TE2402 Linear Algebra & Numerical Methods

Mid-term Test

Name: ______________________________ **Metric No:** ________________________

Instructions: *Answer all the questions below in the space provided.*

Q.1. Given a linear system $\mathbf{A}\ \mathbf{x} = \mathbf{b}$, state under what conditions, the system has: i) a unique solution; ii) multiple solutions; and iii) no solution.

Ans: Let $\tilde{\mathbf{A}} = [\ \mathbf{A}\ \ \mathbf{b}\ ]$, the augmented matrix of the linear system.

- i) the system has a unique solution if rank($\mathbf{A}$) = rank($\tilde{\mathbf{A}}$) = no. of unknowns.
- ii) the system has multiple solutions if rank($\mathbf{A}$) = rank($\tilde{\mathbf{A}}$) < no. of unknowns.
- iii) the system has no solution if rank($\mathbf{A}$) ≠ rank($\tilde{\mathbf{A}}$).

Q.2. Let $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ be a set of vectors with the same length, and let c_1, c_2 and c_3 be coefficients such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = 0$. What can you conclude about these coefficients i) if the vectors are linearly independent? and ii) if they are linearly dependent?

Ans:

- i) All the coefficients are zero; and
- ii) At least one of them is non-zero.

Q.3. What are the two key properties that a vector space $\mathbf{V}$ should hold?

Ans:

- i) For any $\mathbf{v}_1$ and $\mathbf{v}_2$ in $\mathbf{V}$, $\mathbf{v}_1 + \mathbf{v}_2$ should be in $\mathbf{V}$.
- ii) For any scalar k and vector $\mathbf{v}$ in $\mathbf{V}$, $k\mathbf{v}$ is also in $\mathbf{V}$.

Q.4. What are the properties for a basis of a vector space $\mathbf{V}$? Is the basis unique?

Ans:

- i) Vectors form a basis have to be linearly independent; and
- ii) They span the whole vector space.

please turn over

Q.5. Is *rank* defined for only square matrices? Give the definitions of the row rank and column rank of an appropriate matrix.

Ans:

No. Rank is defined for any matrix. Row rank is defined as the number of linearly independent row vectors in a matrix. Column rank is defined as the number of linearly independent column vectors of the matrix.

Q.6. Given two square matrices $\mathbf{A}$ and $\mathbf{B}$ with the same dimension, are the following statements true or false?

i) $\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B}) = \det(\mathbf{B})\det(\mathbf{A})$; and ii) $\mathbf{A}\,\mathbf{B} = \mathbf{B}\,\mathbf{A}$.

Ans:

i) is true and ii) is false.

Q.7. Given an n x n matrix $\mathbf{A}$, is it possible that the following statements might all be true?

i) $\mathbf{A}$ is a singular matrix; ii) $\mathbf{A}$ has a rank of $n-2$; iii) $\mathbf{A}$ has an eigenvalue equal to 0; and iv) $\mathbf{A}$ has an eigenvalue equal to 3.

Ans: Yes.

Q.8. Are the concepts of eigenvalues and eigenvectors applicable to square matrices? Give the formal definitions of the eigenvalues and eigenvectors.

Ans:

Yes. A scalar λ and a nonzero vector $\mathbf{v}$ are said to be respectively the eigenvalue and eigenvector of a given matrix A, if they satisfy $\mathbf{A}\,\mathbf{v} = \lambda\,\mathbf{v}$.

Q.9. If a matrix $\mathbf{S}$ satisfies $\mathbf{S}\,\mathbf{S}^{T} = \mathbf{I}$, is this matrix said to be a symmetric matrix or a skew-symmetric matrix or both or something else? What is the inverse of $\mathbf{S}$?

Ans:

$\mathbf{S}$ is an orthogonal matrix. The inverse of $\mathbf{S}$ is equal to $\mathbf{S}^{T}$.

Q.10. When are two matrices $\mathbf{A}$ and $\mathbf{B}$ said to be similar? How are their eigenvalues related?

Ans:

$\mathbf{A}$ and $\mathbf{B}$ said to be similar if there exists a nonsingular matrix $\mathbf{T}$ such that $\mathbf{A} = \mathbf{T}^{-1}\,\mathbf{B}\,\mathbf{T}$.

They have the same set of eigenvalues.

the end