A Matlab Toolkit for Composite Nonlinear Feedback Control

— Improving transient response in tracking control

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1 Background

Transient performance is one of the important issues in tracking control problems such as target tracking and output regulation (see [1, 2]). In general, quick response and small overshoot are desirable in most of the target tracking control problems. However, it is well known that quick response results in a large overshoot. Thus, most of the design schemes have to make a tradeoff between these two transient performance indices, which is especially true for physical systems with input saturation. To improve the tracking performance, a composite nonlinear feedback (CNF) control technique is developed in [3] for a class of second-order linear systems with input saturation. The technique is extended in [4] to higher order and multiple input systems under some restrictive assumptions. However, both [3] and [4] consider only the state feedback case. Recently, the problem has been solved in [5, 6] for general linear systems with measurement feedback and in [7] for a class of nonlinear systems with input saturation. The CNF control technique has also been successfully applied to design hard disk drive servo systems (see [5, 6, 9]) and adopted to improve control performance for a nonlinear benchmark problem of [10] (see also [11, 12]) and for tracking a general target reference[8].

The CNF control consists of a linear feedback law and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with a small damping ratio for a quick response, while at the same time not exceeding the actuator limits for desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce overshoot caused by the linear part. Basically, the CNF control design philosophy is to combine the good properties associated with largely and slightly damped systems (see Fig. 1): At the initial stage when the system output is far away from the target reference, the CNF control utilizes a control law that gives a small damping ratio in the resulting closed-loop system to produce a fast rise time. At the final stage when the system output is approaching the target, the CNF control makes use of a control law that yields a large damping ratio to minimize overshoot and undershoot in the response. The overall design is achieved through a smooth nonlinear gain.

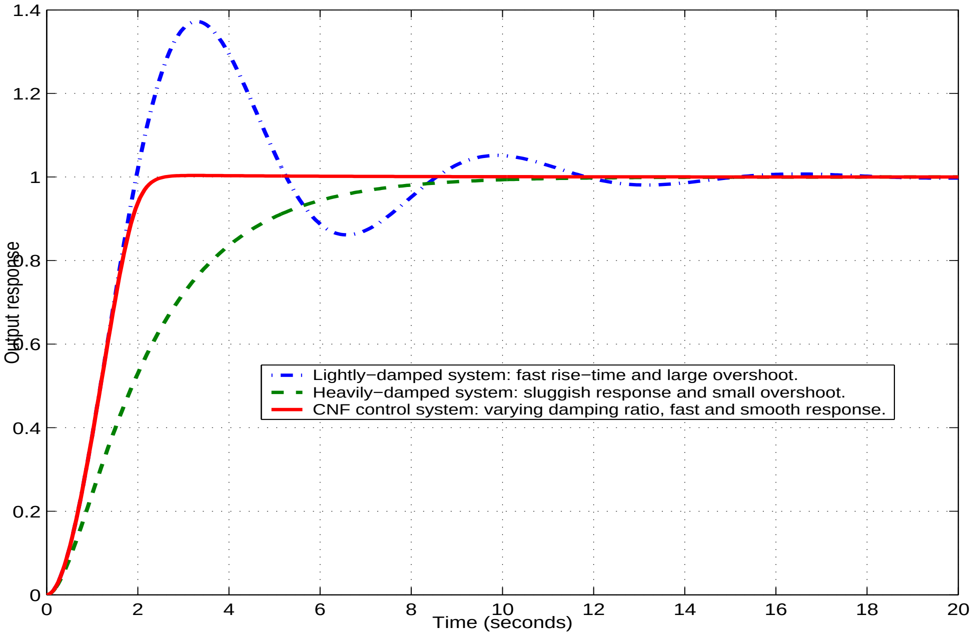

Figure 1: The design philosophy of the CNF tracking control.

The CNF design involves some matrix calculations and parameter selections. Especially with those parameters for forming the nonlinear feedback law, some tuning and re-tuning are needed to obtain the best possible solutions. Hence, it is desirable to have a toolkit to facilitate the design process. This motivated us to develop a MATLAB toolkit with a user-friendly graphical interface for the CNF control design. The toolkit can be utilized to design fast and smooth tracking controllers for general SISO linear systems and a class of nonlinear systems with actuator saturation and other nonlinearities such as friction, as well as external disturbances. The toolkit can display both time-domain and frequency-domain responses on its main panel, and generate three different types of control laws, namely, the state feedback, the full-order measurement feedback and the reduced-order measurement feedback controllers. The usage and design procedure of the toolkit are illustrated by a nonlinear benchmark problem, a practical example on the design of a hard disk drive servo system, and a numerical example for tracking general target references.

2 Theoretical formulation

We recall in this section the theory of the CNF control for plants with nonlinearities and disturbances. To be precise, we consider a given plant characterized by

$$\dot{q} = f(q, \ell(x), h), \quad q(0) = q_0, \tag{1}$$

$$\dot{x} = A\,x + B\,\mathrm{sat}\,(g(y) + u) + Ew, \quad x(0) = x_0, \tag{2}$$

$$y = C_1\,x, \tag{3}$$

$$h = C_2\, x + D_2\, \text{sat}(g(y) + u), \tag{4}$$

where $(q, x) \in \mathbb{R}^m \times \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}^p$, $h \in \mathbb{R}$, and $w \in \mathbb{R}$ are respectively the state, control input, measurement output, controlled output, and disturbance input of the system; $f(\cdot,\cdot,\cdot)$ is a smooth (C^∞) function characterizing the system nonlinear dynamics, $\ell(x)$ represents some appropriate elements of x, $g(y)$ is a nonlinear function representing the plant nonlinearities; and A, B, C_1, C_2, D_2, and E are appropriate dimensional constant matrices. The function sat: $\mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\text{sat}(u) = \begin{cases} -u_{\max}, & \text{if } u < -u_{\max}, \\ u, & \text{if } -u_{\max} \le u \le u_{\max}, \\ u_{\max}, & \text{if } u > u_{\max}, \end{cases}$$

where $u_{\max}$ is the saturation level of the input. Note that when the system nonlinear dynamics $\dot{q} = f(q, \ell(x), h)$ is nonexistent, (1)–(4) are reduced to a linear system with input saturation. The following assumptions are made:

1. (A, B) is stabilizable, and (A, C_1) is detectable;
2. (A, B, C_2, D_2) has no invariant zero at $\text{Re}(s) \ge 0$;
3. w is unknown constant disturbance;
4. h is measurable (h is part of the measurement output);
5. There exists a smooth positive definite function $V(q)$ and class K_∞ functions α_1 and α_2 such that

$$\alpha_1(\|q\|) \le V(q) \le \alpha_2(\|q\|), \quad \frac{\partial V(q)}{\partial q} f(q, 0, r) < 0,$$

for all $q \in \Omega \subseteq \mathbb{R}^m$, where Ω is a compact set containing the origin, and r is the target reference.

Assumptions 1–4 are standard for tracking control problems. Assumption 5 implies that the system nonlinear dynamics is asymptotically stable. We summarize in the following the step-by-step design procedure for constructing a CNF control law for the state feedback case, that is, for the case when $y = x$.

STEP 1: Given a target reference r, we define

$$\dot{z} = \kappa_{\rm i} e = \kappa_{\rm i}(h - r),$$

where $\kappa_{\rm i}$ is the integration gain, and obtain the auxiliary system

$$\dot{\bar{x}} = \bar{A}\, \bar{x} + \bar{B}\, \text{sat}(g(y) + u) + \bar{B}_{\rm r}\, r + \bar{E}\, w, \tag{5}$$

$$\bar{y} = \bar{C}_1\, \bar{x}, \quad h = \bar{C}_2\, \bar{x} + D_2\, \text{sat}(g(y) + u), \tag{6}$$

where

$$\bar{x} = \begin{pmatrix} z \\ x \end{pmatrix}, \quad \bar{x}_0 = \begin{pmatrix} 0 \\ x_0 \end{pmatrix}, \quad \bar{y} = \begin{pmatrix} z \\ y \end{pmatrix},$$

$$\bar{A} = \begin{bmatrix} 0 & \kappa_{\rm i} C_2 \\ 0 & A \end{bmatrix}, \ \bar{B} = \begin{bmatrix} \kappa_{\rm i} D_2 \\ B \end{bmatrix}, \ \bar{B}_{\rm r} = \begin{bmatrix} -\kappa_{\rm i} \\ 0 \end{bmatrix}, \ \bar{E} = \begin{bmatrix} 0 \\ E \end{bmatrix},$$

$$\bar{C}_1 = \begin{bmatrix} 1 & 0 \\ 0 & C_1 \end{bmatrix}, \ \ \bar{C}_2 = [0 \quad C_2] \, .$$

Note that this step is not necessary if (2) does not have external disturbances.

STEP 2: Design a state feedback gain matrix F such that 1) $\bar{A}+\bar{B}F$ is an asymptotically stable matrix, and 2) the closed-loop system $(\bar{C}_2 + D_2 F)(sI - \bar{A} - \bar{B}F)^{-1}\bar{B} + D_2$ has certain desired properties. Let us partition $F = [F_{\rm z} \quad F_{\rm x}]$ in conformity with z and x. The general guideline in designing such an F is to place the closed-loop pole of $\bar{A} + \bar{B}F$ corresponding to the integration mode z to be sufficiently closer to the imaginary axis compared to the remaining eigenvalues, which implies that $F_{\rm z}$ is a relatively small scalar. The remaining closed-loop poles of $\bar{A}+\bar{B}F$ are placed to have a dominant pair with a small damping ratio, which in turn would yield a fast rise time in the closed-loop system response.

Next, according to the property of the target reference r, the linear feedback control law is designed for two cases.

Case 1: For a set-point target reference r, the linear feedback control law is given by

$$u_{\rm L} = F\bar{x} + G\, r,$$

where

$$G = [D_2 - (C_2 + D_2 F_{\rm x})(A + BF_{\rm x})^{-1} B]^{-1}.$$

Further, we define

$$G_{\rm e} = \begin{bmatrix} 0 \\ -(A + BF_{\rm x})^{-1} BG \end{bmatrix} \quad \text{and} \quad \bar{x}_{\rm e} = G_{\rm e}\, r.$$

Case 2: When the target reference r is a time-varying function (for example, a sinusoidal function), a reference generator is first designed to produce the target signal, $r(t)$. The reference generator is constructed based on the nominal plant and is characterized by (see [8])

$$\Sigma_{\rm aux} : \begin{cases} \dot{x}_{\rm e} = A\, x_{\rm e} + B\, u_{\rm e}, \ \ x_{\rm e}(0) = x_{\rm e0} \\ r = C_2\, x_{\rm e} + D_2\, u_{\rm e}, \end{cases} \tag{7}$$

where $x_{\rm e} \in \mathbb{R}^n$, $u_{\rm e} \in \mathbb{R}$ and $r \in \mathbb{R}$ are respectively the state, control input and output of the auxiliary system, $\Sigma_{\rm aux}$. The r is the reference produced to be tracked. A, B,C_2 and D_2 are appropriate dimensional constant matrices of the plant model.

For the auxiliary system $\Sigma_{\rm aux}$, a linear control law is designed as follows,

$$u_{\rm e} = F_{\rm e} x_{\rm e} + r_{\rm s}, \tag{8}$$

where $F_{\rm e}$ is the feedback gain matrix and $r_{\rm s}$ is an external signal source. The auxiliary system (7) can generate arbitrary type of output signal, such as the step signal, ramp signal, and sinusoidal signal by designing $F_{\rm e}$, choosing $r_{\rm s}$, and setting the initial value $x_{\rm e0}$.

The designs of reference generators for a polynomial signal and a single sinusoidal signal have been presented in [8]. For a more general or transcendental signal $r(t)$, $F_{\rm e}$ is chosen such that the eigenvalues of $A+BF_{\rm e}$ are all zero, the initial value $x_{\rm e0}$ is set as

$$x_{\rm e0} = \begin{bmatrix} C_2 + D_2 F_{\rm e} \\ (C_2 + D_2 F_{\rm e})(A + BF_{\rm e}) \\ \vdots \\ (C_2 + D_2 F_{\rm e})(A + BF_{\rm e})^{n-1} \end{bmatrix}^{-1} \begin{pmatrix} r(0) \\ \dot{r}(0) \\ \vdots \\ r^{(n-1)}(0) \end{pmatrix}. \tag{9}$$

and a nonzero external signal $r_{\rm s}$ is chosen as,

$$r_{\rm s}(t) = \frac{1}{N(s)} \times r^{(n)}(t), \tag{10}$$

where $N(s)$ is the numerator of the transfer function $(C_2 + D_2 F_{\rm e})[sI - (A + BF_{\rm e})]^{-1} B + D_2$.

Next, define

$$\bar{x}_{\rm e} = \begin{bmatrix} 0 \\ x_{\rm e} \end{bmatrix}.$$

Now, the linear feedback control law for tracking a general reference $r(t)$ can be given by

$$u_{\rm L} = u_{\rm e} + F(\bar{x} - \bar{x}_{\rm e}). \tag{11}$$

Here we would like to note that the reference generator is used to construct the target state, $x_{\rm e}$, which is an important variable in the CNF control law. For the circumstance when $r(t)$ can be generated by an autonomous linear exosystem, similar ideas have been adopted to reformulate the tracking problem into an equivalent output regulation problem by augmenting the exosystem (see *e.g.*, [12, 14, 15]).

STEP 3: Given a positive definite symmetric matrix $W \in \mathbb{R}^{(n+1)\times(n+1)}$, we solve

$$(\bar{A} + \bar{B}F)'P + P(\bar{A} + \bar{B}F) = -W,$$

for $P > 0$. Such a solution exists as $(\bar{A} + \bar{B}F)$ is asymptotically stable. The nonlinear feedback portion of the CNF control law $u_{\rm N}$ is then given by

$$u_{\rm N} = \rho(e)\bar{B}'P(\bar{x} - \bar{x}_{\rm e}),$$

where $\rho(e)$, with $e = h - r$ being the tracking error, is a smooth, nonpositive and nondecreasing function of $|e|$, to be used to gradually change the system closed-loop damping ratio to improve tracking performance.

We note that the design parameter W is generally used to tune the desired closed-loop poles at the steady state situation. Several forms of nonlinear function $\rho(e)$ have been suggested in [3, 5, 6, 9].

Especially, the one proposed in [9] is a scaled nonlinear function with a better performance robustness to variation of tracking targets

$$\rho(e) = -\beta \mathrm{e}^{-\alpha\alpha_0|e|} \tag{12}$$

with

$$\alpha_0 = \begin{cases} \dfrac{1}{|e(0)|}, & \text{if } e(0) \neq 0, \\ 1, & \text{if } e(0) = 0, \end{cases}$$

in which α and β are positive scalars that can be tuned to improve tracking performances. The detailed interpretations of these parameters can be found in [5] and [6]. Since parameter α and β have a significant impact on the transient performance, it is sensible to auto-tune the values of α and β, to optimize some performance criterion, $e.g.$, the integral of time multiplied absolute-value of error (ITAE),

$$I(\alpha, \beta) = \int_0^\infty t|e(t)|dt \tag{13}$$

which makes a balanced trade-off between overshoot and quick response in the transient performance. An auto-tuning procedure with the Hooke-Jeeves method is proposed in [9].

STEP 4: The linear feedback control law and nonlinear feedback portion derived in the previous steps together with a nonlinearity compensation are now combined to form the CNF control law

$$u = u_{\text{pre}} + u_{\text{L}} + u_{\text{N}} = -g(y) + \begin{cases} F\bar{x} + G\,r + \rho(e)\bar{B}'P(\bar{x} - \bar{x}_{\text{e}}), & \text{for set-point } r, \\ u_{\text{e}} + F(\bar{x} - \bar{x}_{\text{e}}) + \rho(e)\bar{B}'P(\bar{x} - \bar{x}_{\text{e}}), & \text{for general } r(t). \end{cases} \tag{14}$$

For the measurement feedback case, the state variable x, contained in $\bar{x}$ of the CNF state feedback law (14) is replaced by an estimation using either a full-order or reduced-order observer. The detailed derivation is rather involved. We refer interested readers to [6, 8] (see also [13]) for details.

3 Software framework and user guide

The CNF control toolkit is developed under MATLAB together with SIMULINK. The toolkit fully utilizes the graphical user interface (GUI) resources of MATLAB and provides a user-friendly graphical interface. The main interface of the toolkit consists of three panels, the panel for conducting simulation, the panel for setting up system data and the panel for specifying an appropriate controller. We illustrate the design procedure using our CNF control toolkit in the following:

STAGE 1: INITIALIZATION. Once the toolkit is properly executed in MATLAB, a main panel as shown in Fig. 2 will be generated in a popup window. Users have to first enter required data for a system to be controlled and then specify an appropriate controller structure before running simulation on this panel.

STAGE 2: PLANT MODEL SETUP. To enter system data, users need to click on the box labeled with `PLANT` to open the plant model setup panel as shown in Fig. 3. In addition to the state space model of (1)–(4), the toolkit also allows users to specify resonance modes of the plant as well on this panel.

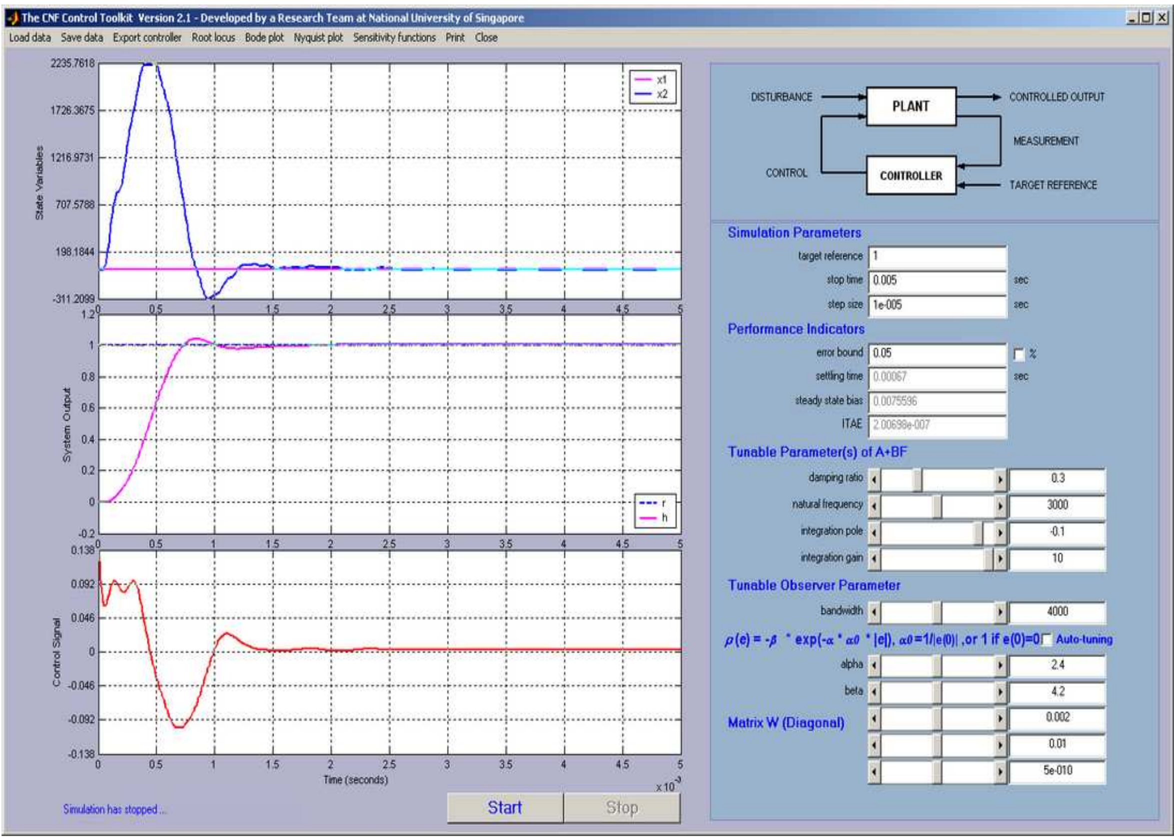

Figure 2: The main panel of the CNF control toolkit. This is the main GUI of the toolkit, on which users can tune controller parameters, run simulation setup and view time-domain responses.

We note that high frequency resonance modes are existent almost in all mechanical systems. Because of the complexity of resonance modes, they are generally ignored or simplified in the controller design stage. However, these resonance modes have to be included in the simulation and evaluation of the overall control system design.

Each time a plant model is keyed in or modified on the panel, the toolkit automatically runs a checkup on the system stabilizability, detectability, invertibility, and other requirements. For a nonlinear system, the toolkit also checks the stability of its nonlinear dynamics. Users will be warned if the solvability conditions for the CNF tracking control are not satisfied and users have to revise the model before proceeding to controller design.

STAGE 3: CONTROLLER SETUP. As the core of this toolkit, the CNF controller design is to be proceeded in a configurable and convenient fashion. A controller setup panel as shown in Fig. 4 is opened when the user activates the box marked with `CONTROLLER` in the main panel. This panel carries a block diagram for an adjustable controller configuration, which automatically refreshes when the user makes any change or re-selection on the controller structure. Users need to decide a controller structure first before proceeding to specify the corresponding controller parameters.

The following options are available for the controller in the CNF control toolkit:

1. If (2)–(4) have some nonlinearities, users can choose the pre-compensation option and enter an appropriate nonlinear function to cancel as many nonlinearities as possible.

2. If (2) has some unknown constant disturbances or other types of disturbances, users can select a controller structure with integrator to remove the steady state bias.

3. If (1)–(4) are the nominal model of a noisy plant, which has high frequency resonance modes, users might enter a pre-designed lowpass or notch filter to minimize their effects on the overall performance.

4. Based on the properties of (1)–(4) and personal interest, users can then choose a controller with either one of the following options:

   (a) State feedback;

   (b) Full order measurement feedback;

   (c) Reduced order measurement feedback.

When it comes to the design of the state feedback gain F, users have three choices. Users can either specify an explicit matrix (obtained through any other design methodologies such as H_∞ control) for F, or employ the H_2 control technique (provided on the panel), or use the pole placement method characterized by the damping ratio and natural frequency of the dominant poles as well as integration pole and gain (if

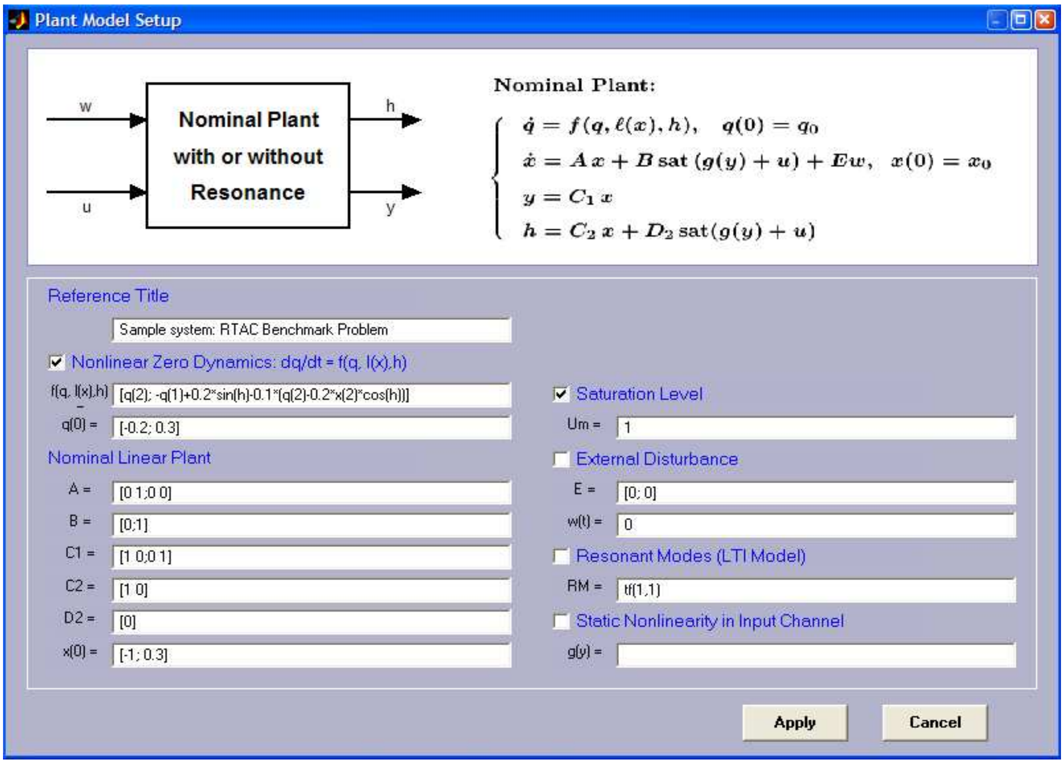

Figure 3: The panel for the plant model setup. In this panel users define their plant model, including nominal model, plant nonlinearities, resonance modes and disturbances, if applicable.

applicable). For the pole placement method, the remaining closed-loop poles are placed three times faster than the dominant pair with a Butterworth pattern.

In the case of measurement feedback control, users also have three choices for the design of the observer gain K. Users can directly enter an pre-designed solution for K, or design it online using the H_2 control technique, or using the pole placement method to organize the observer poles into the pattern of a Butterworth filter with an appropriate bandwidth. In any cases, the corresponding parameters can be further tuned on the main panel to obtain a satisfactory performance for the overall system.

STAGE 4: DESIGN AND SIMULATION. Once the plant model setup and the controller setup are completed, users can then specify directly on the panel the simulation parameters, such as the type and parameters for the target reference, the duration of simulation and the step size. Users can also define the tracking performance indicator and obtain the result for settling time and steady state bias of the controlled output response. The integral of time multiplied absolute-value of error (ITAE) is also provided as a comprehensive performance index. For example, in Fig. 2, the settling time is defined as the time when the controlled output of the closed-loop system enters the neighborhood of ± 0.05 of the target reference. Alternatively, one can define such a neighborhood in terms of the percentage of final target instead of the absolute error bound.

As mentioned in the previous section, the CNF controller consists of two parts, a linear part and a

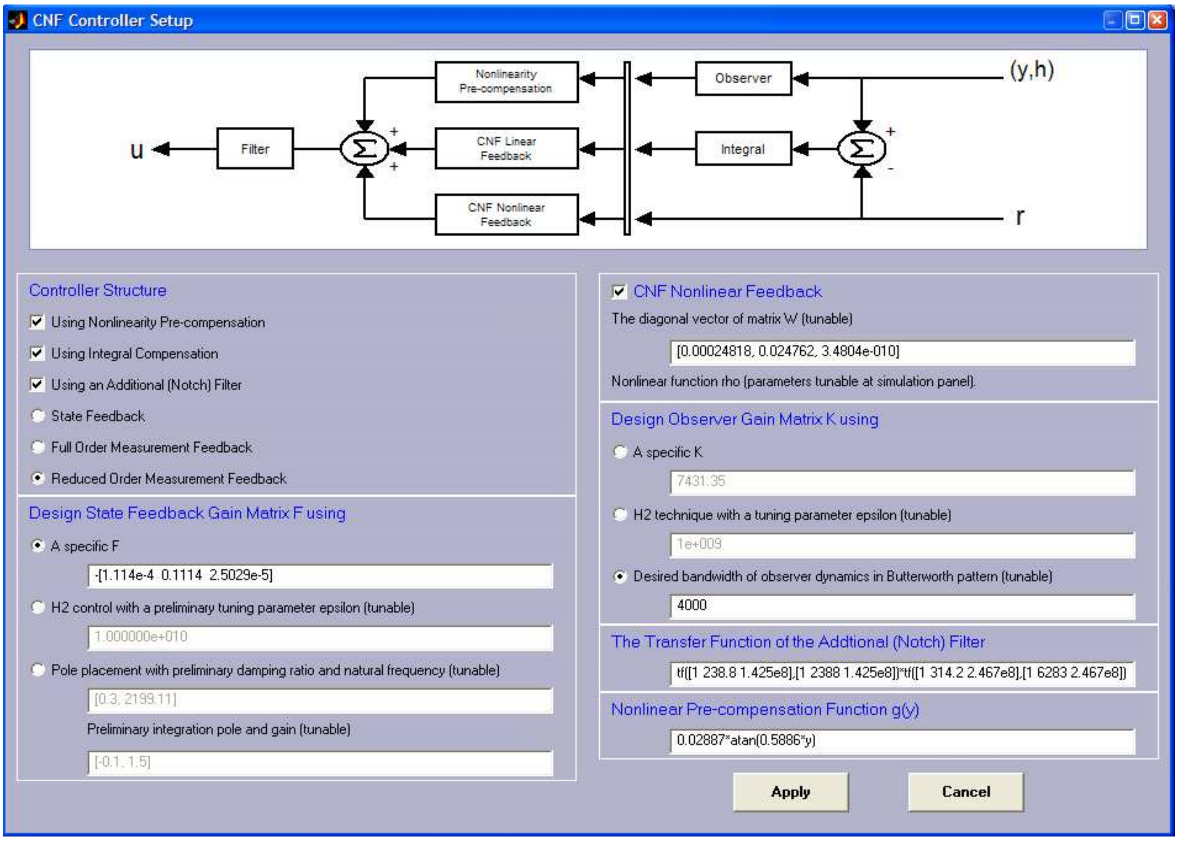

Figure 4: The panel for the CNF controller setup. In this panel users choose the structure of controller and enter initial values for controller parameters, which can be further tuned in the main panel.

nonlinear part. On the main panel, users are able to tune the properties of the linear part by selecting appropriate values of the damping ratio and natural frequency of the dominant modes of the linear state feedback dynamical matrix $A + BF$. The remaining eigenvalues of $A + BF$ are placed 3 times faster than the dominant modes with a Butterworth pattern. Such an arrangement is purely for the simplification of the control system design. The pole corresponding to the integrator part, if applicable, can be tuned on this main panel as well. Alternatively, users are allowed to apply the H_2 design with a tunable parameter ε, or directly specify a state feedback gain matrix (obtained using any design method) in the controller setup panel without any tunable parameters.

Design parameters W and $\rho(e)$ for the nonlinear part of the CNF controller can also be tuned online on the main panel. In particular, the parameters α and β for the nonlinear function $\rho(e)$ of (12) can be either adjusted by users manually, or auto-tuned by the toolkit to minimize the ITAE using the Hooke-Jeeves algorithm (Users just need to tick the checkbox named "Auto-tuning" before starting simulation). In the current version of the CNF control toolkit, the design parameter W is restricted to a diagonal matrix for the simplicity of implementation.

There are three windows on the main panel for displaying the system state variables, the controlled output response and the control input signal, together with a block diagram showing the structure of the overall control system. Using the right button of the computer mouse to click on the window displaying the state variables, output response and control signal, users are prompted by a small text window showing options to re-draw the plots on a new popup window or export the simulation data to the MATLAB workspace.

Finally, we have also implemented the following commands and functions on the main panel for saving and loading data as well as for evaluating the frequency domain properties of the overall control system:

1. Load Data: This function is used to load data previously saved in the CNF control toolkit.

2. Save Data: This command is to save the system and controller data for future use.

3. Export Controller: This function is to export the data of the CNF controller obtained to the MATLAB workspace. The controller data are given by

$$\dot{z} = \kappa_{\rm i}(h - r), \tag{15}$$

$$\dot{x}_{\rm v} = A_{\rm cmp}\, x_{\rm v} + B_{\rm ycmp}\, y + B_{\rm ucmp}\, \mathsf{sat}(\bar{u}), \tag{16}$$

$$\hat{x} = C_{\rm cmp}\, x_{\rm v} + D_{\rm cmp}\, y, \tag{17}$$

$$\bar{u} = \begin{cases} F\begin{pmatrix} z \\ \hat{x} \end{pmatrix} + G\,r + \rho(e)F_{\mathrm{n}}\left[\begin{pmatrix} z \\ \hat{x} \end{pmatrix} - G_{\mathrm{e}}\,r\right], & \text{for set-point } r, \\ F_{\mathrm{e}}x_{\mathrm{e}} + r_{\mathrm{s}} + F\begin{pmatrix} z \\ \hat{x} - x_{\mathrm{e}} \end{pmatrix} + \rho(e)F_{\mathrm{n}}\begin{pmatrix} z \\ \hat{x} - x_{\mathrm{e}} \end{pmatrix}, & \text{for general } r(t). \end{cases} \tag{18}$$

$$\begin{cases} \dot{x}_{\mathrm{e}} = (A + BF_{\mathrm{e}})\,x_{\mathrm{e}} + B\,r_{\mathrm{s}}, \quad x_{\mathrm{e}}(0) = x_{\mathrm{e}0} \\ r = (C_2 + D_2F_{\mathrm{e}})\,x_{\mathrm{e}} + D_2\,r_{\mathrm{s}}, \end{cases} \tag{19}$$

$$r_{\mathrm{s}}(t) = H_{\mathrm{r}}(s) \times r_{\mathrm{n}}(t) \tag{20}$$

$$u = H(s)\bar{u} - g(y), \tag{21}$$

where A_{cmp}, B_{ycmp}, B_{ucmp}, C_{cmp}, D_{cmp}, F, F_{n}, G, G_{e},F_{e},$x_{\mathrm{e}0}$ (if existent) are constant vectors or matrices, and $\rho(e)$ and $g(y)$ are scalar functions. The source signal $r_{\mathrm{s}}(t)$ is generated by passing the nth-order differential signal $r_{\mathrm{n}}(t)$ of the target reference $r(t)$ through a filter with the transfer function $H_{\mathrm{r}}(s)$. If a filter is used to reduce the effects of noise or high frequency resonance modes of a physical plant, $H(s)$ will represent the transfer function of such a filter. All these parameters can be saved under a structured workspace variable (specified by the user) in the MATLAB command window.

4. Root Locus: This function is to generate the root locus of the control system with the CNF controller with respect to the change of the nonlinear function $\rho(e)$.

5. Bode Plot: This function is to generate the Bode magnitude and phase responses of the open-loop system comprising the plant and the controller in the steady state situation when the nonlinear gain is converging to a constant. This function can be used to evaluate the frequency domain properties of the control system, such as gain and phase margins.

6. Nyquist Plot: Similar to the Bode plot given above. Note that for both the Bode and Nyquist plots, users are allowed to specify a frequency range of interest.

7. Sensitivity Functions: This function is to plot the sensitivity and complementary sensitivity functions of the overall design with a pre-specified frequency range.

8. Print: This function is to print the items shown on the main panel.

9. Close: This command is to close up the toolkit.

Note that all frequency domain responses are obtained by ignoring the system nonlinear dynamics, if any, and at the steady state situation when the nonlinear gain function $\rho(e)$ becomes a constant.

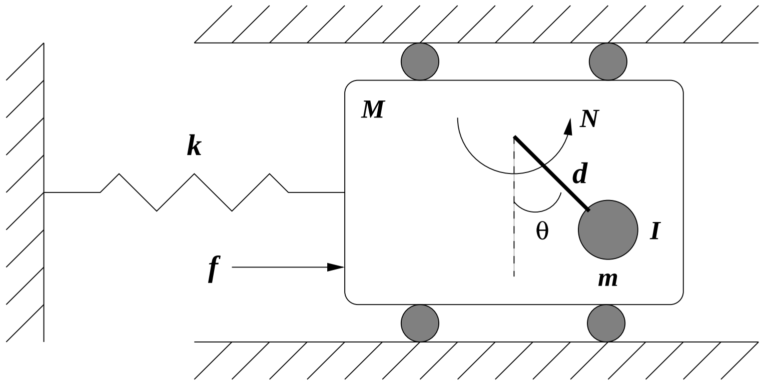

Figure 5: The rotational/translational actuator (RTAC) benchmark problem.

Although the CNF control toolkit is meant to design a composite nonlinear feedback controller, users have the option to choose only the linear portion of the CNF control law for their design. This option is particularly useful for the comparison of performances of the CNF controller and the linear controller.

4 Illustrative examples

We illustrate in this section the CNF control toolkit using two benchmark problems and a numerical example, one benchmark problem is on a rotational and translational actuator (RTAC) system of [10] (see also [11, 12]) and the other is on a hard disk drive (HDD) servo system given in [13].

Example 1 (RTAC System [10, 11]). We first consider the rotational/translational actuator (RTAC) system depicted in Fig. 5. M is the mass of a cart connected to a fixed wall by a linear spring of stiffness k. The proof-mass actuator attached to the cart has mass m and moment of inertia I about its center of mass, which is located at a distance d from the point about which the proof-mass rotates. The motion of the mechanical system (see [10, 11, 12]) is characterized by

$$\ddot{\xi} + b\dot{\xi} + \xi = \epsilon(\dot{\theta}^2 \sin\theta - \ddot{\theta}\cos\theta) + f, \tag{22}$$

$$\ddot{\theta} = -\epsilon\ddot{\xi}\cos\theta + v, \tag{23}$$

where ξ is the normalized displacement of the cart, θ is the angular position of the eccentric mass, f is the disturbance force, b is the coefficient of viscous friction for motion of the cart, and v is the normalized control input. The coupling term between the translational and rotational motion ϵ is given by

$$\epsilon = \frac{md}{\sqrt{(I + md^2)(M + m)}}. \tag{24}$$

Assuming the disturbance $f = 0$ and applying the change in coordinates

$$q_1 = \xi + \epsilon\sin\theta, \quad q_2 = \dot{\xi} + \epsilon\dot{\theta}\cos\theta, \quad x_1 = \theta, \quad x_2 = \dot{\theta}, \tag{25}$$

we obtain a transformed system

$$\dot{q}_1 = q_2, \quad \dot{q}_2 = -q_1 - bq_2 + \epsilon(\sin x_1 + bx_2 \cos x_1), \tag{26}$$

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = u, \tag{27}$$

where

$$\begin{aligned} u = {} & \frac{\epsilon \cos x_1 \left[q_1 - (1+x_2^2)\epsilon \sin x_1\right]}{1-\epsilon^2 \cos^2 x_1} + \frac{v}{1-\epsilon^2 \cos^2 x_1} \\ & + \frac{b\epsilon \cos x_1 (q_2 - \epsilon x_2 \cos x_1)}{1-\epsilon^2 \cos^2 x_1}. \end{aligned} \tag{28}$$

Our objective is to design a state feedback control law for the RTAC system to regulate the angular position of the proof body $x_1 = \theta$ to zero quickly without any overshoot. The problem can be nicely formulated into the framework of the CNF design with

$$\dot{q} = f(q, x_2, h), \tag{29}$$

$$\dot{x} = Ax + B\,\mathrm{sat}(u), \quad y = x, \quad h = C_2 x, \tag{30}$$

where

$$q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{31}$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_2 = [1 \quad 0], \tag{32}$$

$$f(q, x_2, h) = \begin{pmatrix} q_2 \\ -q_1 - bq_2 + \epsilon(\sin h + bx_2 \cos h) \end{pmatrix}. \tag{33}$$

We can easily verify that $\dot{q} = f(q, 0, 0)$ is an asymptotically stable system. For simulation purpose, we assume that $b = 0.1$, $\epsilon = 0.2$, $u_{\max} = 1$, and the initial conditions $q_1(0) = -0.2$, $q_2(0) = 0.3$, $x_1(0) = -1$, and $x_2(0) = 0.3$. Using our CNF toolkit with initial damping ratio of 0.3 and natural frequency of 2 rad/sec together with an identity matrix W, we obtain a CNF state feedback control law

$$u = -[4 \quad 1.2]\,x + 4r + \rho(e)\,[0.125 \quad 0.5208] \left[x - \begin{pmatrix} 1 \\ 0 \end{pmatrix} r \right] \tag{34}$$

where $\rho(e) = -9\mathrm{e}^{-2.5|h-r|}$ and $r = 0$. The simulation result in Fig. 6 shows that the angular position of the proof body is settled from -1 to 0 very quickly with almost no overshoot.

Example 2 (HDD Servo System [13]). Next, we study a benchmark problem on an HDD servo system design given in [13]. The typical structure of HDDs with a voice-coil-motor (VCM) actuator is shown in Fig. 7. The rotating disks coated with a thin magnetic layer or recording medium are written with data that are arranged in concentric circles or tracks. Data are read or written with a read/write (R/W) head, which consists of a small horseshoe-shaped electromagnet. The dynamics model of an HDD servo system with VCM actuator can be illustrated by a block diagram as in Fig. 8. For an HDD servo system,

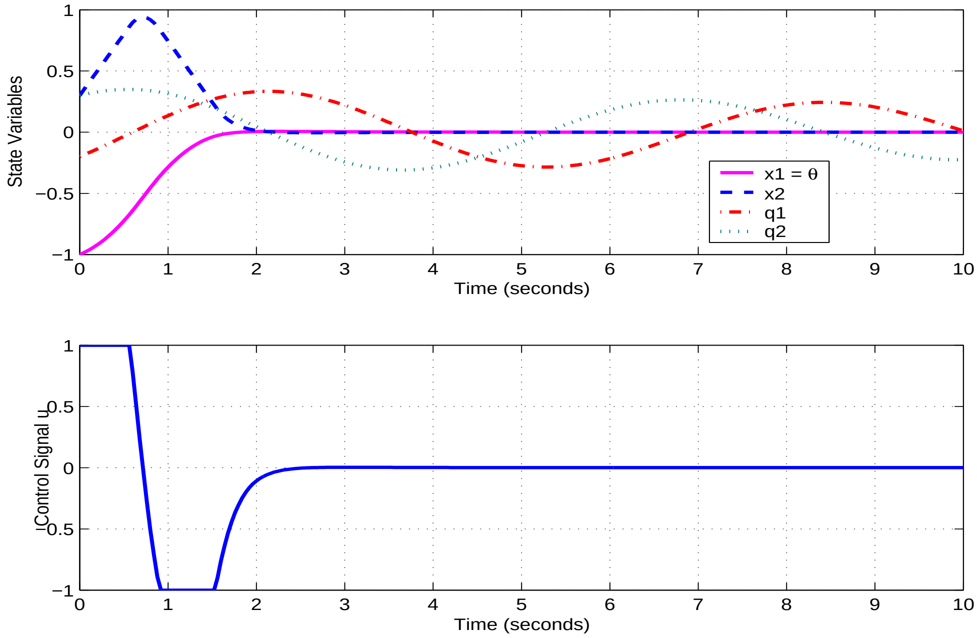

Figure 6: Response of state variables and control signal of the RTAC benchmark problem. The controlled output θ has a 2% settling time of 1.56 seconds with very little overshoot.

the task of its controller is to move the actuator R/W head to a desired track as fast as possible and to maintain the R/W head as close as possible to the target track center when data are being read or written. To ensure reliable data reading and writing, it is required that the deviation of R/W head from the target track center should not exceed 5% of track pitch.

The nominal plant of the HDD VCM actuator is characterized by the following second-order system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 6.4013\times 10^7 \end{bmatrix} \Big(\mathrm{sat}(u) + w\Big), \quad \tilde{y} = [\,1 \quad 0\,]\, x, \tag{35}$$

where the state variable x consists of the displacement and velocity of the R/W head of the VCM actuator, u is the control input limited within ± 3 V, and w is an unknown disturbance with $|w| \leq 3$ mV. For simplicity and for simulation purpose, we assume that the unknown disturbance $w = -3$ mV. The system output y (in μm), which is also the only measurement available for control, is the displacement of the VCM R/W head and is given by

$$y = \left[\prod_{i=1}^{4} G_{\mathrm{r},i}(s) \right] \tilde{y}, \tag{36}$$

where the transfer functions of the resonance modes are

$$G_{\mathrm{r},1}(s) = \frac{0.912s^2 + 457.4s + 1.433(1+\delta)\times 10^8}{s^2 + 359.2s + 1.433(1+\delta)\times 10^8},$$

$$G_{\mathrm{r},2}(s) = \frac{0.7586s^2 + 962.2s + 2.491(1+\delta)\times 10^8}{s^2 + 789.1s + 2.491(1+\delta)\times 10^8},$$

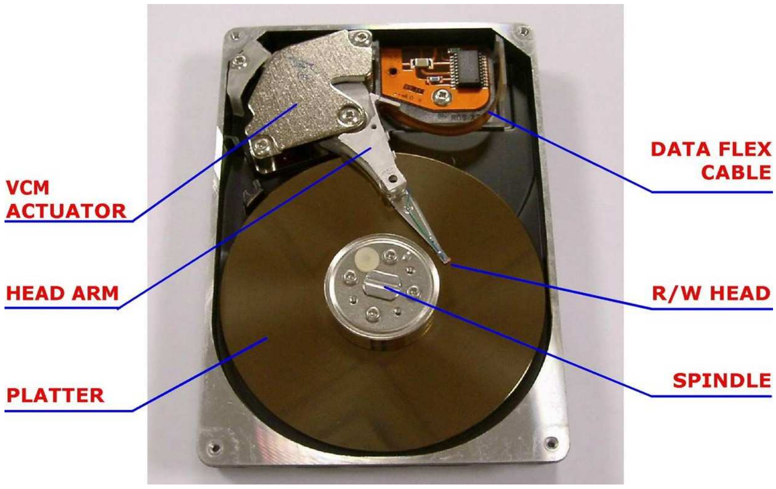

Figure 7: A typical HDD with a VCM actuator.

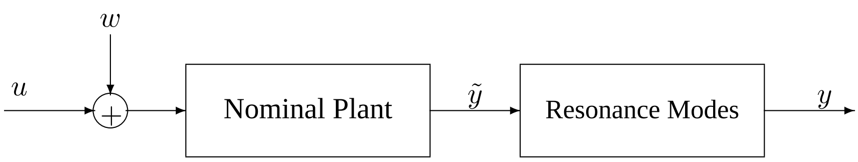

Figure 8: Block diagram of the dynamical model of HDD servo system with a VCM actuator. Resonance modes are major difficulties in pushing up HDD servo bandwidth.

$$G_{\mathrm{r},3}(s) = \frac{9.917(1+\delta)\times 10^8}{s^2 + 1575s + 9.917(1+\delta)\times 10^8},$$

$$G_{\mathrm{r},4}(s) = \frac{2.731(1+\delta)\times 10^9}{s^2 + 2613s + 2.731(1+\delta)\times 10^9},$$

and where $-20\% \le \delta \le 20\%$ represents the variation of the resonance modes of the actual actuators whose resonant dynamics are changing from time to time and also from disk to disk in a batch of million drives.

The problem is to design a controller to track a reference $r = 1$ μm and meet the following specifications:

1. the overshoot of the output response is less than 5%;
2. the steady state error is zero;
3. the gain margin and phase margin of the overall design are respectively greater than 6 dB and 30°; and
4. the maximum peaks of the sensitivity and complementary sensitivity functions are less than 6 dB.

Using the CNF control toolkit with few online adjustments on the design parameters, we manage to obtain a reduced-order CNF controller that yields a very satisfactory performance. The controller is given by

$$\begin{aligned}\begin{pmatrix}\dot{x}_\mathrm{i}\\ \dot{x}_\mathrm{v}\end{pmatrix} &= \begin{bmatrix}0 & 0\\ 0 & -4000\end{bmatrix}\begin{pmatrix}x_\mathrm{i}\\ x_\mathrm{v}\end{pmatrix} - \begin{bmatrix}-10\\ 1.6\times 10^7\end{bmatrix} y \\ &\quad + \begin{bmatrix}0\\ 6.4013\times 10^7\end{bmatrix}\mathrm{sat}(\tilde{u}) - \begin{bmatrix}10\\ 0\end{bmatrix} r,\end{aligned}$$

$$\begin{aligned}\tilde{u} &= \rho(e)\,[\,0.7113 \quad 0.037 \quad 2.9438\times 10^{-5}\,]\begin{pmatrix}x_\mathrm{i}\\ y-r\\ x_\mathrm{v}+4000y\end{pmatrix} \\ &\quad - [\,0.0014 \quad 0.1406 \quad 2.8121\times 10^{-5}\,]\begin{pmatrix}x_\mathrm{i}\\ y-r\\ x_\mathrm{v}+4000y\end{pmatrix},\end{aligned}$$

where $\rho(e) = -4.2\mathrm{e}^{-2.4|e|}$. Finally, the control signal to the VCM actuator is generated by $u = G_{\mathrm{notch}}(s)\cdot\tilde{u}$ with

$$\begin{aligned}G_{\mathrm{notch}}(s) &= \left(\frac{s^2 + 238.8s + 1.425\times 10^8}{s^2 + 2388s + 1.425\times 10^8}\right) \\ &\times \left(\frac{s^2 + 314.2s + 2.467\times 10^8}{s^2 + 6283s + 2.467\times 10^8}\right) \\ &\times \left(\frac{s^2 + 628.3s + 9.87\times 10^8}{s^2 + 12570s + 9.87\times 10^8}\right).\end{aligned} \tag{37}$$

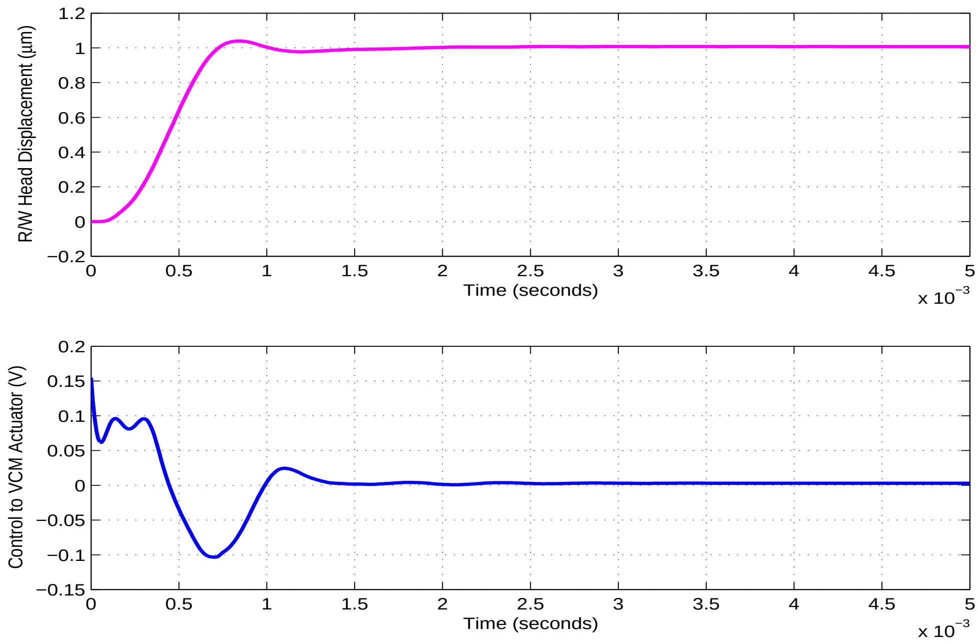

Figure 9: Controlled output response and control signal of the HDD servo system. The overall output response has a fast settling time, small overshoot, and zero steady state error.

The simulation results obtained with $\delta = 0$ given in Figs. 9 to 13 show that all design specifications are achieved. In particular, the resulting 5% settling time is 0.67 ms, the gain margin is 6.54 dB and the phase margin is 40.8°, and finally, the maximum values of the sensitivity and complementary sensitivity functions are less than 5 dB. The overall control system can still produce a satisfactory result and satisfy all the design specifications by varying the resonance modes with the value of δ changing from -20% to 20%.

Example 3 (Tracking of general target references). In this example we demonstrate the design of CNF controller for tracking general target references. The plant is a 3rd-order system characterized by

$$\dot{x} = Ax + B\,\mathrm{sat}(u), \quad y = h = Cx, \tag{38}$$

where

$$A = \begin{bmatrix} -1.5 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 1 \quad 0]\,, u_{\max} = 100, \quad x(0) = 0. \tag{39}$$

In the following, CNF controllers will be designed to track three target references respectively, *i.e.*, (a) Polynomial signal; (b) Sinusoidal signal; (c) Transcendental signal.

Case a: Tracking a polynomial signal $r(t) = 1 + 0.1t + 0.01t^2$.

Using the toolkit, we first specify the target reference as the given polynomial signal. We then choose the dominant poles of $A + BF$ as having a damping ratio of 0.3 and a natural frequency of 5 rad/sec, the observer bandwidth of 10 rad/sec, and the weighting matrix $W = diag(1, 1, 100)$. The tracking

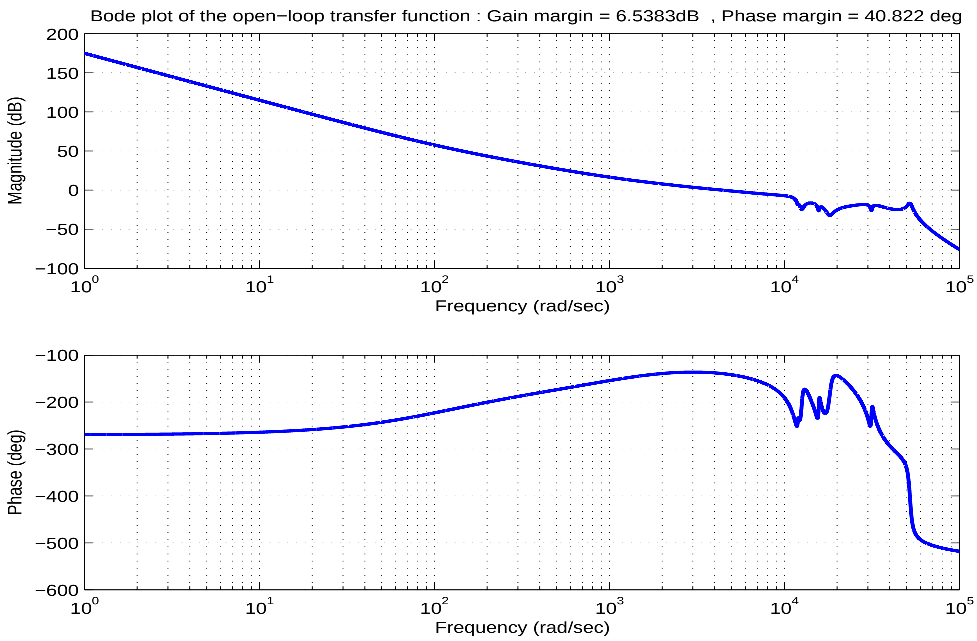

Figure 10: Bode plot of the open-loop transfer function of the HDD servo system with the CNF controller. The gain margin and phase margin obtained from the Bode plot are a primary indication on the robustness of the control system with respect to plant uncertainties and input disturbances.

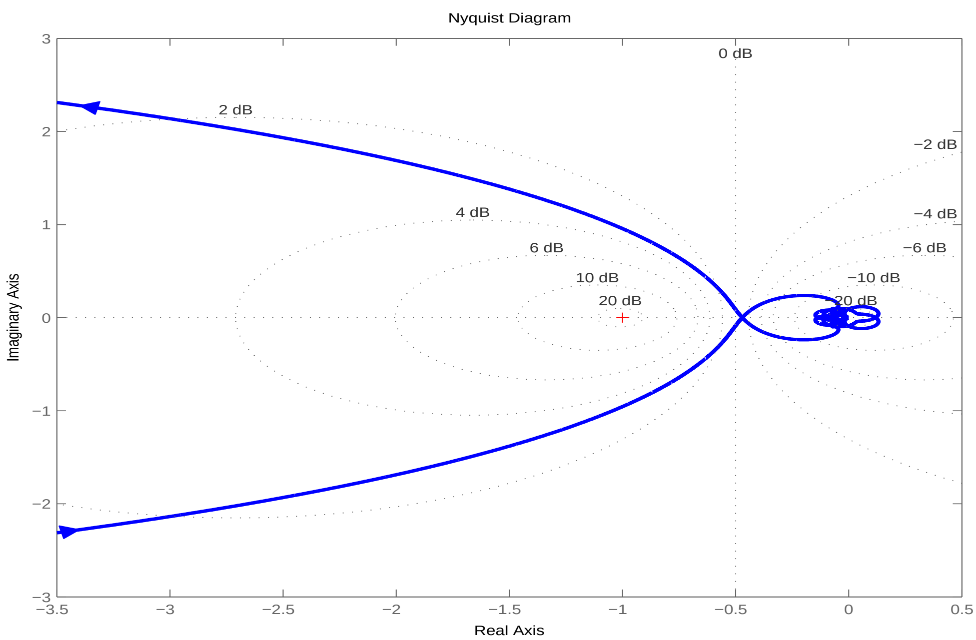

Figure 11: Nyquist plot of the open-loop transfer function of the HDD servo system with the CNF controller. Such a plot is particularly useful for double-checking the true stability margins for systems with high frequency resonance modes.

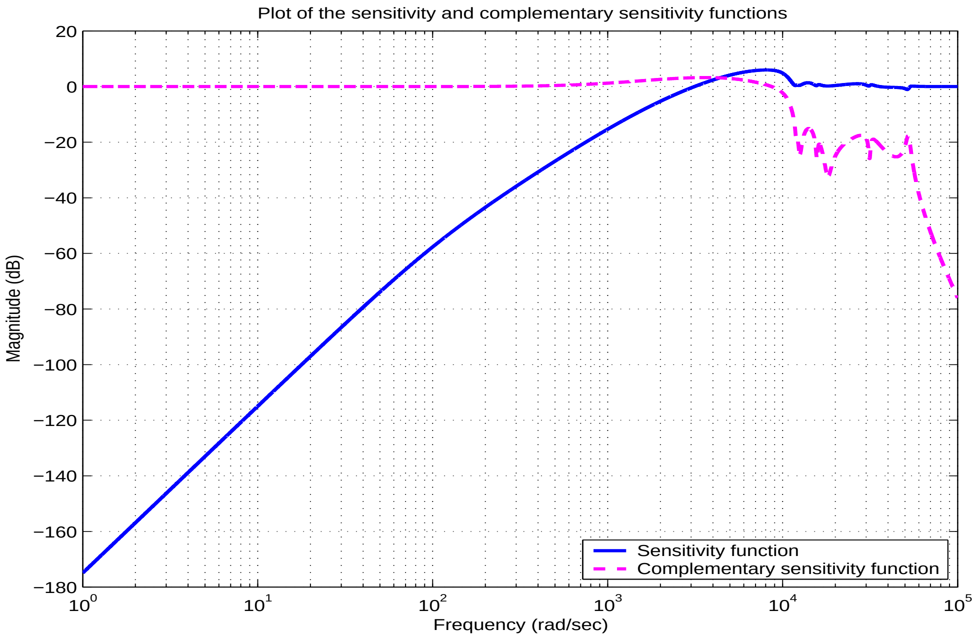

Figure 12: Sensitivity and complementary sensitivity functions of the HDD servo system with the CNF controller. These results can be used to examine the robustness of the control system with respect to output disturbances and measurement noise.

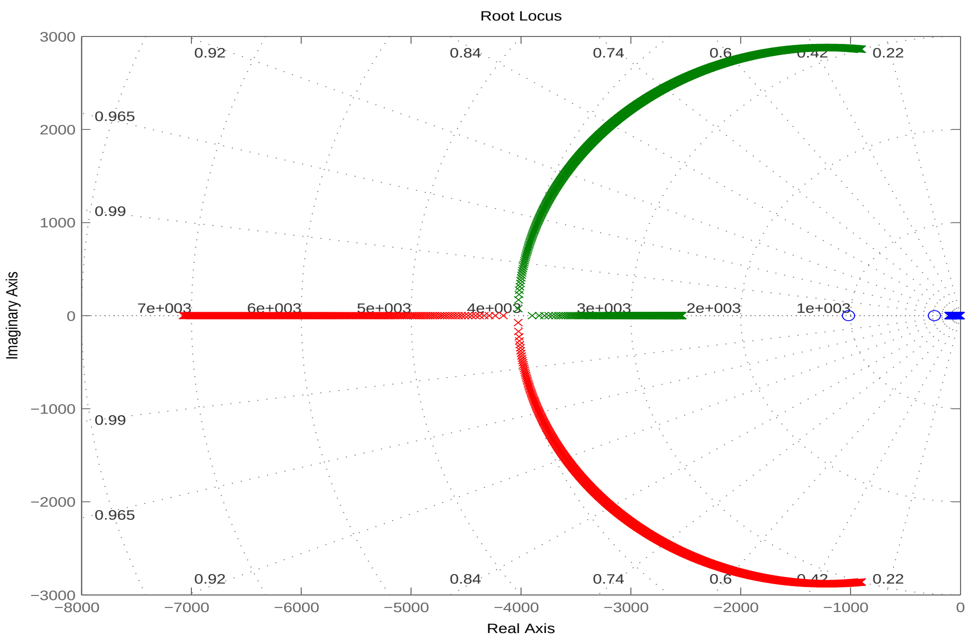

Figure 13: Root locus of the closed-loop HDD servo system versus the nonlinear function $\rho(e)$. This figure shows the evolution of the closed-loop poles as the value of nonlinear function $\rho(e)$ changes from 0 at the beginning to a negative constant at the end.

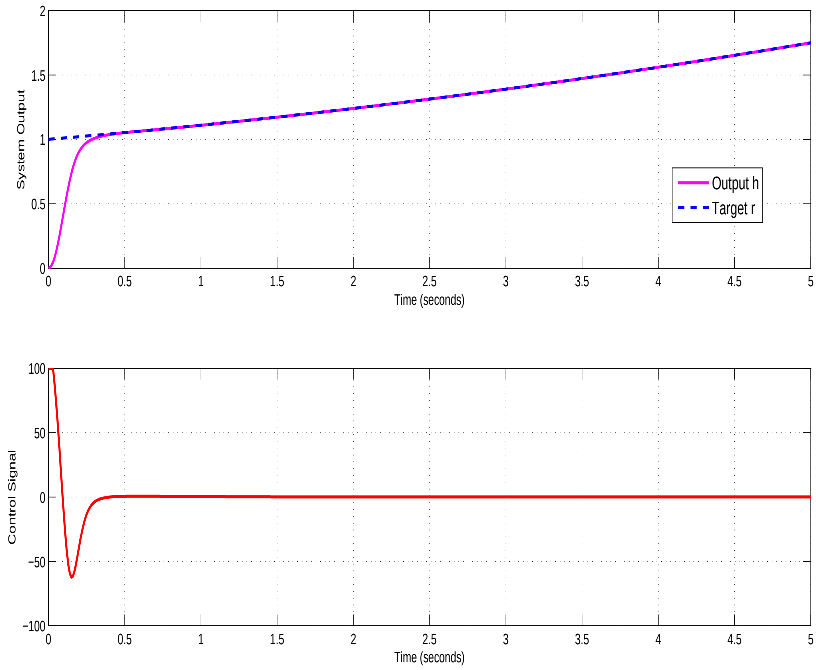

Figure 14: Tracking a polynomial signal.

controller, which consists of a reduced-order state observer, a reference generator, and a CNF control law, is given as follows,

$$\Sigma_{\rm o}: \left\{ \begin{array}{l} \dot{v} = \begin{bmatrix} 25.858 & 16.042 \\ -70.711 & 40 \end{bmatrix} v + \begin{bmatrix} 241.49 \\ -505.69 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathrm{sat}(u), \\ \hat{x} = \begin{bmatrix} -0.7071 & 0 \\ 0.7071 & 0 \\ 0 & 1 \end{bmatrix} v + \begin{bmatrix} 11.343 \\ -10.343 \\ 40 \end{bmatrix} y, \end{array} \right. \tag{40}$$

$$\Sigma_{\rm g.a}: \left\{ \begin{array}{l} \dot{x}_{\rm e} = \begin{bmatrix} -1.5 & 1 & 0 \\ 0 & 0 & 1 \\ 3.375 & -2.25 & 1.5 \end{bmatrix} x_{\rm e}, \quad x_{\rm e}(0) = \begin{bmatrix} 0.3853 \\ 0.6147 \\ 0.0632 \end{bmatrix} \\ r = [\,1 \quad 1 \quad 0\,]\, x_{\rm e}, \end{array} \right. \tag{41}$$

and

$$\begin{aligned} u \;=\; & [\,3.375 \quad -2.25 \quad 1.5\,]\, x_{\rm e} - [\,307.125 \quad 45.25 \quad 16.5\,]\,(\hat{x} - x_{\rm e}) \\ & + \rho(e)\,[\,-26.039 \quad 17.363 \quad 4.083\,]\,(\hat{x} - x_{\rm e}) \end{aligned}$$

where $\rho(e) = -9\mathrm{e}^{-2|e|}$ with $e = h - r$.

The simulation results given in Fig. 14 show that the controller can track the polynomial signal fast and accurately.

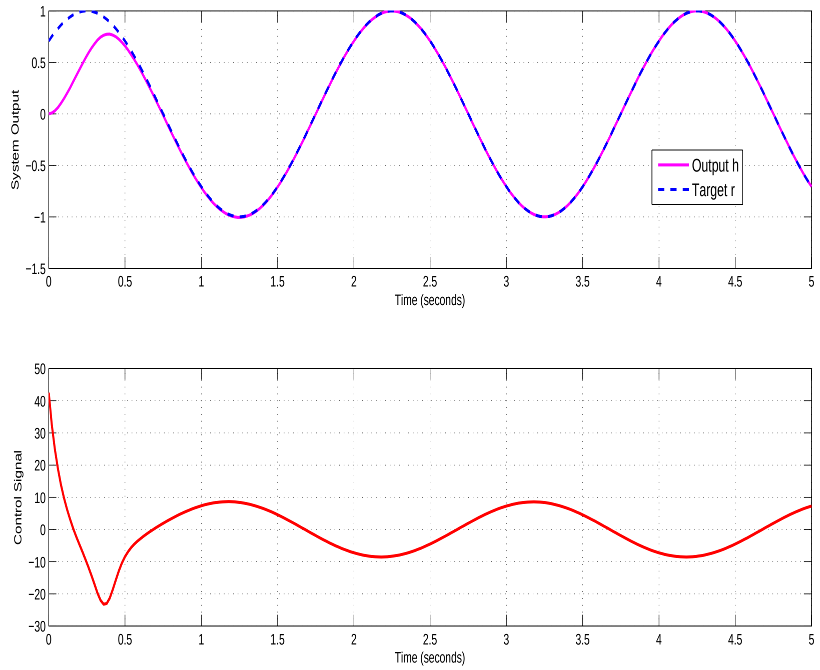

Figure 15: Tracking a sinusoidal signal.

Case b: Tracking a sinusoidal signal $r(t) = \sin(\pi t + \frac{\pi}{4})$.

For this case, we adopt the same values for the controller parameters except for those of the $\rho(e)$ function. As a result, the observer in (40) is also used here. However, the reference generator is a new one and is given by

$$\Sigma_{\mathrm{g.a}} : \begin{cases} \dot{x}_{\mathrm{e}} = \begin{bmatrix} -1.5 & 1 & 0 \\ 0 & 0 & 1 \\ 18.18 & -12.12 & 1.5 \end{bmatrix} x_{\mathrm{e}}, \quad x_{\mathrm{e}}(0) = \begin{bmatrix} -0.0281 \\ 0.7353 \\ 1.444 \end{bmatrix} \\ r = [\,1 \quad 1 \quad 0\,]\; x_{\mathrm{e}}, \end{cases} \tag{42}$$

The control law is

$$\begin{aligned} u \;&=\; [\,18.18 \quad -12.12 \quad 1.5\,]\; x_{\mathrm{e}} - [\,307.125 \quad 45.25 \quad 16.5\,]\,(\hat{x} - x_{\mathrm{e}}) \\ &\quad +\rho(e)\,[\,-26.039 \quad 17.363 \quad 4.083\,]\,(\hat{x} - x_{\mathrm{e}}) \end{aligned}$$

where $\rho(e) = -10\mathrm{e}^{-5\sqrt{2}|e|}$.

The simulation results are given in Fig. 15, and a good tracking performance is achieved.

Case c: Tracking a transcendental signal $r(t) = 0.2t + 0.5\mathrm{e}^{\sin(\pi t)}$.

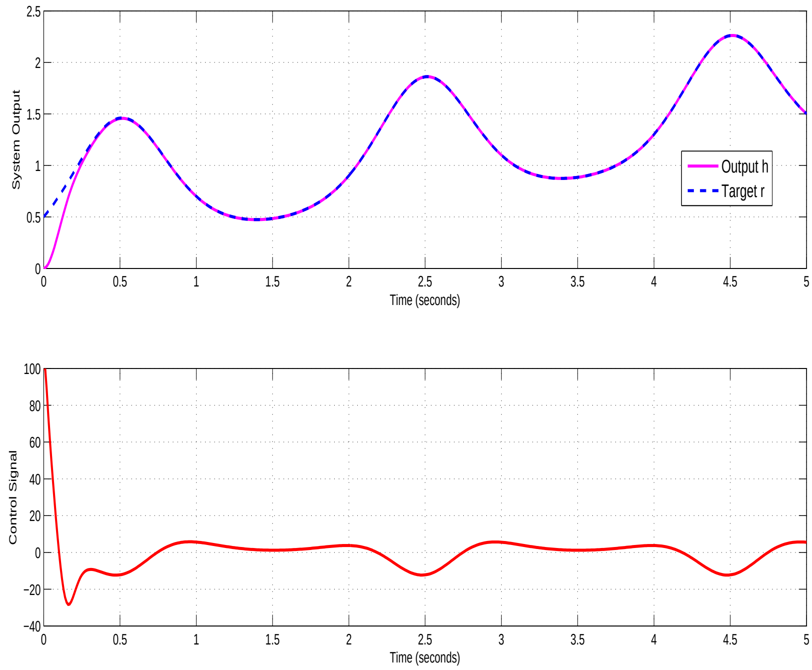

Figure 16: Tracking a transcendental signal.

For the given signal, the reference generator is designed as follows,

$$\Sigma_{\mathrm{g.c}}: \begin{cases} \dot{x}_{\mathrm{e}} = \begin{bmatrix} -1.5 & 1 & 0 \\ 0 & 0 & 1 \\ 3.375 & -2.25 & 1.5 \end{bmatrix} x_{\mathrm{e}} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r_{\mathrm{s}}, \quad x_{\mathrm{e}}(0) = \begin{bmatrix} 0.2325 \\ 0.2675 \\ 1.852 \end{bmatrix} \\ r = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} x_{\mathrm{e}}, \end{cases} \tag{43}$$

where

$$r_{\mathrm{s}}(t) = H_{\mathrm{r}}(s) \times r_{\mathrm{n}}(t), \ H_{\mathrm{r}}(s) = \frac{1}{s+2.5}, \ r_{\mathrm{n}}(t) = -\frac{\pi^3}{4}\sin(2\pi t)[\sin(\pi t)+3]\mathrm{e}^{\sin(\pi t)}. \tag{44}$$

Using the same observer of (40), the tracking control law is given by

$$\begin{aligned} u \;\; = \;\; & \begin{bmatrix} 3.375 & -2.25 & 1.5 \end{bmatrix} x_{\mathrm{e}} + r_{\mathrm{s}} - \begin{bmatrix} 307.125 & 45.25 & 16.5 \end{bmatrix} (\hat{x} - x_{\mathrm{e}}) \\ & + \rho(e) \begin{bmatrix} -26.039 & 17.363 & 4.083 \end{bmatrix} (\hat{x} - x_{\mathrm{e}}) \end{aligned}$$

where $\rho(e) = -9\mathrm{e}^{-4|e|}$ with $e = h - r$.

The simulation results given in Fig. 16 clearly show that the controller can track the transcendental signal with a satisfactory performance.

5 Concluding remarks

We have presented in this article a CNF control toolkit for SISO linear and nonlinear systems with actuator and other nonlinearities, external disturbances, and high frequency resonance. The toolkit is built on the MATLAB and SIMULINK environment with a user-friendly interface. All design parameters can be easily tuned online on the panels of the toolkit. The CNF control toolkit can be utilized to design servo systems that require fast target tracking and good robustness. The toolkit can be downloaded for use in research and academical purposes from `http://hdd.ece.nus.edu.sg/~bmchen/`. So far, the CNF control toolkit has been requested and distributed to more than 150 users in 38 countries including China, USA, Canada, Russia, Singapore, Germany, and France.

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