Q.3 Shown in the figure below is the expressions of the primary and secondary voltages of an ideal transformer. Assume that the transformer reluctance $\mathcal{R} = 25$, the numbers of the turns of the primary and secondary windings are $N_1 = 10, N_2 = 5$, respectively.

(a) Given the waveforms of $i_1(t)$ and $i_2(t)$ as the figure below, sketch the waveforms of the induced voltages, $v_1(t)$ and $v_2(t)$.

Solution:

$$L_1 = \frac{N_1^2}{\mathcal{R}} = \frac{100}{25} = 4, \quad L_2 = \frac{N_2^2}{\mathcal{R}} = \frac{25}{25} = 1, \quad M = \sqrt{L_1 L_2} = 2$$

$$v_1(t) = 4 \frac{di_1(t)}{dt} + 2 \frac{di_2(t)}{dt}, \quad v_2(t) = 2 \frac{di_1(t)}{dt} + \frac{di_2(t)}{dt}$$
Sketch the waveforms directly on the graphs below.

\[ v_1(t) = 4 \frac{di_1(t)}{dt} + 2 \frac{di_2(t)}{dt} \]

\[ v_2(t) = 2 \frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} \]
(b) For AC environments, show that the primary and secondary voltages in the phasor form are given as those in the figure below.

Hint: Assume \( i_1(t) = \sqrt{2} r_1 \cos(\omega t + \theta_1) \) and \( i_2(t) = \sqrt{2} r_2 \cos(\omega t + \theta_2) \).

(5 Marks)

**Proof:**

\[
i_1(t) = \sqrt{2} r_1 \cos(\omega t + \theta_1) \quad \Rightarrow \quad I_1 = r_1 e^{j\theta_1}, \quad i_2(t) = \sqrt{2} r_2 \cos(\omega t + \theta_2) \quad \Rightarrow \quad I_2 = r_2 e^{j\theta_2}
\]

\[
v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} = -L_1 \sqrt{2} r_1 \omega \sin(\omega t + \theta_1) - M \sqrt{2} r_2 \omega \sin(\omega t + \theta_2) = L_1 \sqrt{2} r_1 \omega \cos(\omega t + \theta_1 + 90^\circ) + M \sqrt{2} r_2 \omega \cos(\omega t + \theta_2 + 90^\circ)
\]

\[
V_1 = \omega L_1 r_1 e^{j(\theta_1 + 90^\circ)} + \omega Mr_2 e^{j(\theta_2 + 90^\circ)} = j \omega L_1 r_1 e^{j\theta_1} + j \omega Mr_2 e^{j\theta_2} = j \omega L_1 I_1 + j \omega M I_2
\]

\[
v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} = -M \sqrt{2} r_1 \omega \sin(\omega t + \theta_1) - L_2 \sqrt{2} r_2 \omega \sin(\omega t + \theta_2) = M \sqrt{2} r_1 \omega \cos(\omega t + \theta_1 + 90^\circ) + L_2 \sqrt{2} r_2 \omega \cos(\omega t + \theta_2 + 90^\circ)
\]

\[
V_2 = \omega Mr_1 e^{j(\theta_1 + 90^\circ)} + \omega L_2 r_2 e^{j(\theta_2 + 90^\circ)} = j \omega Mr_1 e^{j\theta_1} + j \omega L_2 r_2 e^{j\theta_2} = j \omega M I_1 + j \omega L_2 I_2
\]
(c) The transformer is connected to a load inductor with inductance $L = 2 \, \text{H}$, as shown in the figure below. Recall that the transformer reluctance $\mathcal{R} = 25$, the numbers of the turns of the primary and secondary windings are $N_1 = 10$, $N_2 = 5$, respectively. Determine the ratio of the primary and secondary currents (in phasor).

Why is the usual transformer property below no longer valid in such a situation?

$$\frac{I_1}{I_2} = -\frac{N_2}{N_1}$$

(10 Marks)

Solution:

$$Z_L = j\omega L = j2\omega$$

$$V_2 = j2\omega I_1 + j\omega I_2$$

By KVL,

$$V_2 + I_2 Z_L = j2\omega I_1 + j\omega I_2 + j2\omega I_2 = j2\omega I_1 + j3\omega I_2 = 0$$

$$2I_1 + 3I_2 = 0$$

$$\frac{I_1}{I_2} = -\frac{3}{2} \neq -\frac{N_2}{N_1} = -\frac{1}{2}$$

This is because $|Z_L| = |j2\omega| = 2\omega > |j\omega L_2| = |j\omega| = \omega$. The condition $|j\omega L_2| >> |Z_L|$ is invalid.
Q.4 An LED display panel shown in the figure below has four LED light bars labeled \( A, B, C \) and \( D \), respectively. You are required to design an appropriate digital logic circuit to display Letters \( F \) and \( L \) using the panel.

(a) Construct a truth table for your design. 

(b) Obtain logical expressions for \( F \) and \( L \), respectively. 

(c) Draw logic circuit implementations for \( F \) and \( L \) using only 2-input NOR gates (i.e., each NOR gate only has two input channels). For each letter, you cannot use more than 8 NOR gates for the implementation. 

(d) What other English letters can be displayed by the panel? 

\[ \text{Solution to Q4 (a):} \]

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(6 Marks)
Solution to Q4 (b):

\[ F = A \cdot B \cdot \overline{C} \cdot D, \quad L = A \cdot \overline{B} \cdot C \cdot D \]

(6 Marks)

Solution to Q4 (c):

\[ F = A \cdot B \cdot \overline{C} \cdot D = A \cdot B \cdot \overline{C} \cdot D = (A + B) \cdot (C + D) = A + B + C + D \]

(5 Marks)

\[ L = A \cdot B \cdot C \cdot D = A \cdot B \cdot C \cdot D = (A + B) \cdot (C + D) = A + B + C + D \]

(5 Marks)

Solution to Q4 (d):

Letters, \( C, E \) and \( I \).

(3 Marks)