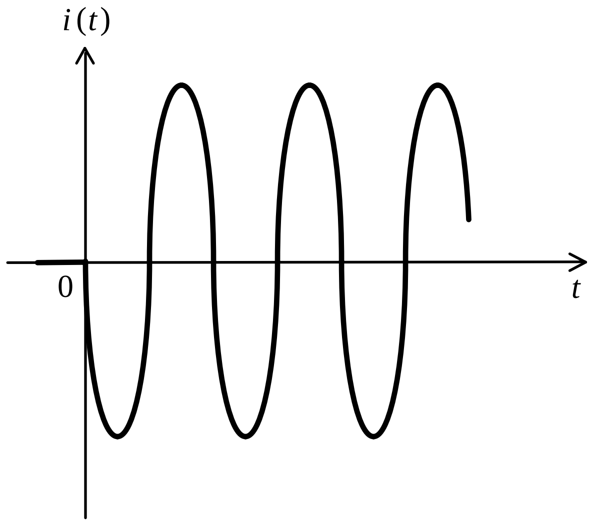


(e) Undamped situation when $R = 0$

When $R = 0$, the Q factor and current is

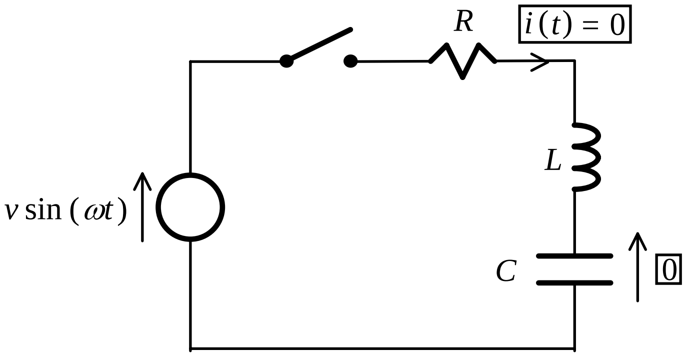
$$Q = \frac{\omega L}{R} = \infty$$

$$i(t) = \frac{-10e^{-\frac{\omega}{2Q}t}}{L\omega\sqrt{1-\frac{1}{4Q^2}}}\sin\left(\omega\sqrt{1-\frac{1}{4Q^2}}t\right) = \frac{-10}{L\omega}\sin(\omega t),\, t \geq 0$$



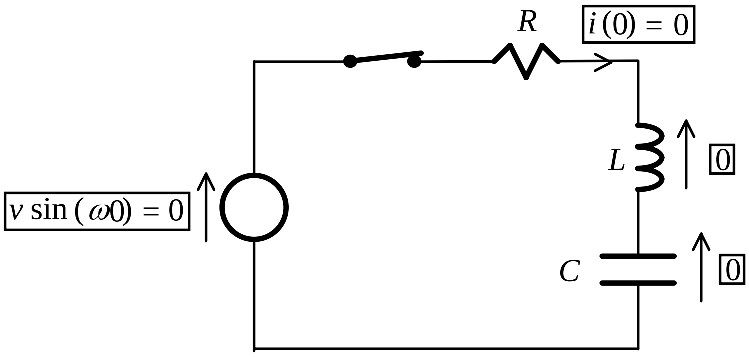
Q.4

Voltages and current for $t < 0$ before the switch is closed

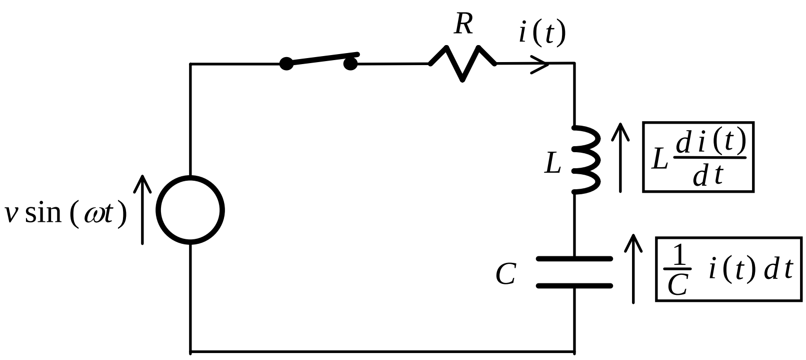


Voltages and current at $t = 0$ just after the switch is closed

As the voltage across the capacitor and the current through the inductor must be continuous:



Current for $t \geq 0$ after the switch is closed



$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int i(t)dt = v\sin(\omega t) = \mathrm{Re}\left[-jve^{j\omega t}\right]$$

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{C}i(t) = \mathrm{Re}\left[\omega ve^{j\omega t}\right]$$

with initial conditions

$$i(0) = 0$$

Initial voltage across inductor $= 0 \;\Rightarrow\; L\left.\dfrac{di(t)}{dt}\right|_{t=0} = 0$

Steady state current

The steady state response $i_{ss}(t)$ is given by

$$i_{ss}(t) = \mathrm{Re}\left[I_{ss}e^{j\omega t}\right],\; t \geq 0$$

$$L\frac{d^2\,\mathrm{Re}\left[I_{ss}e^{j\omega t}\right]}{dt^2} + R\frac{d\,\mathrm{Re}\left[I_{ss}e^{j\omega t}\right]}{dt} + \frac{1}{C}\mathrm{Re}\left[I_{ss}e^{j\omega t}\right] = \mathrm{Re}\left[\omega v e^{j\omega t}\right]$$

$$\mathrm{Re}\left[\left(L\frac{d^2 e^{j\omega t}}{dt^2} + R\frac{de^{j\omega t}}{dt} + \frac{e^{j\omega t}}{C}\right)I_{ss}\right] = \mathrm{Re}\left[\left(j^2\omega^2 L + j\omega R + \frac{1}{C}\right)I_{ss}e^{j\omega t}\right] = \mathrm{Re}\left[\omega v e^{j\omega t}\right]$$

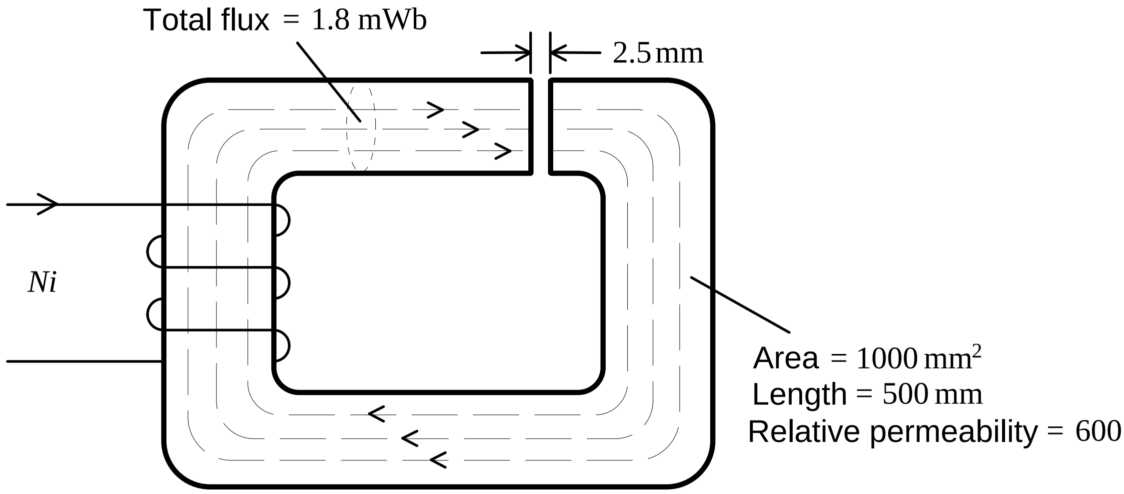
$$\left(j^2\omega^2 L + j\omega R + \frac{1}{C}\right)I_{ss} = \omega v \;\Rightarrow\; I_{ss} = \frac{v}{j\left(j\omega L + R + \dfrac{1}{jC}\right)} = \frac{v}{jZ}$$

$$i_{ss}(t) = \mathrm{Re}\left[\frac{v}{jZ}e^{j\omega t}\right] = \mathrm{Re}\left[\frac{v}{j|Z|e^{j\mathrm{Arg}(Z)}}e^{j\omega t}\right] = \frac{v}{|Z|}\sin\left[\omega t - \mathrm{Arg}(Z)\right]$$

$$= \frac{v}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}\sin\left[\omega t - \tan^{-1}\left(\frac{\omega L - \dfrac{1}{\omega C}}{R}\right)\right],\; t \geq 0$$

F.10 Magnetic Circuits

Q.1



Reluctance of iron core $= \Re_{core} = \dfrac{500\times10^{-3}}{\left(600\times4\pi\times10^{-7}\right)\left(1000\times10^{-6}\right)}$

$$= 0.6635\times10^{6}\ \text{At/Wb}$$

Reluctance of air gap $= \Re_{gap} = \dfrac{2.5\times10^{-3}}{\left(4\pi\times10^{-7}\right)\left(1000\times10^{-6}\right)}$

$$= 1.99\times10^{6}\ \text{At/Wb}$$

MMF $= Ni = \left(1.8\times10^{-3}\right)\left(R_{core}+R_{gap}\right)$

$$= \left(1.8\times10^{-3}\right)\left(0.6635\times10^{6}+1.99\times10^{6}\right) = 4776\,\text{At}\left(\text{Amp. turns}\right)$$

Q.2

Force of attraction

MMF for 2 air gaps $= \dfrac{2}{3}(4)(300) = 800\,\text{At}$

Reluctance of air gaps $= \Re_{gap} = \dfrac{2\left(1.5\times10^{-3}\right)}{\left(4\pi\times10^{-7}\right)(0.05)} = 4.77\times10^{4}\ \text{At/Wb}$

Flux in circuit $= \varPhi = \dfrac{800}{4.77\times10^{4}} = 0.0168\,\text{Wb}$

To determine the force, suppose the gap length is decreased by δx and the current is changed by δi while the flux remains the same. Since there is no back emf induced and no power is supplied by the electrical circuit:

$$\text{Decrease in reluctance} = \delta\Re_{gap} = \frac{2(\delta x)}{(4\pi\times10^{-7})(0.05)} = \frac{10^8\delta x}{\pi}$$

$$\text{Decrease in energy stored} = \frac{\delta\Re_{gap}\Phi^2}{2} = \frac{0.0168^2 10^8\delta x}{2\pi} = 4.49\times10^3\delta x$$

$$\text{Work done by movement of armature} = (\text{attraction force})\delta x$$

From energy conservation:

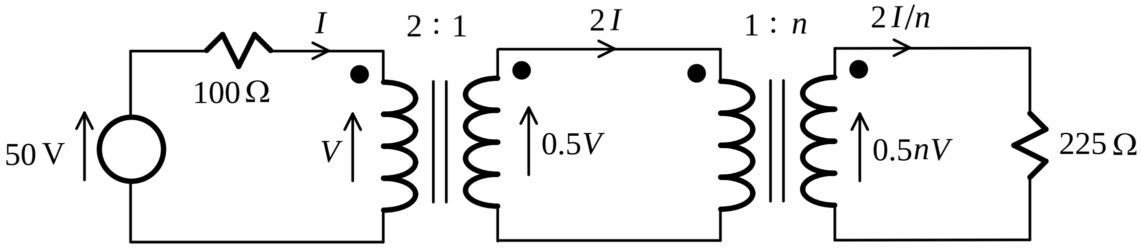
$$\text{Attraction force} = \frac{4.49\times10^3\delta x}{\delta x} = 4.49\times10^3\ \text{N}$$

Doubling of current

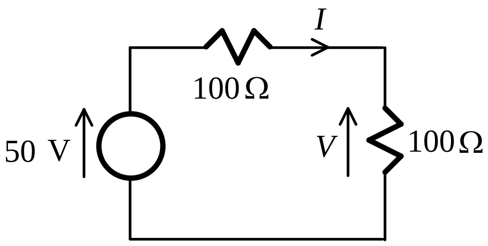
If the current is doubled, the flux Φ will be doubled. Since the energy stored and the force are proportional to Φ^2, they will increase by 4 times to $4(4.49) = 18\,\text{kN}$ theoretically.

However, in a practical device, the iron may get saturated as the current is increased. The flux may therefore not increase by a factor of 2 and the actual force will be smaller.

Q.3



For maximum power transfer, the source must be seeing a load resistance of 100Ω:

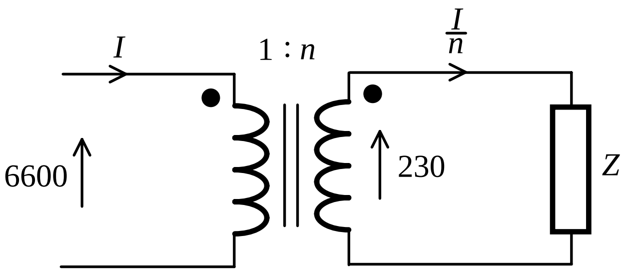


$V = 25\text{V}$

$$I = \frac{50}{(100+100)} = 0.25\,\text{A}$$

$$\frac{0.5nV}{2I/n} = \frac{n^2V}{4I} = 225 \;\Rightarrow\; n^2 = \frac{225}{25} = 9 \;\Rightarrow\; n = 3$$

Q.4



$$n = \frac{230}{6600} = 0.0348$$

$$|I| = \frac{\text{rated current}}{2} = \frac{100000/6600}{2} = 7.58\,\text{A}$$

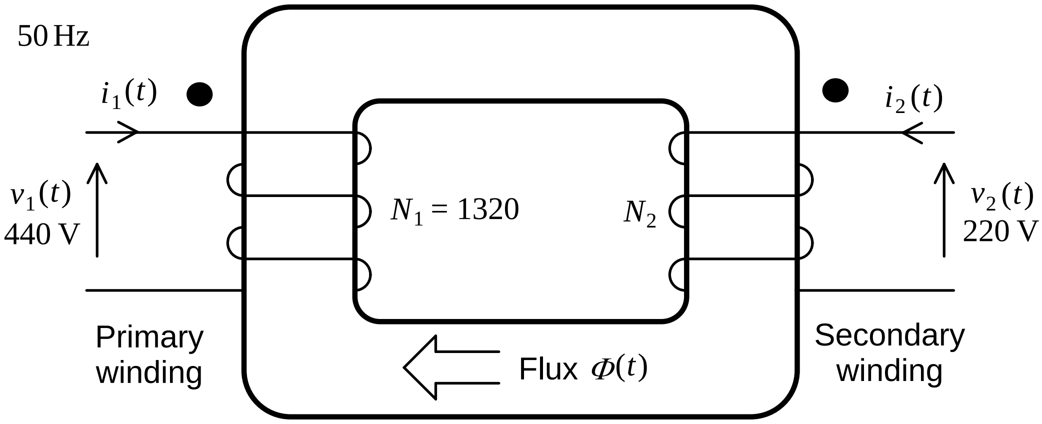
$$0.8\,\text{lagging p.f.} = \left\langle \begin{matrix} \cos[\text{Arg}(I)] = 0.8 \\ \text{Arg}(I) < 0 \end{matrix} \right\rangle = \left\langle \begin{matrix} \text{Arg}(I) = \pm 36.9^0 \\ \text{Arg}(I) < 0 \end{matrix} \right\rangle \;\Rightarrow\; \text{Arg}(I) = -36.9^0$$

$$Z = \frac{230}{I/n} = \frac{230}{7.58e^{-j36.9^0}/0.0348} = 1.06e^{j36.9^0}\ \Omega$$

Load impedance referred to primary $=$ load impedance seen by source

$$= \frac{6600}{I} = \frac{6600}{7.58e^{-j36.9^0}} = 870.7e^{j36.9^0}\ \Omega$$

Q.5



$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{220}{440} = \frac{1}{2} \;\Rightarrow\; N_2 = \frac{1320}{2} = 660$$

Since the currents are sinusoidal at $50\,\mathrm{Hz}$, the flux $\Phi(t)$ will also be sinusoidal:

$$\Phi(t) = \Phi_{max}\cos(100\pi t + \theta)$$

Thus, the primary voltage will be

$$v_1(t) = N_1\frac{d\Phi(t)}{dt} = -100\pi\, N_1\Phi_{max}\sin(100\pi t + \theta)$$

The rms value of this is

$$\frac{100\pi\, N_1\Phi_{max}}{\sqrt{2}} = 440 \;\Rightarrow\; \Phi_{max} = \frac{440\sqrt{2}}{100\pi(1320)} = 0.0015\,\mathrm{Wb}$$

Cross sectional area of core $= \dfrac{\Phi_{max}}{\text{maximum flux density}} = \dfrac{\Phi_{max}}{0.6} = 25\,\mathrm{cm}^2$

Q.6

Efficiencies under different operating conditions

	Full load, unity pf	$\frac{3}{4}$ full load, unity pf
Apparent power delivered (VA)	50000	$50000 \times 0.75 = 37500$
Actual power delivered (W)	$50000 \times 1 = 50000$	$37500 \times 1 = 37500$
Copper loss (W)	630	$630 \times 0.75^2 = 354$
Iron loss (W)	350	350
Total loss (W)	$630 + 350 = 980$	$354 + 350 = 704$
Power supplied (W)	$50000 + 980 = 50980$	$37500 + 704 = 38204$
Efficiency	$\frac{50000}{50980} = 98.1\%$	$\frac{37500}{38204} = 98.2\%$

	Full load, 0.8 pf	$\frac{3}{4}$ full load, 0.8 pf
Apparent power delivered (VA)	50000	$50000 \times 0.75 = 37500$
Actual power delivered (W)	$50000 \times 0.8 = 40000$	$37500 \times 0.8 = 30000$
Copper loss (W)	630	$630 \times 0.75^2 = 354$
Iron loss (W)	350	350
Total loss (W)	$630 + 350 = 980$	$354 + 350 = 704$
Power supplied (W)	$40000 + 980 = 40980$	$30000 + 704 = 30704$
Efficiency	$\frac{40000}{40980} = 97.6\%$	$\frac{30000}{30704} = 97.7\%$

All-day efficiency

	Energy loss $(\mathrm{kW\,hr})$	Energy delivered $(\mathrm{kW\,hr})$
$0000 - 0600$ hr no load	$6 \times (0.35) = 2.1$	0
$0600 - 1200$ hr full load, unity pf	$6 \times (0.35 + 0.63) = 5.88$	$6 \times 50 = 300$
$1200 - 1400$ hr $\frac{3}{4}$ full load, unity pf	$2 \times (0.35 + 0.63 \times 0.75^2) = 1.41$	$2 \times 50 \times 0.75 = 75$
$1400 - 1800$ hr full load, 0.8 pf	$4 \times (0.35 + 0.63) = 3.92$	$4 \times 50 \times 0.8 = 160$
$1800 - 2200$ hr $\frac{3}{4}$ full load, 0.8 pf	$4 \times (0.35 + 0.63 \times 0.75^2) = 2.82$	$4 \times 50 \times 0.75 \times 0.8 = 120$
$2200 - 2400$ hr no load	$2 \times (0.35) = 0.7$	0
Total	16.83	655

All-day efficiency $= \dfrac{655}{655 + 16.83} = 97.5\%$