Intelligent Control of Mechatronic Systems

T. H. Lee
(in collaboration with colleagues & students)

Centre for Intelligent Control
Department of Electrical & Computer Engineering
National University of Singapore

eleleeth@nus.edu.sg
• Student projects in Mechatronics…
• Design - complexity of structure and control
• Simple “intelligence”
Mechatronics R&D projects: prototypes being refined in different stages

Fukuda & co-workers (Nagoya)
Lewis & co-workers (Arlington)
Moore & co-workers (Utah)
Grimble & co-workers (Strathclyde)
Tso & co-workers (Hong Kong)
C.W. de Silva & co-workers (UBC)
Ship welding robotic system
Robotic Telepresence System

Fukuda & co-workers (Nagoya)
Ang & co-workers (NUS)
Daniel & co-workers (Oxford)
J.H.Kim & co-workers (KAIST)
Other Systems (at CIC, NUS)

Gladiator Robots

Some of the other fun things our students work on…

Pole-balancing Robot
1. Introduction
2. Preliminary
3. Intelligent Control of Servo Mechanisms
4. Intelligent Control of Robotic Systems
5. Conclusions and Further Research
1. The synergistic combination of precision mechanical engineering, electronic control and systems thinking in the design of products and processes.


2. The combination of mechanical engineering and electronics, as used in the design and development of new manufacturing techniques. [From mecha(nics) + (elec)tronics]

The question of exactly what is "mechatronics" has been discussed for several years now and a reasonably consistent view has resulted.

There was a valuable workshop on Mechatronics, held at Stanford University in 1994, which brought together people who were actively involved in a range of courses, labs and programs in mechatronics.
Several quotes from presenters at the workshop on the definition of mechatronics are given below.

1. “Although there is not a universal accepted definition of Mechatronics, most definitions refer in some way to the integration of electronics and software into mechanical systems. Many also refer to the integration of these disciplines through out the design process.” (Edward Carryer, Stanford University)

2. “It (mechatronics) integrates the classical fields of mechanical engineering, electronic engineering and computer science/information technology at the design stage of a product or system.” (Memis, Loughborough, UK)
3. “Mechatronics is an application of the concept of concurrent engineering to the design of electromechanical systems. This design philosophy is exemplified by an interdisciplinary and integrated design approach where electrical, electronic, computer, and mechanical subsystems are simultaneously designed to function as an integrated single system.” (Charles Ume, Georgia Tech)

4. “Mechatronics, …, is the synergistic combination of mechanical engineering, electronics, control engineering and computer science. The key element in mechatronics is the integration of these areas through the design process.” (Kevin Craig, RPI)
1. “Intelligent controls respond to uncertainty and act to determine the best solutions.”

2. “Intelligent Control is a new cross scientific & technical field based on Artificial Intelligence & Automation Control etc.”

3. Intelligent controllers are envisioned emulating human mental faculties such as adaptation and learning, planning under large uncertainty, coping with large amounts of data etc in order to effectively control complex processes; ...
4. “Intelligent Control is a fusion of a number of research areas in Systems and Control, Computer Science, and Operations Research among others, coming together, merging and expanding in new directions and opening new horizons to address the problems of this challenging and promising area.”

http://robotics.ee.nus.edu.sg/tcic
5. “Intelligent control systems are typically able to perform one or more of the following functions to achieve autonomous behavior: planning actions at different levels of detail, emulation of human expert behavior, learning from past experiences, integrating sensor information, identifying changes that threaten the system behavior, such as failures, and reacting appropriately.”

http://robotics.ee.nus.edu.sg/tcic
A CONTROLLER IS SAID INTELLIGENT IF ITS GAINS AND/OR STRUCTURES ARE NOT FIXED!
Two Classes of Mechatronic Systems

- **Servo-Mechanisms**
  - Friction Compensation
  - Precision Motion Control

- **Robotic Systems**
  - Rigid Body Robots
  - Flexible Joint Robots
  - Flexible Link Robots
  - Smart Materials Robots

Fukuda & co-workers (Nagoya)
Lewis & co-workers (Arlington)
Moore & co-workers (Utah)
Grimble & co-workers (Strathclyde)
Tso & co-workers (Hong Kong)
C.W. de Silva & co-workers (UBC)
MORE SYSTEMS (at CIC, NUS)

Prahlad & students (just won 1st Prize at FIRA Competition, Austria; 7 Oct 2003)
MORE SYSTEMS (where funding is...)

[Image of a satellite in space with another smaller image of a satellite or space station.]
Preliminary and Methodology
Stability – Basic requirement of control design

Curve 1: asymptotically stable
Curve 2: stable
Curve 3: unstable

Narendra & co-workers (Yale)
Landau & co-workers (Grenoble)
Consider the nonlinear system
\[ \dot{x} = f(x, u) \]
Choosing \( V(x) \) as a Lyapunov function candidate, its time derivative is
\[ \dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x, u(x)) \]
Choose \( V(x) \)?
Design \( u = u(x) \)?

\[ \dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x, u(x)) \leq -W(x) \]
\( W(x) \) is positive definite.

The closed-loop system is stable.
For affine nonlinear system
\[ \dot{x} = f(x) + g(x)u \]
The inequality becomes
\[ \dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x) + \frac{\partial V(x)}{\partial x} g(x)u(x) \leq -W(x) \]

The closed-loop system is stable.

For the case where \( \dot{V}(x) \) is only negative semi-definite

LaSalle’s Invariant Principle \( \Rightarrow \) Asymptotic stability
Lemma

Let \( r(t) = G_r(p)e(t) \), where \( G_r^{-1}(s) \) is strictly proper & exponentially stable.

- If \( r \in L^2 \) then \( e \in L^2 \cap L^\infty \), \( \dot{e} \in L^2 \), \( e \) is continuous and \( e \to 0 \) as \( t \to \infty \).
- If, \( r \to 0 \) as \( t \to \infty \), then \( \dot{e} \to 0 \).

Common choices:

\[
G_r(s) = sI + \Lambda
\]

\[
G_r(s) = sI + K_1 + \frac{K_2}{s}
\]

\[
G_r(s) = sI + K_1 + \frac{K_2}{s} + \frac{K_3}{s^2}
\]

Has the advantage of being a nice systematic approach.

However, usually lacks freedom in selectively cancelling the undesirable terms and retaining the nice well-behaved parts.
SINGULAR PERTURBATION

- Multiple time-scale structures inherent in many practical systems

- Discontinuous dependence of system properties on the perturbation parameter

\[
\begin{align*}
\dot{x} &= f(t, x, z, \varepsilon) \\
\varepsilon \dot{z} &= g(t, x, z, \varepsilon)
\end{align*}
\]

Kokotovic & co-workers (UIUC)
Spong & co-workers (UIUC)

Tikhonov’s Theorem

The discontinuity of solutions caused by singular perturbations can be avoided if analyzed in separate time scales.
Intelligent Control of Servo Systems
4. Friction Compensation

Friction exists in any relative motion
Friction Compensation

Friction exists almost everywhere
Friction compensation is achievable with a reasonable accurate model!

- Static friction (Stiction)
  \[
  f_m = \begin{cases} 
  u, & |u| < f_s \\
  f_s \delta(\dot{x}) \text{sgn}(u), & |u| \geq f_s 
  \end{cases}
  \text{small motion} \rightarrow f_m = f_t x \delta(\dot{x})
  \]

- Coulomb friction (Dry friction)
  \[
  f_m = f_c \text{sgn}(\dot{x}), \quad f_c = \mu |f_n|
  \]

Canudas de Wit & co-workers (Grenoble)
Ge, Chen, Tan, Lee & co-workers (NUS)
Friction Model

- **Viscous friction**
  \[ f_m = f_v \dot{x} \]

- **Drag friction**
  \[ f_m = f_d |\dot{x}| \ddot{x} \]

- **Exponential model**
  \[ f_m(\dot{x}) = f_c \text{sgn}(\dot{x}) + (f_s - f_c) \exp(-\frac{\dot{x}}{\dot{x}_s})^\delta + f_v \dot{x} \]

- **Lorentzian model**
  \[ f_m(\dot{x}) = f_c \text{sgn}(\dot{x}) + (f_s - f_c) \frac{1}{1 + (\dot{x} / \dot{x}_s)^2} + f_v \dot{x} \]
Friction Model

A classical model including Stiction, Coulomb, viscous, drag friction and square root friction in LIP form

\[ f_m(x, \dot{x}) = S^T(x, \dot{x})P \]

where

\[ S(x, \dot{x}) = \begin{bmatrix} x\delta(\dot{x}) & \text{sgn}(\dot{x}) & \dot{x} & |\dot{x}| & \frac{1}{2} \text{sgn}(\dot{x}) \end{bmatrix} \]

\[ P = [f_t \ f_c \ f_v \ f_d \ f_r]^T \]

Friction Model

Friction is continuous though may be extremely highly nonlinear.

Neural network model

\[ f(x, \dot{x}) = S^T(x, \dot{x})P + \varepsilon(x, \dot{x}) \]

where

- \( S(x, \dot{x}) = [s_1, s_2, \ldots, s_l]^T \), basis function
- \( P \), corresponding weight vector
- \( \varepsilon(x, \dot{x}) \), reconstruction error.
NN Approximation

- Linearly parameterized neural networks (LPNNs)
  \[ y = W^T S(x) \]
  - Gaussian RBF neural networks
  - High order neural networks

- Multilayer neural networks (MNNs)

Dynamic Friction Model

LuGre model

Enable position dependence
Design dual observer for unknown $\alpha(\cdot,\cdot)$
(more later…)

$$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 \dot{x}$$
$$\dot{z} = \dot{x} - \alpha(\dot{x})|\dot{x}| z$$

where

- $z$, unmeasured average deflection of the bristles
- $\sigma_0, \sigma_1$ and $\sigma_2$, friction parameters
- $\alpha(\dot{x})$, a finite positive function.

Adaptive Friction Compensation

Servo mechanism

\[ m\ddot{x} + F = u \]

Uncertain \( F \)

Model and Approximation based control
A unified adaptive controller

\[ u = \ddot{m} \dot{x}_r + \tilde{f}_m + k_1 r + u_r + k_i \int_0^t r d\tau, \quad k_1 > 0 \]

where \( \tilde{f}_m = S^T \hat{P}, \ u_r = k_2 \tanh \left( \frac{r}{\varepsilon_r} \right), \ \varepsilon_r > 0 \)

Parameter update law

\[ \dot{\hat{P}} = \Gamma Sr, \quad \Gamma = \Gamma^T > 0 \]

The tracking error converges to a small neighbourhood of zero whose size is adjustable by the design parameters \( k_1, k_2 \) and \( \varepsilon_r \).
Approximation based control is entering its accepted paradigm though not mature as yet.

- Approximation based control has been developed for complex systems whose accurate model is difficult to obtain.

- Neural networks and fuzzy systems are used as the main parameterization tools for unknown nonlinear systems without requiring the systems parameterization form.
Approximation Based Control

- NN approximation of $\alpha(x, \dot{x})$

$$\alpha(x, \dot{x}) = W^{*T} S(x, \dot{x}) + \varepsilon, \quad \varepsilon \leq \varepsilon_b$$

- Define

$$\delta = z + \frac{m}{\sigma_1} r$$

- Dual observer to estimate the internal friction state $z$

$$\dot{\hat{\delta}}_1 = \dot{x} - \frac{\sigma_0}{\sigma_1} \hat{\delta}_1 + \frac{m\sigma_0}{\sigma_1^2} r + \xi - r$$

$$\dot{\hat{\delta}}_2 = \dot{x} - \frac{\sigma_0}{\sigma_1} \hat{\delta}_2 + \frac{m\sigma_0}{\sigma_1^2} r + \xi + \sigma_1 r\dot{x}$$

Approximation Based Control

- **Control law**
  \[ u = (\sigma_0 + \sigma_1)\dot{x} + m\ddot{x}_r + u_{ar} \]
  where \[ u_{ar} = -c_1 r + \sigma_0 \hat{\delta}_1 - \sigma_1 \tilde{\alpha}(x, \dot{x})|\dot{x}| \hat{\delta}_2 - \frac{m\sigma_0}{\sigma_1} r \]
  \[ - \frac{\sigma_1^2 \sigma_2 \alpha_m r \dot{x}^2}{\sigma_0} - k\sigma_1 |\dot{x}| |\hat{\delta}_2| \text{sgn}(r) \]

- **Adaptation law**
  \[ \dot{W} = \Gamma S(x, \dot{x})\sigma_1 |\dot{x}| \hat{\delta}_2 r \]

The tracking error converges to zero and all the signals in the closed-loop are bounded.
Servo systems

\[ \ddot{x} = -\frac{K_1}{M} \dot{x} + \frac{K_2}{M} u + \frac{K_2}{M} f(x, \dot{x}) \]

A 3-tier composite control structure

- Feedforward control
- Feedback control
- Non-linear Radial Basis Function (RBF) based compensator.
Overall control

\[ u = u_{FF} + u_{PID} + u_{RBF} \]

where

\[ u_{FF} = \frac{M}{K_2} \dddot{x}_d + \frac{K_1}{K_2} \ddot{x}_d, \quad u_{PID} = -(r_0 + 1)B^T P x \]

\[ u_{RBF} = -\tilde{f}(x, \dot{x}) = \sum_{i=0}^{m} \phi_i(x, \dot{x})\hat{w}_i, \quad \dot{\hat{w}}_i = r_1 x^T PB\phi_i - r_2 \hat{w}_i \]

The tracking error converges to zero and all the signals in the closed-loop are bounded.
Intelligent Control of Robotic Systems
With the application of neural network, the exact model structure of robots need not to be known.

Nonlinear dynamics of rigid body robots

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \]

Using NN estimation, we have

\[ \hat{D}_{NN}(q) = \{\hat{W}_D\}^T \cdot \{\Xi_D(q)\} \]

\[ \hat{C}_{NN}(q, \dot{q}) = \{\hat{W}_C\}^T \cdot \{\Xi_C(q, \dot{q})\} \]

\[ \hat{G}_{NN}(q) = \{\hat{W}_G\}^T \cdot \{\Xi_G(q)\} \]
GL Matrix and Operator

- **GL row vector and its transpose**
  \[ \{W_i\} = \{W_{i1} \cdots W_{ik}\}, \{W_i\}^T = \{W_{i1}^T \cdots W_{ik}^T\} \]

- **GL matrix**
  \[ \{W\} = \begin{bmatrix} \{W_1\} \\ \vdots \\ \{W_n\} \end{bmatrix} = \begin{bmatrix} W_{11} & \cdots & W_{1k} \\ \vdots & \ddots & \vdots \\ W_{n1} & \cdots & W_{nk} \end{bmatrix} \]

- **GL operator**
  \[ \left[ \{W\}^T \bullet \{X\} \right] = \begin{bmatrix} W_{11}^T X_{11} & \cdots & W_{1k}^T X_{1k} \\ \vdots & \ddots & \vdots \\ W_{n1}^T X_{n1} & \cdots & W_{nk}^T X_{nk} \end{bmatrix} \]
Control of Rigid Robots

- A general controller

\[ \tau = \dot{D}_{NN}(q)\ddot{q}_r + \dot{C}_{NN}(q, \dot{q})\dot{q}_r + \dot{G}_{NN}(q) + K_P r + K_I \int_0^t r d\tau + \tau_r \]

- Parameter adaptation laws

\[
\begin{align*}
\dot{\hat{W}}_{Dk} &= \Gamma_{Dk} \{\xi_{Dk}(q)\} \ddot{q}_r r_k \\
\dot{\hat{W}}_{Ck} &= \Gamma_{Ck} \{\xi_{Ck}(q, \dot{q})\} \dot{q}_r r_k \\
\dot{\hat{W}}_{Gk} &= \Gamma_{Gk} \xi_{Gk}(q) r_k
\end{align*}
\]

Tracking error converges to zero by appropriate selecting the control parameters.
The controller presented is just one of the many controllers available

Many different controllers can be obtained using different techniques including

- passivity based control,
- model reference based adaptive control,
- feedback linearization control …

Many industrial manipulators, particularly those equipped with harmonic drives for speed reduction, exhibit joint flexibility.

The simplified dynamic of flexible joint robots

\[
D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = K(q_m - q)
\]

\[
J\ddot{q}_m + K(q_m - q) = u
\]

1. Oscillation suppression control

To introduce joint damping $K_m (\dot{q}_m - \dot{q})$ effectively into the closed-loop system to damp out the joint oscillations.

A general form of the control law

$$u = K_m (\dot{q}_m - \dot{q}) + u_s$$

$u_s$ is the slow time control to be designed later.
Define $z = K_m(q_m - q)$, $K = \frac{K_1}{\varepsilon^2}$, $K_v = \frac{K_2}{\varepsilon}$, $\varepsilon$ a small parameter $\Rightarrow$

Singular perturbed model of FJRs

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = z$$

$$\varepsilon^2\ddot{z} + \varepsilon J^{-1}K_2\dot{z} + J^{-1}K_1z = J^{-1}K_1u_s - K_1\dot{q}$$

Slow subsystem

$$(D(q) + J)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u_s$$

which is same as the dynamics of rigid robots.
Define $\eta = \bar{z} - \bar{z}$, with $\bar{z}$ being constant in $\tau_t = t / \varepsilon$.

Fast subsystem

$$\frac{d^2 \eta}{d \tau_t^2} + K_2 \dot{\eta} + K_1 (J^{-1} + D^{-1}) \eta = 0$$

which is uniformly exponentially stable in $(t, q)$ if $K_1, K_2$ and $J$ are positive definite diagonal matrices.

2. Motor tracking error suppression control/hierarchical control model

The motor tracking errors $e_m = q_{md} - q_m$ are modeled as the fast variables instead of the joint elastic forces.

The composite control law

$$u = u_s + J(\ddot{q}_{md} + K_V \dot{e}_m + K_p \bar{e}_m)$$

$u_s$ is the slow time control to be designed later.
The control law is rewritten as
\[ u = u_s + J(\ddot{q}_{md} + K_V \dot{e}_m + K_p e_m) + \alpha(q_d^T, \dot{q}_d^T, \ldots, q_d^{(4)T}) \]
where \( \alpha(q_d^T, \dot{q}_d^T, \ldots, q_d^{(4)T}) = -J(\ddot{q}_m + K_V \dot{q}_m + K_p q_m). \)

Define \( z_1 = Ke_m, \ z_0 = K(q_{md} - q), \ K_P + KJ^{-1} = \frac{K_1}{\varepsilon^2}, \)

\[ K_V = \frac{K_2}{\varepsilon}, \ \varepsilon \text{ a small parameter} \implies \]

Singular perturbed model of FJRs
\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = z_0 - z_1 \]
\[ \varepsilon^2 \ddot{z}_1 + \varepsilon K_2 \dot{z}_1 + K_1 z_1 = (K_1 - \varepsilon^2 K_P)(z_0 - u_s - \alpha) \]
Control of Flexible Joint Robots

Slow subsystem

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u_s + \alpha \]

Control law for slow subsystem

\[ \tau = \hat{D}_{\text{NN}}(q)\ddot{q}_r + \hat{C}_{\text{NN}}(q, \dot{q})\dot{q}_r + \hat{G}_{\text{NN}}(q) - \hat{\alpha}_{\text{NN}}(q^T_d, \dot{q}^T_d, \cdots, q^{(4)T}_d) \]

\[ + K_P r + K_I \int_0^t r d\tau + K_r \text{sgn}(r) \]

with

\[ \dot{\hat{W}}_{Dk} = \Gamma_{Dk} \bullet \{ \xi_{Dk}(q) \} \dot{q}_r r_k \]

\[ \dot{\hat{W}}_{Ck} = \Gamma_{Ck} \bullet \{ \xi_{Ck}(q, \dot{q}) \} \dot{q}_r r_k \]

\[ \dot{\hat{W}}_{Gk} = \Gamma_{Gk} \xi_{Gk}(q) r_k \]

\[ \dot{\hat{W}}_{\alpha k} = \Gamma_{\alpha k} \xi_{\alpha k}(q^T_d, \dot{q}^T_d, \cdots, q^{(4)T}_d) r_k \]
Define $\eta = z_1 - \overline{z}_1$, with $\overline{z}_1$ being constant in $\tau_t = t / \varepsilon$.

Fast subsystem

$$\frac{d^2 \eta}{d\tau_t^2} + K_2 \frac{d\eta}{d\tau_t} + K_1 \eta = 0$$

which is uniformly exponentially stable.
If the slow subsystem has a unique solution defined on an interval \( t \in [0, t_1] \) and if the boundary-layer system is exponentially uniformly stable in \((t, q)\), then there exists \( \varepsilon^* \) such that for all \( \varepsilon < \varepsilon^* \)
\[
z(t) = \bar{z}(t) + \eta(t) + O(\varepsilon), \quad q(t) = \bar{q}(t) + O(\varepsilon)
\]
hold uniformly for \( t \in [0, t_1] \). -Tikhonov's Theorem.

The statement is valid for a finite time interval.
The statement is valid for an infinite time interval if the slow subsystem is exponentially stable.
Some works are documented:
Flexible link robots (FLRs) have many advantages than traditional rigid robots.

Difficulties to control flexible link robots

- Distributed parameter system
- Infinite dimension
- Nonminimum phase property from base input to tip position output

Energy-based Control of FLRs
Other Flexible Systems
Energy-based Control of FLRs

Geometry of a N-link FLR system
Work done by external inputs

\[ W = \sum_{i=1}^{N} \int_{0}^{t} \tau_i(t) \dot{\theta}_i(t) \, dt \]

From the energy-work relationship, we have

\[ [E_k(t) + E_p(t)] - [E_k(0) + E_p(0)] = \sum_{i=1}^{N} \int_{0}^{t} \tau_i(t) \dot{\theta}_i(t) \, dt \]

Thus we can obtain

\[ \dot{E}_k(t) + \dot{E}_p(t) = \sum_{i=1}^{N} \tau_i(t) \dot{\theta}_i(t) \]
Energy-based Control of FLRs

Robust Control Law

\[ \tau_i = -k_{pi}(\theta_i - \theta_{di}) - k_{di}\dot{\theta}_i - k_{si}y_i''(t, x_s) \text{sgn}(\dot{\theta}_i) \]

\[ \int_0^t |\dot{\theta}_i| y_i''(s, x_s) ds \quad i = 1, 2, \ldots, N \]

where \( k_{pi}, k_{di} > 0 \), \( y_i''(t, x_s) \) is the strain of each link at location \( x_s \).

The energy-based robust control law can guarantee the stability of the closed-loop multi-link flexible robotic system.

- The controller is constructed independent of system parameters.

- The controller can be easily extended to general control where a general function $f_i(x,t)$ is used instead of $y_i''(x_s,t)$. 
Parameter tuning using genetic algorithm

- Genetic algorithm is parameter search procedures based upon the mechanics of natural genetics. It combines the Darwinian’s principle of survival-of-the-fittest with a random, yet structured information exchange among population of artificial chromosomes.

- Basic operation includes initial population, reproduction, crossover and mutation.

Energy-based Control of FLRs

The optimization of the fitness function is the basic goal of GA.

- Modified function of integral of squared errors (MISE)
  \[
  MISE = \int_0^T K[L \theta_f - p(L, t)]^2 dt
  \]

- Modified function of integral time-multiplied absolute value of errors (MITAE)
  \[
  MITAE = \int_0^T K t |L \theta_f - p(L, t)| dt ,
  \]
  where \( K = \begin{cases} 
  1.0, & \text{when } |p(L, t)| \leq L \theta_f \text{ (no overshoot)} \\
  10, & \text{when } |p(L, t)| > L \theta_f \text{ (overshoot)} 
  \end{cases} \)
Adaptive NN Control of SMRs

- For conventional flexible robots
  - Infinite number of degrees of freedom
  - Finite number of actuators

- Use of smart materials (e.g. piezo)
  - Achieve the transformation between mechanical deformation and electrical field
  - Serve as actuators and sensors
  - Apply additional control abilities to traditional flexible link robot
  - Control loops increase
Adaptive NN Control of SMRs

- Dynamic model of the smart materials robot

\[
\begin{bmatrix}
M_{rr} & M_{rf} \\
M_{fr} & M_{ff}
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_r \\
\ddot{q}_f
\end{bmatrix}
+ \begin{bmatrix}
H_r \\
H_f
\end{bmatrix}
+ \begin{bmatrix}
0 \\
K_{ff} q_f
\end{bmatrix}
= \begin{bmatrix}
\tau \\
F_f w
\end{bmatrix}
\]

Let \( K_{ff} = k\tilde{K}, \xi = k\tilde{K}q_f, \varepsilon^2 = 1/k \)

- Singular perturbed model

\[
\ddot{q}_r = -D_{rr} H_r - D_{rf} H_f - D_{rf} \xi + D_{rr} \tau + D_{rf} F_f w
\]

\[
\varepsilon^2 \ddot{\xi} = -D_{fr} H_r - D_{ff} H_f - D_{ff} \xi + D_{fr} \tau + D_{ff} F_f w
\]

Setting $\varepsilon = 0$, we obtain

$$\bar{\xi} = D_{ff}^{-1} \left( -D_{fr} H_r + D_{fr} \bar{\tau} \right) - H_f + F_f \bar{w}$$

- **Slow subsystem**

$$\bar{M}_{rr} \ddot{q}_r + \bar{C}_{rr} \dot{q}_r = \bar{\tau}$$

which corresponds to the rigid body dynamic model.
Define $z_1 = \xi - \bar{\xi}, z_2 = \epsilon \dot{\xi}, \tau_t = t/\epsilon, \tau_f = \tau - \bar{\tau}$

- Fast subsystem

$$\frac{dz_1}{d\tau_t} = z_2$$

$$\frac{dz_2}{d\tau_t} = -\bar{D}_{ff} z_1 + \bar{D}_{fr} \tau_f + \bar{D}_{ff} F_f w$$

Since it is difficult to design $\tau_f$, the fast subsystem will be controlled using $w$ only.
Active Stabilization of fast subsystem

- Fast subsystem becomes

\[
\frac{d^2 z_1}{d\tau^2} + D_{ff} z_1 = D_{ff} F_f w
\]

- Control law

\[
w = -\varepsilon \kappa U F_f^+ \dot{\xi}
\]

where \( \kappa > 0, F_f U = [F_{f1}, F_{f2}], F_{f1} \neq 0, \)

\[
F_f^+ = \begin{bmatrix}
F_{f1}^{-1} - F_{f1}^{-1} F_{f2} F_{f3} \\
F_{f3}
\end{bmatrix}.
\]

The fast subsystem is uniform exponential stable.
If the slow subsystem has a unique solution defined on an interval $t \in [0, t_1]$ and if the boundary-layer system is exponentially uniformly stable in $(t, q)$, then there exists $\varepsilon^*$ such that for all $\varepsilon < \varepsilon^*$

$$\zeta(t) = \tilde{\zeta}(t) + z_1(\tau_1) + O(\varepsilon), \quad q_r(t) = \tilde{q}_r(t) + O(\varepsilon)$$

hold uniformly for $t \in [0, t_1]$. -- Tikhonov's Theorem

Ship welding robotic system
MORE SYSTEMS (at CIC, NUS)

Prahlad & students (just won 1st Prize at FIRA Competition, Austria; 7 Oct 2003)

Ge & students

Tan & students
1. Intelligent control has been successfully applied to mechatronic systems using neural networks and approximation theory as the main tools.

2. A few selected techniques have been presented for a few typical systems, servo mechanisms and robotic systems.

3. Several manifestations of intelligent control have been proposed by various scientists. Different intelligent control methods should be employed as appropriate in the implementation of various functions at different levels of the intelligent systems.
~ Thank You! ~